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REVIEW

Optimization based trajectory planner for multilane roundabouts with connected automation

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ABSTRACT

This paper proposes a trajectory planner for multilane roundabouts. The goal is to enhance the existing roundabout in terms of capacity with the help of connected automation. The trajectory planner is formulated as a bi-level optimization. The upper level is a passing sequence optimization, which determines the passing sequence of the approaching vehicles to achieve delay minimization. The lower level is a speed optimal control aiming at minimizing fuel consumption while honoring the pass sequences generated from the upper level optimization. The proposed planner is evaluated in simulation on a typical four-arm roundabout with two entrance lanes in each arm. Compared with the traditional yield-regulated control, the proposed approach can reduce the delay by 69.6% to 87.8% and fuel consumption by 24.7% to 28.6%. Compared with the first-come-first-service (FCFS) control, the proposed approach can reduce the delay by 33.5% to 81.9% and fuel consumption by 7.5% to 18.8%. The change in benefit is due to the variation of demand.

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Connected and automated vehicles; fuel consumption; roundabout; speed optimization; trajectory optimization

1. Introduction

Roundabouts are considered as a traffic safety enhancement method (Lochrane et al., 2014). Because they have lower accident rates and accident severities compared to signalized intersections (Duan et al., 2019; Saccomanno et al., 2008). The benefit in safety is achieved by reducing the number of conflict points, changing conflict points from crossing conflicts to merging conflicts, and reducing vehicle speed (Azimi et al., 2013; Lu et al., 2013)

However, one main disadvantage bounds the application of roundabout. Roundabout has limited capacity and leads to high delay and fuel consumption under congested traffic. The limited capacity results from the operation mode of the roundabout. Traditional roundabout adopts rule-based management based on yield regulation (Polus & Vlahos, 2005). The entering traffic must yield to the vehicles inside the roundabout and wait for a gap to enter the roundabout. As the traffic volume increases, the headways inside the roundabout drop below the accepted gap of entering vehicles. Thus, vehicles are forced to wait at the entrance lane, which causes massive delays. Conventionally, there are currently two ways to improve roundabout capacity. The first choice is the metering signals, which produce gaps for vehicles from entrance lanes by regulating the flow from other entrance lanes (Duan et al., 2019; Hummer & Ph, 2013; Natalizio, 2005). Thus, it is an enhancement of the yield regulation. However, metering only works during peak periods when the traffic flow falls into a small band of demand volumes (Natalizio, 2005). The second approach is equipping the roundabout with signals at four entrances (Jiang et al., 2019; Ma et al., 2013). This method changes the operation mode of the roundabout from desecrated rule-based control to centralized control. However, it involves high construction costs and may reduce safety and effectiveness (Jiang et al., 2019).

Fortunately, the connected and automated vehicle (CAV) technologies provide a possibility to further improve data collection and vehicle control. The state

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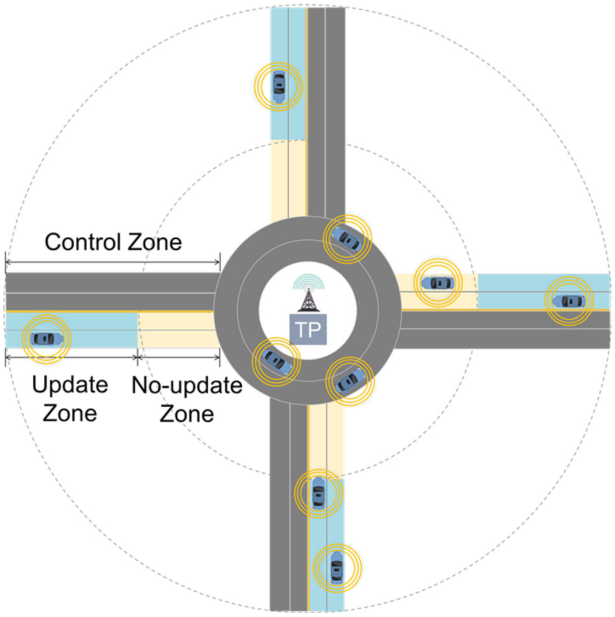

Figure 1. A typical four-arm roundabout with two approaches.

information of vehicles can be collected through dedicated short-range communications (DSRC). At the same time, the rich information facilitates the control process, such as trajectory planning (Feng et al., 2018). Previous studies fall into two categories. The first category can be concluded as rule-based or strategy-based, which sets specific rules to assign the rights of way of conflicting traffic flows at roundabouts. The First-Come-First-Service (FCFS) (Azimi et al., 2013) (Martin-Gasulla & Elefteriadou, 2019) and First-Come-First-Out (FCFO) (Zhao et al., 2018) are adopted to determine the time vehicle arrival at the roundabout and therefore the driving sequence is settled. However, the optimality of these researches cannot be guaranteed due to the nature of rule-based control (Yu et al., 2019). And this limitation becomes apparent in heavy-volume traffic situations (Malikopoulos et al., 2018; Meng et al., 2017). The second category is optimization-based. Methods belonging to this category address coordination of the CAVs by optimization model. For example, game theory is applied to guide two conflicting CAVs to pass the roundabout corporately (Banjanovic-Mehmedovic et al., 2016; Tian et al., 2018). However, these two methods only consider two vehicles. The performance of applying to numerous vehicles remains to be explored. Besides, previous studies have also failed to consider multiple lanes within the roundabout and therefore lacked management of vehicles to enter and leave the roundabout.

