Bayesian Data Analysis
A PHY 451/551, I CSI 451/551, I INF 451/551
HW2w
Show all work

1. A Fair 20-sided Die
Consider a fair 20-sided die (a icosahedron) with sides indexed by *i*.

a. What are the possible states of the die? Are these states mutually exclusive and exhaustive?

b. What is the probability of rolling $i=7$ on the fair 20-sided die?

c. What is the probability of rolling *i* such that *i* is odd?

d. What is the probability of rolling *i* such that *i* is prime?

e. What is the average value (also called the expected value) of *i* ?

f. Is it possible to actually observe the expected value on a given roll of the die? Why or why not?

g. You want to generate a random integer from one to five. Devise a way to accomplish this with one roll of the die, and show by presenting the explicit calculations that the probabilities are both uniform and sum to unity.

2. Independent Pair of Fair 8-Sided Dice
Consider an independent pair of fair 8-sided dice with sides indexed by *i* and *j*.

a. Since they are independent, how does $p(i \mid j, I)$ relate to $p(i \mid I)$?

b. What is the probability of rolling *i*=2 on the first 8-sided die?
That is, what is $p(i = 2 \mid I)$?

c. What quantity does $p(i = 2,\ j = 4 \mid I)$ represent? And what is its value?

d. What is the average value (also called the expected value) of *i* ?

e. Is it possible to ever observe this expected value? Why or why not?

f. What is the expected value of *i*+*j* ?

g. What is the most probable value of *i*+*j* ?

3. Coupled Dice

Imagine now that this pair of 6-sided dice (faces indexed by *i*) is connected via a spring that is attached to the $i = 1$ face and the $j = 6$ face so that when the spring is compressed, the one die sits directly on top of the other so that the faces showing 2, 3, 4, and 5 line up. Since the spring is in the way, we will never be able to roll a 1 or a 6 on either die. Moreover, it is less probable for the spring to twist. Since the 2 face is opposite to the 5 face and the 3 face is opposite to the 4 face, the spring will have to be twisted quite a bit to attain those rolls. The states where opposite faces appear on each die (2-5 and 3-4) is one half as possible than adjacent faces appearing on each die (2-3, 2-4, 3-5, 4-5), which, again due to the twisted spring, is one-half as possible as like faces (2-2, 3-3, 4-4, 5-5). That is:

$p(i = 1 \mid I) = 0$ and $p(i = 6 \mid I) = 0$ for each die

$p(i = 2,\ j = 2 \mid I) = 2 \times p(i = 2,\ j = 3 \mid I) = 4 \times p(i = 2,\ j = 5 \mid I)$

a. Write out the probabilities for all the possible cases.

b. Show that the probabilities sum to unity.

c. What is the expected value of $i+j$?

d. What is the probability $p(i \mid I)$ for all values of i ?

e. What is the expected value of i ?