Winter 18 – AMS206B Homework 5

Due: Friday Feb. 23nd

1. Let (X_1, X_2, X_3) have trinomial distribution with density

$$f(x_1, x_2, x_3|\theta_1, \theta_2) \propto \theta_1^{x_1}\theta_2^{x_2}(1-\theta_1-\theta_2)^{x_3}.$$

   Derive Jeffreys prior for (θ_1, θ_2).

2. (Robert Problem 3.9) Let $x \mid \theta \sim \mathrm{Bin}(n, \theta)$ and $\theta \sim \mathrm{Be}(\alpha, \beta)$. Determine whether there exits values of α, β such that $\pi(\theta \mid x)$ is the uniform prior on [0, 1], even for a single value of x.

3. (Robert Problem 3.10) Let $x \mid \theta \sim \mathrm{Pa}(\alpha, \theta)$, a Pareto distribution, and $\theta \sim \mathrm{Be}(\mu, \nu)$. Show that if $\alpha < 1$ and $x > 1$, a particular choice of μ and ν gives $\pi(\theta \mid x)$ as the uniform prior on [0,1].

4. (Robert Problem 3.31) Consider $x \mid \theta \sim \mathrm{N}(\theta, \theta)$ with $\theta > 0$.
   (a) Determine the Jeffreys prior $\pi^J(\theta)$.
   (b) Say whether the distribution of x belongs to an exponential family and derive the conjugate priors on θ.
   (c) Use Proposition 3.3.14 to relate the hyperparameters of the conjugate priors with the mean of θ.

5. (Berger Problem 3-12) Determine the Jefferys noninformative prior for the unknown parameter in each of the following distributions:
   (a) $\mathrm{Poi}(\theta)$
   (c) $\mathrm{NB}(m, \theta)$ (m is given)

6. (Berger Problem 3-22) Suppose X, the failure time of an electronic component, has density (on $(0, \infty)$) $f(x \mid \theta) = \theta^{-1}\exp\{-x/\theta\}$. The unknown θ has an $\mathrm{IG}(1, 0.01)$ prior distribution. Calculate the (marginal) probability that the component fails before time 200.