To summarize, previous studies have not been able to improve the efficiency of roundabouts while reducing fuel consumption with various traffic demands. A centralized optimization method of vehicles that can be applied to multilane roundabouts is missing. Therefore, the objective of this paper is to develop a centralized trajectory planner for a two-lane roundabout, which optimizes the operating efficiency and fuel efficiency simultaneously. The main contributions can be summarized as follows:

1. Propose an optimization-based control strategy to optimize vehicles' trajectories in the entrance lane and inside the roundabout in a CAV environment.
2. Both fuel consumption and vehicle delay are minimized in the trajectory planner and the importance of these two goals can be adjusted by weight.
3. Comparing the proposed method with two baselines by numerical experiments draw insights of the contribution of centralized control and trajectory optimization.

The exposition of this paper is organized as follows: the next section presents the problem and notations; Section 3 describes the methodology of the proposed approach; Section 4 conducts the case study. Conclusions are delivered in Section 5.

2. Problem description

The trajectory planner is formulated as a bi-level optimization. The upper level is a passing sequence optimization and the lower level is a speed optimal control. Passing sequence optimization determines the driving schedule of vehicles passing the roundabout. After the sequence is determined, the speed profile of each vehicle is optimized to honor the pass sequences.

Figure 1 below illustrates a typical four-arm roundabout with two entrance lanes against two circulating lanes. All vehicles are CAVs. A central planner of vehicle trajectories is set called Trajectory Planner (TP). TP knows the road layout and can collect information of all vehicles by Vehicle-to-Infrastructure (V2I) about their states, such as vehicle position and speed (Cao et al., 2017; Jiang et al., 2021; Wang et al., 2016). The control zone refers to the area where vehicles start to communicate with TP. The control zone is divided into update zones and no-update zones. The update zone is at upstream of the no-update zone. Inside the update zone, vehicles provide their state information to TP and receive an optimized trajectory to guide them across the roundabout safely

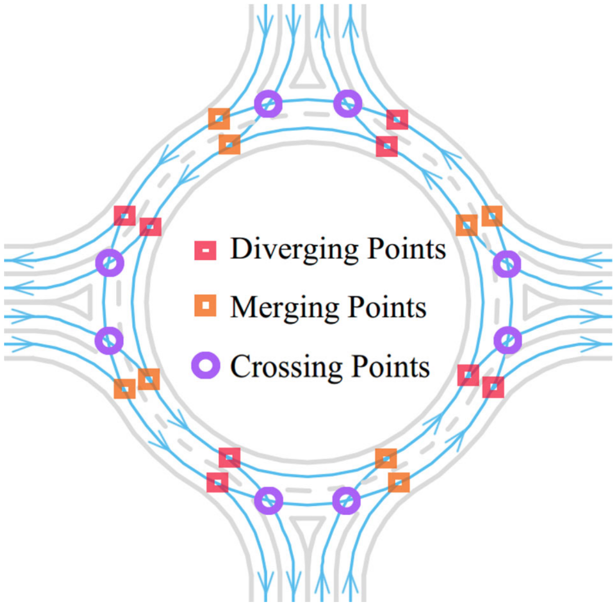

Figure 2. Geometric layout of critical points in a roundabout.

and efficiently. The vehicle state information includes position, speed, acceleration, and future turning. Inside the no-update zone, vehicles continue to send their state information to TP. However, they keep their previously optimized trajectory and are not given further instructions. The no-update zone is designed to mitigate the computational burden and has a minimal negative impact on optimality (Yu et al., 2018).

Given the geometric characteristics of the roundabout and the information of vehicles inside the control zone, this paper aims at minimizing delay and improving fuel efficiency through trajectory optimization. This is achieved by optimizing the passing sequence of vehicles and their speed profile.

To simplify the entire optimization process, several assumptions are listed below:

1. All vehicles are controllable by TP and obey the trajectory without feedback error and communication delay.
2. The planned turning of a vehicle does not change after the vehicle enters the control zone.
3. Vehicles travel at constant speeds inside roundabouts.

3. Methodology

The objective of the proposed bi-level optimization takes efficiency and energy-saving into account simultaneously, which can be formulated as:

$$\min\ J = w_1 J_1 + w_2 J_2. \tag{1}$$

where J_1 is the objective of the upper level and J_2 is the objective of the lower level. w_1 and w_2 are weighting coefficients. A planning horizon strategy is designed to conduct optimization.

3.1. Passing sequence optimization model

As the upper level of trajectory optimization, this section generates the passing sequence of vehicles to maximize efficiency (Meng et al., 2017; Xu et al., 2019), which is represented by the arrival state of vehicles at the stop line. Because each vehicle keeps a constant speed inside the roundabout, their arrival time at each place inside the roundabout can be directly got by the arrival state. Therefore, the control variables of this model are the time and speed at which each vehicle reaches the stop line.

1. *Objective function*
   The objective of this model is minimizing the sum of vehicle delay D_i, which is calculated by Eq. (3).

$$\mathrm{P1}: J_1 = \sum_{i \in \mathrm{I}} D_i \tag{2}$$

$$D_i = (t_i^s - t_o) - \Delta t_i^f,\ \forall i \in \mathrm{I} \tag{3}$$

   where t_i^s is the moment vehicle i pass the stop line, which means vehicle i enters the roundabout. t_o is the time to implement optimization, which is a constant in one optimization progress. Δt_i^f is the free-flow travel time of vehicle i, which is independent to the control variables. Thus, the objective function can be simplified as:

$$J_1 = \sum_{i \in \mathrm{I}} t_i^s. \tag{4}$$

2. *Collision avoidance*
   When two vehicles consecutively pass the stop line in the same entrance lane, a constrain utilizing Newell's car-following model (Newell, 2002) is applied to keep a safe time headway between the two vehicles.

