Supersonic Flow Over Cone Using Schlieren Technique

Wyatt O. Welch[1]
San Diego State University, San Diego, California

The goal of this experiment is to visualize and understand the fundamentals behind supersonic flow and its primary features utilizing Schlieren imaging technique. This experiment will be performed in a Mach 2.0 wind tunnel, with a 15° half-angle cone in the test section in order to create an oblique shock wave and expansion wave.

I. Objective

The primary objectives of this lab is to understand the behavior of supersonic flow over a cone through the use of Schlieren imaging, which visualizes flow density gradients (as will be discussed in length in the theory section). We will focus on key flow features within the oblique shock that forms, such as expansion waves, which form off the cone. By conducting the experiment, we can gleam firsthand visualization and experience of compressible flow, basing it off the theoretical lessons from class. Along with observation, we will provide comparisons between observed knowledge on shockwave angles, and the theoretical predictions.

The goal for this report is to describe the theory of both supersonic flow, and the Schlieren visualization technique we utilize in the experiment. We will use the data gathered to measure the shockwave angle and compare it with the theoretical values. From the imaging, it should be clear where the shockwave and expansion regions are.

II. Theory

Attaching a cone to a supersonic wind tunnel leads to a number of interesting interactions in the flow. When compressible flow generates as speeds are too great for air molecules to move out of the way of each other, a shock wave can form with the development of a shock wave. The geometry of the cone and the upstream mach number are the key elements in determing both the angle of shock, and its key properties.

Cones in supersonic flow can produce conical oblique forming from the tip. This flow experiences a sudden jump in pressure, temperature, and density as it reacts to the immense speeds and interactions between other atoms. Across the shock, the mach number decreases as a result of the new temperature, this may result subsonic behavior but depends on the initial mach number, and the angle the shock forms. This shock angle β, is always greater than the cone half-angle θ, and the relationship between the two comes from conical shock relation theory, provided through readings.

This cone is supported by a smaller rod which holds the cone in place. Because of this, there is a convex corner at the end of the cones growth, which causes expansion waves to form. The corner causes flow to turn away from the corner, which speeds the fluid as it escapes viscous forces, thereby leading to noticeable expansions in the flow under the cone. In theory, if the cone angle is too great for a given speed, the flow does not have the speed to sharply change directions when meeting the cone whilst remaining attached, causing a detached shock, or bow shock. This leads to more complex flow and multi-variable dependent flow aspects.

The Schlieren Technique is an extremely powerful tool used to visualize fluid density in a flow, which is also applicicable in low and high speed flows. The basic technique involves deflecting light as it passes through a density gradient (such as a shock wave). The refraction of light changes with density, so as ligh pases through air with changing density, it bends in different angles, and can be visualized. The method to visualize the refractions is to use a knife edge at the focal point (or uniform meet) in a mirror. By doing so, rays of light used are blocked, which changes the brightness of the gradients when seen through a high speed camera. The amount of dark/light seen from the fluid is based on the amount of deflection from the refraction.

[1] Aerospace Engineering Student, College of Engineering, 5500 Campanile Drive, San Diego, CA 92182.

III. Equipment

SDSU's supersonic wind tunnel has the capability to run at speeds up to Mach 4.5. It is designed to study high speed aerodynamic wind flow, and can be equipped with models in order to study the impact supersonic flow has over objects. This machine employs the use of a compressor, pressure vessels to store the compressed air, a regulator valve to control pressure release, a stilling chamber to spread the flow, and a nozzle that directs the flow into the chamber / test section. The apparatus works by compressing the air and storing it in pressure vessels, which runs to the regulator valve, and from there onwards. The stilling chamber and screens within spreads the flow, which then converges into a nozzle and diverges into the 6"x6" test section at supersonic speeds. The exit includes a muffler to safely release the escaping air.

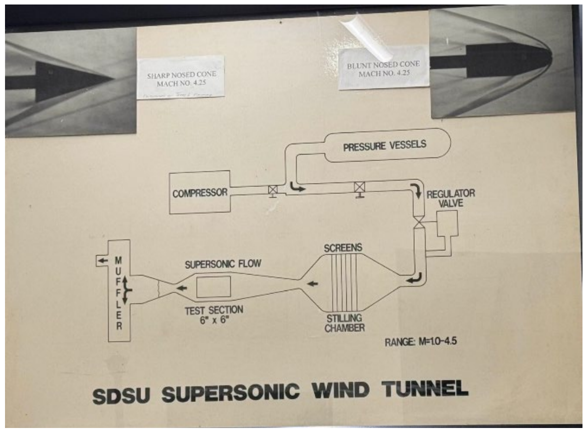

Figure 1: SDSU Supersonic Wind Tunnel design

In order to visualize the flow, schlieren imaging was used to capture the shock wave around the test model, which was a pointed cone with a half-angle of 15 degrees resting in the test section of the wind tunnel. The schlieren system used in this lab was a large concave mirror, and a light source, along with a high speed camera. A wooden beam gave support and stability for the mirror and the camera. This setup allowed for visualiziation of the density gradients in the test section.

