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PHY/CSI/INF 451/551: Bayesian Data Analysis / Signal Processing

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Homework 4 Written

1. Bayes Theorem and the Weather.

(a) Hypothesis and the data

Hypothesis: R - On days when it does rain
$\overline{R}$ - On days when it does not rain

Data: DC - There are dark clouds that roll in during the morning
$\overline{DC}$ - There are not dark clouds that roll in during the morning

(b) Bayes Theorem for this problem

$$p(R|DC, I) = \frac{p(DC|R, I)p(R|I)}{p(DC|I)}$$

(c) Expression for the evidence

$$p(DC|I) = p(DC, R|I) + p(DC, \overline{R}|I) =$$

(d) Using Bayes Theorem to solve the problem

$$\begin{array}{ll} p(R|I) = 0.29 & p(\overline{R}|I) = 0.71 \\ p(DC|R, I) = 0.9 & p(DC|\overline{R}, I) = 0.25 \end{array}$$

$$p(RO|DC, I) = \frac{p(DC|R, I)p(R|I)}{p(DC|I)}$$

$$= \frac{p(DC|R, I)p(R|I)}{p(DC|R, I)p(R|I) + p(DC|\overline{R}, I)p(\overline{R}|I)}$$

$$= \frac{0.9 \times 0.29}{0.9 \times 0.29 + 0.25 \times 0.71}$$

$$= \frac{0.261}{0.4385} = 0.59521$$

2. You are a laptop repair person.

(a) Hypothesis and the data

Hypothesis: FP - Laptops power supply has failed
$\overline{FP}$ - Laptops power supply is OK

Data: S - Plugging it in will produce smoke
$\overline{S}$ - Plugging it doesn't in will produce smoke

(b) Bayes Theorem for this problem

$$p(FP|S,I) = \frac{p(S|FP,I)p(FP|I)}{p(S|I)}$$

(c) Expression for the evidence

$$p(S|I) = p(S,\overline{FP}|I) + p(S,FP|I) =$$

(d) Using Bayes Theorem to solve the problem

$$\begin{array}{ll} p(FP|I) = 0.3 & p(\overline{FP}|I) = 0.7 \\ p(S|FP,I) = 0.45 & p(S|\overline{FP},I) = 0.05 \end{array}$$

$$p(FP|S,I) = \frac{p(S|FP,I)p(FP|I)}{p(S|I)}$$

$$= \frac{p(S|FP,I)p(FP|I)}{p(S|FP,I)p(FP|I) + p(S|\overline{FP},I)p(\overline{FP}|I)}$$

$$= \frac{0.45 \times 0.3}{0.45 \times 0.3 + 0.05 \times 0.7}$$

$$= \frac{0.135}{0.17} = 0.7941$$

3. The blue M&M was introduced in 1995.

Symbols: Y - Yellow M&Ms
G - Green M&Ms

$B1 = 94$ - Bag 1 from 1994
$B2 = 96$ - Bag 2 from 1996
$B1 = 94$ - Bag 1 from 1994
$B2 = 96$ - Bag 2 from 1996

$$p(B1 = 94, B2 = 96|I) = 0.5 \quad p(B1 = 96, B2 = 94|I) = 0.5$$

Solution using Bayes Theorem

$$p(YB1, GB2|B1 = 94, B2 = 96, I) = 0.2 \times 0.2 = 0.04$$

$$p(YB1, GB2|B1 = 96, B2 = 94, I) = 0.14 \times 0.1 = 0.014$$

Bayes Expression

If B1 is from 1994, so B2 is from 1996. Therefore, I will not put that B2=94 or B2=96 in the probabilities condition for not confuse.

$$p(B1 = 94|YB1, GB2, I) = \frac{p(YB1, GB2|B1 = 94, I)p(B1 = 94|I)}{p(YB1, GB2|I)}$$

$$= \frac{p(YB1, GB2|B1 = 94, I)p(B1 = 94|I)}{p(YB1, GB2|B1 = 94, I)p(B1 = 94|I) + p(YB1, GB2|B1 = 96, I)p(B1 = 96|I)}$$

$$= \frac{0.04 \times 0.5}{0.04 \times 0.5 + 0.014 \times 0.5}$$

$$= \frac{0.02}{0.027} = 0.7407$$