$$t_i^s - t_j^s \geq \tau + d/v_j^s - M\gamma_{i,j} - M(1 - \eta_{i,j}),\ \forall i,j \in \mathrm{I}, i \neq j \tag{5}$$

$$t_j^s - t_i^s \geq \tau + d/v_i^s - M\gamma_{i,j} - M\eta_{i,j},\ \forall i,j \in \mathrm{I}, i \neq j \tag{6}$$

   In Eqs. (5) and (6), τ and d are constants in Newell's car-following model. v_i^s is the speed of vehicle i while passing the stop line. $\gamma_{i,j}$ and $\eta_{i,j}$ indicate the relationship among vehicles at the

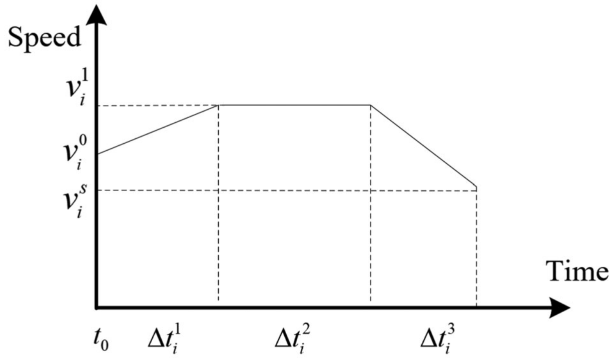

Figure 3. The limit condition of vehicle arrival.

entrance lane. $\gamma_{i,j} = 1$ if vehicle i and j drives at different entrance lane; $\gamma_{i,j} = 0$, otherwise. $\eta_{i,j} = 1$ if vehicle i passes the stop line before vehicle j; $\eta_{i,j} = 0$, otherwise. M is a large enough integer. I is the set of vehicles inside the control zone.

The optimized trajectory of vehicles in the update zone cannot conflict with the defined trajectory of vehicles already in the no-update zone:

$$t_i^s - t_k^s \geq \tau + d/v_i^s, \forall k \in \mathrm{K}, i \in \mathrm{I} \tag{7}$$

where t_k^s is the latest time point of vehicles in defined zone passing the stop line.

Consistent with reality, right-turning vehicles enter the roundabout from the right entrance and left-turning vehicles enter from the left entrance. For vehicles going straight, both two entrance lanes are optional. 24 conflict points are resulted inside a four-arm roundabout with two entrance lanes against two circulating lanes, including diverging points, merging points, and crossing points as shown in Figure 2. The location of potential conflict points results from the crossing between the routes of different vehicles.

The safety constraints between conflicting vehicles inside roundabout are guaranteed by Eqs. (8)–(10).

$$t_i^n - t_j^n \geq \tau + d/v_j^s - \mathrm{M}\delta_{i,j}^n, \forall n \in \mathrm{N}, i,j \in \mathrm{I}, i \neq j \tag{8}$$

$$t_j^n - t_i^n \geq \tau + d/v_i^s - \mathrm{M}(1 - \delta_{i,j}^n), \forall n \in \mathrm{N}, i,j \in \mathrm{I}, i \neq j \tag{9}$$

$$t_i^n = t_i^s + d_i^n/v_i^s, \forall n \in \mathrm{N}, i \in \mathrm{I} \tag{10}$$

In Eqs. (8) (9), t_i^n is the time point vehicle arrives at the conflict point n and is calculated by Eq. (10). $\delta_{i,j}^n$ is denoted as the indicator of the passing sequence between two vehicles in the update zone. $\delta_{i,j}^n = 1$, if vehicle i passes the conflict point n before vehicle j;$\delta_{i,j}^n = 0$, otherwise. For the convenience of modeling, $\delta_{i,j}^n$ is set as zero if $i = j$. In Eq. (10), d_i^n is the distance between the stop line passed by vehicle i and the conflict point n along the trajectory of vehicle i.

The optimized trajectory of vehicles in update zone cannot conflict with the defined trajectory of vehicles already in the no-update zone:

$$t_i^n - t_d^n \geq \tau + d/v_i^s, \forall n \in \mathrm{N}, i \in \mathrm{I} \tag{11}$$

where t_k^s is the latest time point of vehicles in defined zone passing the conflict point n.

3. *Arrival time constrains*

To ensure the feasibility of the speed optimization model in Section 3.2, the earliest bounds of vehicle arrival times are given in Eqs. (12)–(17). When the vehicle accelerates to the maximum speed limit and passes the stop line at the maximum speed limit in the roundabout, the vehicle spends the shortest time in the entrance lane.

$$t_i^{\min} \leq t_i^s, \forall i \in \mathrm{I} \tag{12}$$

where $t_i^{\min}$ denotes the earliest arrival time of vehicle i. The vehicle trajectory aimed at the lowest fuel consumption can be simplified to a combination of three driving processes (Long et al., 2020), as shown in Figure 3. Thus, $t_i^{\min}$ is calculated by Eqs. (13)–(17).

