Winter 18 – AMS206B Homework 1

Due: Jan-29 (Mon).

1. In Kokomo, IN, 65% are conservative, 20% are liberals and 15% are independents. Records show that in a particular election, 82% of conservatives voted, 65% of liberals voted and 50% of independents voted. If a person from the city is selected at random and it is learned that he/she did not vote, what is the probability that the person is liberal?

2. Suppose $\boldsymbol{X} = (X_1, \ldots, X_n)$ is a sample from uniform distribution, $\text{Unif}(0, \theta)$. Let θ have Pareto $\text{Pa}(\theta_0, a)$ distribution where $\theta_0 > 0$ and $a > 0$ are fixed. That is,

$$\pi(\theta \mid \theta_0, a) = \frac{a}{\theta_0}\left(\frac{\theta_0}{\theta}\right)^{(a+1)}, \text{ for } \theta \geq \theta_0.$$

   Show that the posterior is Pareto, $\text{Pa}(\max\{\theta_0, x_1, \ldots, x_n\}, a+n)$.

3. Let $X \sim \text{Gamma}(n/2, 2\theta)$ (that is, $X/\theta \sim \chi^2_n$) where $E(X) = n\theta$.

$$f(x \mid \theta) = \frac{1}{\Gamma(n/2)(2\theta)^{n/2}} x^{n/2-1} \exp\left(-\frac{x}{2\theta}\right), \qquad x > 0.$$

   Let $\theta \sim \text{IG}(\alpha, \beta)$, inverse gamma distribution. Find the posterior distribution of θ.

4. Suppose that $\boldsymbol{X} = (X_1, \ldots, X_n)$ is a sample from negative binomial, $\text{NB}(m, \theta)$ distribution, that is, $X_i \overset{iid}{\sim} \text{NB}(m, \theta)$, $i = 1, \ldots, n$, with pmf

$$f(x \mid \theta) = \binom{x+m-1}{x}(1-\theta)^m \theta^x, x = 0, 1, 2, \ldots.$$

   Consider a Beta distribution as a prior distribution for θ, $\theta \sim \text{Be}(\alpha, \beta)$. Find the posterior distribution of θ.