Winter 18 – AMS206B Homework 2

Due: Monday Feb 5th.

Note: The questions are taken from Chapters 1 and 2 from JB. Be careful with different notation, e.g. $\mathcal{A}$ in JB denotes the action space which is denoted by $\mathcal{D}$ in CR.

1. An insurance company is faced with taking one of the following 3 actions:

   - a_1: increase sales force by 10%
   - a_2: maintaining present sales force
   - a_3: decrease sales force by 10%.

   Depending upon whether or not the economy is good (θ_1), mediocre (θ_2), or bad (θ_3), the company would expect to lose the following amounts of money in each case:

State of	Action Taken		
Economy	a_1	a_2	a_3
θ_1	-10	-5	-3
θ_2	-5	-5	-2
θ_3	1	0	-1

   (a) Determine if each action is admissible or inadmissible.

   (b) The company believes that θ has the probability distribution $\pi(\theta_1) = 0.2$, $\pi(\theta_2) = 0.3$, $\pi(\theta_3) = 0.5$. Order the actions according to their Bayesian expected loss (equivalent to Bayes risk, here), and state the Bayes action.

   (c) Order the actions according to the minimax principle and the find the minimax nonrandomized action.

2. The owner of a ski shop must order skis for the upcoming season. Orders must be placed in quantities of 25 pairs of skis. The cost *per pair* of skis is \$50 if 25 are ordered, \$45 if 50 are ordered, and \$40 if 75 are ordered. The skis will be sold at \$75 per pair. Any skis left over at the end of the year can be cold (for sure) at \$25 a pair. If the owner runs out of skis during the season he will suffer a loss of "goodwill" among unsatisfied customers. he rates this loss at \$5 per unsatisfied customer. For simplicity, the owner feels that demand for the skis will be 30, 40, 50 or 60 pair of skis, with probabilities 0.2, 0.4, 0.2, and 0.2, respectively.

   (a) Describe $\mathcal{A}$, Θ and $L(\theta, a)$.

   (b) Which actions are admissible?

   (c) What is the Bayes action?

   (d) What is the minimax nonrandomized action?

3. An automobile company is about to introduce a new type of car into the market. It must decide how many of these new cares to produce. Let a denote the number of cars decided upon. A market survey will be conducted, with information obtained pertaining to θ, the proportion of the population which plans on buying a car and would tend to favor the new model. The company has determined that the major outside factor affecting the purchase of automobiles is the state of the economy. Indeed, letting Y denote an appropriate measure of the state of the economy, it is felt that $Z = (1+Y)(10^7)\theta$ cars could be sold. Y is unknown but is thought to have a Unif$(0,1)$ distribution.

   Each car produced will be sold at a profit of \$500, unless that supply (a) exceeds the demand (Z). If $a > Z$, the extra $a - Z$ cars can be sold at a loss of \$300 each. The company's utility function for money is linear (say, $U(m) = m$) over the rage involved. Determine $L(\theta, a)$.

   Note that the answer is

$$L(\theta,a) = \begin{cases} -500a & \text{if } \frac{a}{10^7} \le \theta, \\ \frac{(4225)(10^7)}{4\theta}\left[\frac{8}{13(10^7)}a - \theta\right]^2 - \frac{(2625)(10^7)\theta}{4} & \text{if } \theta \le \frac{a}{10^7} \le 2\theta, \\ 300a - (1200)(10^7)\theta & \text{if } \frac{a}{10^7} \ge 2\theta. \end{cases}$$

4. Let X be Unif$(\theta, \theta+1)$, and suppose that it is desired to test $H_0 : \theta = 0$ versus $H_1 : \theta = 0.9$ these being the only two values of θ considered possible). Consider the test which rejects H_0 if $x \ge 0.95$, and accepts H_0 otherwise.

   (a) Calculate the probabilities of Type I and Type II error for this test.

   (b) If $0.9 < x < 1$ is observed, what is the intuitive (conditional) probability of actually making an error in use of the test.

   (c) Determine the likelihood function of θ for each x.

   (d) Interpret the "message" conveyed by the likelihood function for each x.