$$t_i^{\min} = t_i^0 + \Delta t_i^1 + \Delta t_i^2 + \Delta t_i^3, \forall i \in \mathrm{I} \tag{13}$$

$$\Delta t_i^1 = \min\left\{\frac{V_{\max} - v_i^0}{a_{\max}}, \frac{v_i^1 - v_i^0}{a_{\max}}\right\}, \forall i \in \mathrm{I} \tag{14}$$

$$\Delta t_i^2 = \max\left\{\left(\Delta d - \frac{V_{\max}^2 - v_i^{0^2}}{2a_{\max}} - \frac{v_i^{s^2} - V_{\max}^2}{2a_{\max}}\right)/V_{\max}, 0\right\}, \forall i \in \mathrm{I} \tag{15}$$

$$\Delta t_i^3 = \min\left\{\frac{V_{\max}^r - v_i^1}{a_{\min}}, \frac{V_{\max}^r - V_{\max}}{a_{\min}}\right\}, \forall i \in \mathrm{I} \tag{16}$$

$$v_i^1 = \sqrt{\frac{(2a_{\max}a_{\min}\Delta d + a_{\max}v_i^{0^2} + a_{\min}v_i^{s^2})}{(a_{\max} + a_{\min})}}, \forall i \in \mathrm{I} \tag{17}$$

In Eqs. (14)–(17), $a_{\max}$ is the maximum acceleration and $a_{\min}$ is the minimum acceleration.

4. *Vehicle kinetics limits*

For the consideration of safety and fuel efficiency, speed is designed to change in a designated range. Along the entrance lane, only a freeway speed limit is set as:

$$0 \le v_1 \le V_{\max}. \tag{18}$$

Inside the roundabout, the upper limit of vehicle speed inside the roundabout $V^r_{\max}$ is set for safety for vehicles while turning. Besides, previous research (Rakha et al., 2011) proved that fuel efficiency decreases extremely when the vehicle speed is less than 4.47 m/s (10 mph). Therefore, a lower speed limit $V^r_{\min}$ is set inside the roundabout. The speed limit can be specified as:

$$V^r_{\min} \le v^s_i \le V^r_{\max}. \tag{19}$$

3.2. Speed optimization model

The lower level of the TP generates optimal speed trajectories for each vehicle to minimize fuel consumption. This section builds the relationships between the vehicle state when arrival at the stop line and the optimized trajectory. At the entrance lane, the initial state of the vehicle is known and the terminal state at the stop line can be derived from the cooperative driving optimization model. Therefore, speed optimization can be considered as an optimal control problem where the terminal state and time are known.

The state variable of vehicle i is $\boldsymbol{x}_i(t) = [x_i(t), v_i(t)]^T$, where $x_i(t)$ is the distance traveled by vehicle i at time t. $v_i(t)$ is the speed of vehicle i at time t. Therefore, the state equation can be expressed as:

$$\dot{x}_i(t) = [v_i(t), u_i(t)]^T \tag{20}$$

where the control variable $u_i(t)$ is vehicle acceleration.

1. *Cost function*
   The cost function is the sum of the fuel consumption of all vehicles in the following:

$$\text{P2}: \quad J_2 = \sum_{i \in \mathrm{I}} \sum_{t=t_0}^{t_i^f} L(x_i(t), u_i(t))\Delta t \tag{21}$$

   where $L(x_i(t), u_i(t))$ is the running cost. The estimation of fuel consumption adopts the VT-Micro model (Rakha et al., 2004) for it has been extensively applied. Moreover, the running cost function can also be modified into other instantaneous fuel consumption methods without affecting the proposed approach (Liu et al., 2021; Ma et al., 2017; Qin et al., 2021; Xu et al., 2019; Zhou et al., 2021; Zohdy & Rakha, 2016).

$$L(x_i(t), u_i(t)) = \mathrm{MOE_e}(x_i(t), u_i(t)) \tag{22}$$

$$MOE_e(x_i(t), u_i(t)) = \{ \sum_{e^{m=0}}^{3}\sum_{n=0}^{3}(L^e_{m,n} \times v_i(t)^m \times u_i(t)^n), u_i(t) \ge 0 \; \sum_{e^{m=3}}^{3}\sum_{n=0}^{3}(M^e_{m,n} \times v_i(t)^m \times u_i(t)^n), u_i(t) < 0 \tag{23}$$

   where $L^e_{i,j}$, $\mathrm{M}^e_{i,j}$ are the model regression coefficients.

2. *Initial and terminal state constraints*
   When the optimization is started, the state of the vehicle inside the update zone is defined as the initial state:

$$x_i(t^0_i) = \left[x_i(t^0_i), v_i(t^0_i)\right]^{\mathrm{T}}, \forall i \in \mathrm{I} \tag{24}$$

   where $x_i(t^0_i)$ is the distance between vehicle i and the edge of the update zone.
   At the end of the planning horizon, each vehicle i is supposed to arrive at the stop line at the given time with the given speed. Thus, the terminal state constraints are formulated in Eqs. (25)–(27).

$$t^f_i = t^s_i, \forall i \in \mathrm{I} \tag{25}$$

$$x_i(t^s_i) = L_c, \forall i \in \mathrm{I} \tag{26}$$

$$\mathrm{v}_i(t^s_i) = v^s_i, \forall i \in \mathrm{I} \tag{27}$$

3. *Safety constraint*
   When two vehicles travel in the same lane in the same arm, a rear-end safety constraint should be applied for safety concerns.

$$x_j(t+\tau) \le x_i(t) - d + \mathrm{M}\gamma_{i,j} + \mathrm{M}\eta_{i,j}, \forall i,j \in I, t \in \left[t^0_i, t^f_i\right] \tag{28}$$

$$x_j(t+\tau) \le x_i(t) - d + \mathrm{M}\gamma_{i,j} + \mathrm{M}(1-\eta_{i,j}), \forall i,j \in \mathrm{I}, t \in \left[t^0_i, t^f_i\right] \tag{29}$$

4. *Vehicle kinetics limits*
   The constraint on acceleration is set to ensure the acceleration solutions are feasible considering the limit engine power and brake condition.

$$a_{\min} \le u_i(t) \le a_{\max}, \forall i \in \mathrm{I}, t \in \left[t^0_i, t^f_i\right] \tag{30}$$

The constraint of speed keeps consistent with the limitations in Section 3.1(4).

$$v_i(t) \leq V_{\max}, \forall i \in \mathrm{I}, t \in \left[t_i^0, t_i^f\right] \tag{31}$$

3.3. Planning horizon procedure

When vehicles enter the control zone, they send their state information to the TP. Before receiving the optimized trajectory, vehicles maintain current speed. Every time the optimization process is started, all vehicles in the update zone are included in the optimization and get the optimized trajectory. Ongoing adjustments may be made while the vehicle is in the control area because new entering vehicles affect the optimization results. When the front part of the vehicle reaches the no-changing zone, the vehicle must already have received a trajectory. Then, they follow this final trajectory before leaving the roundabout.

The update of optimization needs to ensure two points: 1) each vehicle must be able to participate in the optimization at least once and get the trajectory during driving in the update zone, 2) the optimization frequency should not be too high, which leads to high computation cost and frequent changes in vehicle trajectory. Therefore, this paper designs two trigger conditions for the start of optimization.

An iterative algorithm to solve the bi-level optimization problem and the planning horizon procedure of proposed approach are summarized as follows:

Step 1: Initialize start time $t_s = 0$ and $t_o = 0$, where t_s is the simulation time and t_o is the time to implement optimization.
Step 2: Collect vehicles' information (displacement, speed) inside the update zone at time t_o. Preset the arrival time of each vehicle $t_f^i = t_{\min}$.
Step 3: Solve P2.
Step 4: If the model is infeasible, then update the arrival time $t_f^i = t_f^i + \Delta t$ and go to Step 3. Otherwise, get the t_i^f and v_i^s of each vehicle and go to the next step.
Step 5: Solve the model P1 with the defined t_i^f and v_i^s. Get the arrival time of each vehicle t_f^i.
Step 6: The algorithm is terminated if the difference between the two objective function values is less than pre-set the error tolerance ψ. If the condition to terminate the iteration is not met, then go to step 3 with the new arrival time. Otherwise, update the trajectory of vehicles based on the optimization results.
Step 7: Update simulation time $t_s = t_s + \Delta t$. Update the position of vehicles. Update the number of new vehicles that drive into the control zone N_{op}.
Step 8: If t_s reaches the final solution time, then end the process. Otherwise, go to the next step.
Step 9: If a vehicle runs out of the update zone without getting an optimized trajectory $\chi_{\max} > L_u$ or $N_{op} > N_{\max}$, then update $t_o = t_s$. Go to step 2.

4. Case study

4.1. Experimental design

A typical four-arm roundabout with two entrance lanes against two circulating lanes is adopted. The settings of the evaluations are specified here:

1. All three directions of movements (left, straight, and right) are considered for each arm. The turning ratio is 0.2:0.3:0.5, which is equal in four arms.
2. The length of the control zone L_c is 275 m, which is shorter than the reliable communication range of DSRC (Stancil et al., 2010). The length of the update zone L_c is 200 m.
3. The vehicle arrival follows the Poisson distribution (Jiang et al., 2017; Li et al., 2014; Yu et al., 2018). The speeds of vehicles when they enter the control zone v^e follow a normal distribution $N(13, 1)$. Unsafe gaps when vehicles enter the network are avoided.

To validate the effectiveness and flexibility of the proposed method in delay and fuel consumption, two baselines are tested:

1. **Baseline1 [Yield-regulated control]:** It is a rule-based modern roundabout without central control. All vehicles drive according to the traditional roundabout rules. This scenario is calibrated and its delay result is similar to the results from HCM 6th (Transportation Research Board, 2016).
2. **Baseline2 [FCFS control]:** the first-come-first-serve control is chosen as the representative of the rule-based central control strategy. Because the first-come-first-serve (FCFS) is the current common practice in the aforementioned researches.
3. **Proposed [Proposed approach]:** It is an optimization-based central control with the capability of passing sequence optimization. By comparing to Baseline 2, the advantage of passing sequence optimization is presented.

Table 1. Basic traffic demand and *V/C* ratio.

		Traffic demand in pcu/h	
Scenarios	*V/C*	Left lane	Right lane
1	0.2	112	126
2	0.4	224	252
3	0.6	336	379
4	0.8	448	505
5	1	560	631
6	1.2	672	757

Note that vehicles adopt the same physical characters and kinetic parameters in three strategies, such as minimum headway, accelerate, reaction time, and so on. Thus, all the benefits in efficiency and fuel consumption all result from the differences in control strategies.

Four scenarios are designed for the simulation. The traffic demands for each entrance lane are given in Table 1, which are scaled proportionally by the volume/capacity (V/C) ratio.

A simulation platform is constructed using Python, MATLAB, and VISSIM. Gurobi 9.0.3 is applied in solving P1 and P2 (Gurobi Optimization Inc., 2019). VISSIM simulates traffic (Fu et al., 2021; Rayaprolu et al., 2013; Xie et al., 2017) and produces input for the master control program in Python. The simulation of Baseline 1 is calibrated to match the real world. This is achieved by adjusting the priority rules at the weaving area of the roundabout. The proposed trajectory planner is tested on a computer with Intel®Core™i5-1.80 GHz.

The values for other parameters adopted are presented in Table 2.

Table 2. Attributes in the simulation.

Parameter	Value	Unit
Simulation time horizon	900	s
Warming-up time	600	s
The upper limit of speed in the approach lane $V_{\max}$	15	m/s
The upper limit of speed in the roundabout $V^r_{\max}$	8.94	m/s
The lower limit of speed in the roundabout $V^r_{\min}$	4.47	m/s
Maximum acceleration $a_{\max}$	2	m/s^2
Minimum acceleration $a_{\min}$	−4	m/s^2
Time headway τ in Newell's car-following model	2.2	s
Space headway d in Newell's car-following model	5	m
Time step Δt	0.1	s
Pre-set the error tolerance ψ	2%	

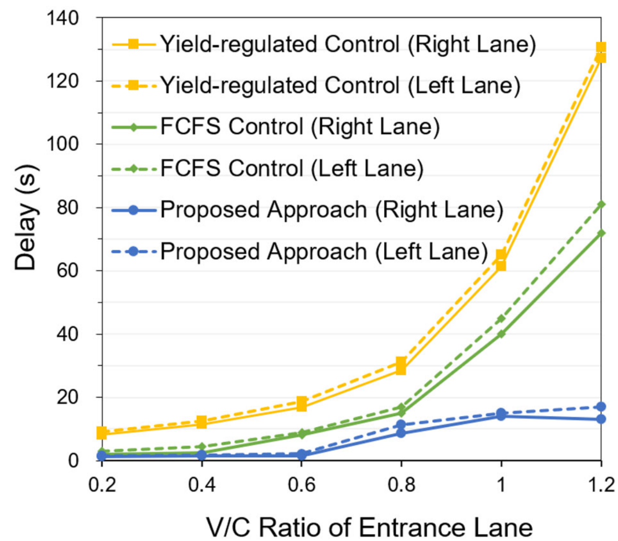

Figure 4. Delay improvement.

4.2. Simulation results and discussions

1. *Efficiency improvement*

Figure 4 shows the delay results of the three different controls. The delay increases with demand when all three kinds of controls are applied. The delay of the traditional yield-regulated control increases obviously when the demand increases from the low level ($v/c = 0.2$) to a high level ($v/c = 1.2$), which is consistent with previous researches (Transportation Research Board, 2016). In the results of FCFS control, delays also show the same significant increase. However, in every scenario, delays of FCFS control are lower than those of yield-regulated control, by an average of 52.5%. In comparison, the proposed approach generates significantly lower delay at all demand levels. The average delay of the proposed approach is remarkably lower than the two baselines. It is further observed that the proposed approach has a better ability to accommodate high demand. When the roundabout is oversaturated ($V/C = 1.2$), the proposed approach decreases the delay by up to 87.8%. This demonstrates the improved capacity of the entrance lane of the proposed approach.

Both the proposed approach and FCFS control can significantly reduce the delay of a roundabout. The benefit of the efficiency of two centralized control results from two factors:

1. Centralized control allows vehicles to enter roundabouts at high speeds through speed optimization, as shown in Figure 5. As a result, the average speed of vehicles within the roundabout is higher. The time required for vehicles to pass through the roundabout is shorter. This brings more efficient use of space within the roundabout.
2. Centralized control can proactively create gaps for vehicles in the entrance lane. Conventionally, vehicles in the entry lane must wait for an accepted gap passively.

The proposed approach is more effective than FCFS control. The proposed approach keeps the delay

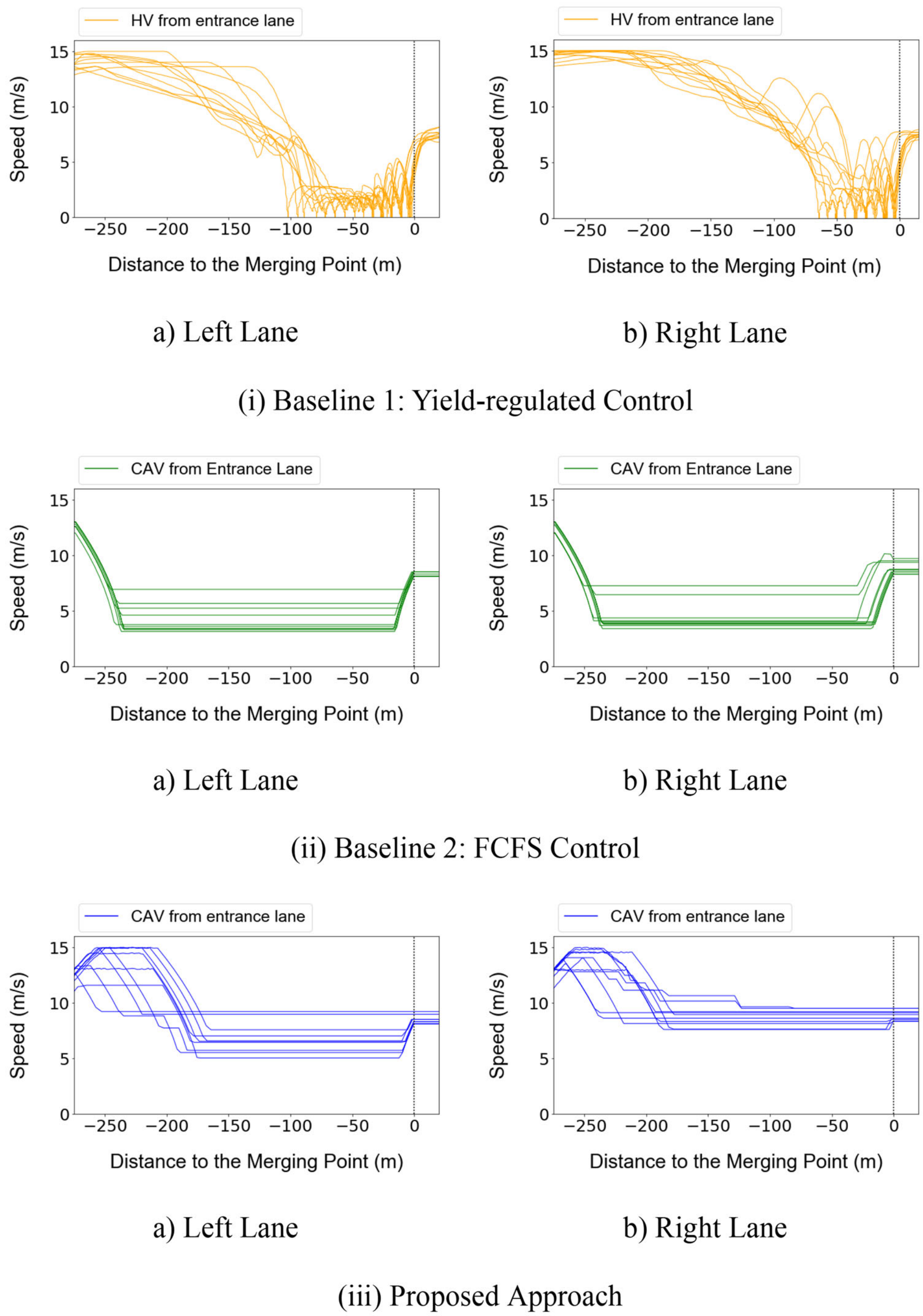

Figure 5. Speed of the first 15 vehicles entering the control zone after 100 s from the east entrance of three control strategies.

below the 20 s even in an oversaturated state, thus increasing the capacity of the roundabout. However, the delay results of FCFS control increase markedly with the increase of the V/C ratio. Because the proposed approach optimized the passing sequence of vehicles. Consequently, the utilization of space resources of the roundabout can be maximized. Figure 6 shows a lower average headway between vehicles of the proposed approach. In contrast, the delay result of FCFS control fluctuates widely because the effect of FCFS depends entirely on the passing sequence in which the vehicles arrive at the control zone.

The results shown in Figure 4 also indicate that the delay of the left lane has the same trends as the right lane. Nevertheless, in all three control modes, the delay of the left lane is slightly greater than the right lane. Of the traditional yield-regulated control, the delay of the left lane is 7.5% higher than the right lane. This results from the smaller capacity of the left lane, which is in agreement with the calculation

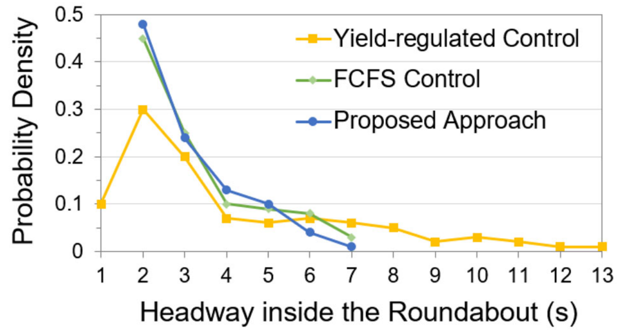

Figure 6. Headway distribution.

formula of delay in HCM 6th (Transportation Research Board, 2016). Moreover, with the two centralized control methods (FCFS control and proposed approach), the left lane has 16.3% and 27.9% higher delays than the right turn lane, respectively. This is because vehicles in the left lane plan to enter the inner lane of the roundabout, so a vehicle in the left lane entering the roundabout requires a gap in both lanes of the roundabout. vehicles in the right lane use the outer lane of the roundabout, so a vehicle in the right lane entering the roundabout requires a gap in the outer lane of the roundabout. Thus, the conditions for vehicles in the left lane to enter the roundabout are more difficult than for vehicles in the right lane to enter the roundabout. Besides, left-turning vehicles travel longer distances within the roundabout. Consequently, each vehicle experiences longer delays due to the speed limit at the roundabout.

1. *Fuel consumption improvement*

Figure 7 shows the fuel consumption results of the three kinds of controls. Comparing with the traditional yield-regulated control and the FCFS control, the proposed approach reduces fuel consumption by 28.1% and 13.4%, respectively. This improvement is owing to the following reasons: (1) Full stop is avoided by centralized control as shown in Figure 8, thereby conserving momentum. (2) High efficiency in operation results in shorter travel times. That is why fuel consumption of the proposed approach is still 13.4% fewer than the FCFS control.

5. Conclusions and future research

This paper proposes a trajectory planner for multilane roundabouts. The goal is to enhance the existing roundabout in terms of capacity with the help of connected automation. The trajectory planner is formulated as a bi-level optimization. The upper level is a passing sequence optimization, which determines the

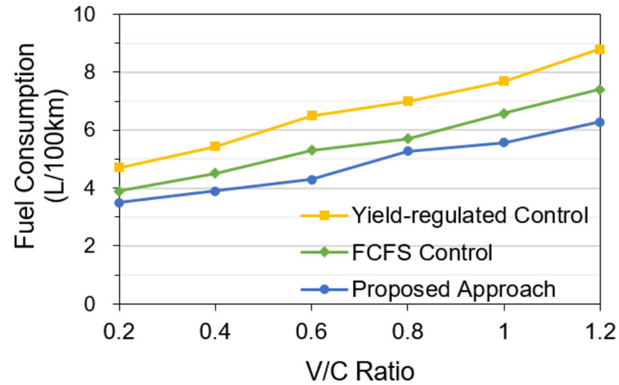

Figure 7. Average fuel consumption of vehicles of three control strategies.

passing sequence of the approaching vehicles to achieve delay minimization. The lower level is a speed optimal control aiming at minimizing fuel consumption while honoring the pass sequences generated from the upper level optimization. The proposed planner is evaluated in simulation on a typical four-arm roundabout with two entrance lanes in each arm. Compared with the traditional yield-regulated control, the proposed approach is able to reduce delay by 69.6% to 87.8% and fuel consumption by 24.7% to 28.6%. Compared with the First-Come-First-Service (FCFS) control, the proposed approach can reduce delay by 33.5% to 81.9% and fuel consumption by 7.5% to 18.8%. The change in benefit is due to the variation of demand. Detailed investigation on the evaluation reveals that:

- The capacity limit of traditional roundabout results from its yield regulation. The proposed planner substitutes the yield regulation and therefore increases the capacity and results in smaller delay under heavy traffic.
- Trajectory optimization is able to minimize stop-and-go maneuver at the stop bar, thereby reduce delay and fuel consumption.
- The proposed planner is able to reduce average headway inside the roundabout. This confirms the proposed planner's strong capability in allocation of right-of-way resources within the roundabout,

In this paper, a 100% CAV environment is assumed. However, human-driven vehicles, connected vehicles, and CAVs will coexist in the near future. The control for mixed traffic environment is worth researching. Future research could investigate enhance the proposed method to be compatible with a partially connected and automated environment.

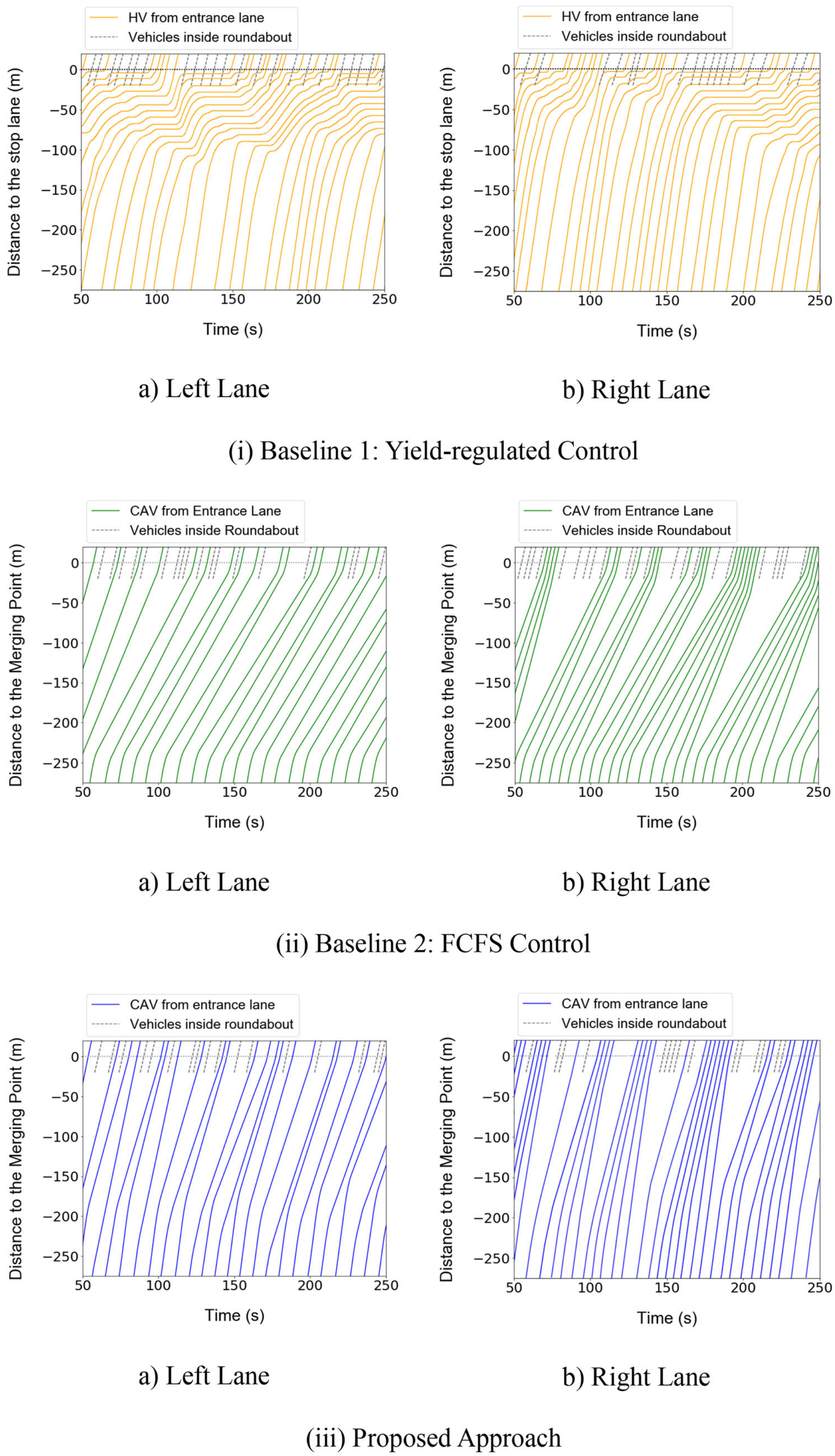

Figure 8. Trajectory of vehicles from the east entrance of three control strategies.

Disclosure statement

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