IV. Setup

Wind Tunnel Setup

- Ensure all tanks and compressors are shut off and system is at atmospheric pressure
- Mount cone model in test section and ensure it is secure in place
- Close test section window and lock it
- Confirm all seals and fasteners are in place
- Turn on dehydrator and wait for all water in the system to be filtered out
- Power on compressor and fill air canisters
- Once pressurized, prepare valve system
- Set the parameters in table 1 for the controller

PL	0	RL	0	MODE	A	KP	1.1	ΔT	1
ΔPV	100	PV	--	MH	100	Ti	0.01	D/R	REV
DF	0.25	CV	--	ML	0	Td	0	VD	REV
LC	OFF	SV	17	ΔMV	25	Rt	1	MVF	-25
RH	100	MV	-25	DVL	100	BS	0	PH	100

Table 1: Control Parameters for Mach 2 run

Schlieren Imaging Setup

- Mount large concave mirror behind wind tunnel facing inward
    - Mount to wooden beam for stability
- Position light source facing mirror reflecting along optical path
- Place knife edge at focal point to create schlieren contrast
- Setup high speed camera behind knife edge
    - Ensure focuse and exposure
- Allign all components and verify system captures the baseline images without issue

V. Procedure

Mach 2 Run Procedure

- Perfom final setup checks
- Open supply valve to the fast valve controller
- Open main valve
- Don appropriate hearing protections
- Clear exhaust area of people/loose objects
- Turn ligts off for optimal imaging
- Activate wind tunnel with green button next to controller
- Wait until apparatus is done releasing air and compressor is empty before removing hearing protections
- Use red button as emergency stop if needed
- Perform procedure in reverse to clean up wind tunnel

VI. Results and Analysis

The results of this lab are simple, and can be visualized through the use of the Schlieren technique. With this method, a recording of the flow, waves, and fans can be taken throughout the experiment. With this, we are able to observe key images for analysis such as the shock wave and the expansion fan generated by the geometry of the cone.

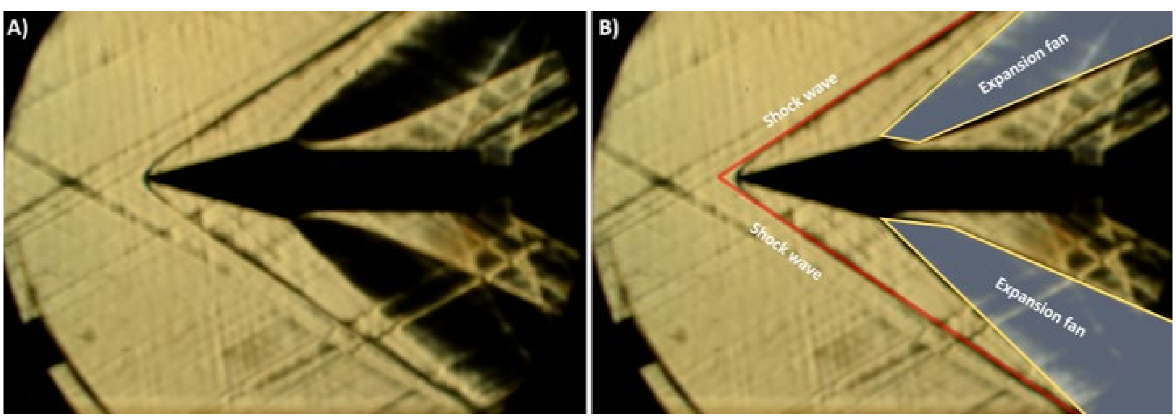

Figure 1: Schlieren Imaging of 15 degree cone in Mach 2 Flow. Featuring schock wave at leading edge, and expansion fan at trailing edge.

From figure 1, it is clear that an angled shock wave forms at the leading edge of the cone, and an expansion fan forms due to the inward cut of the shape. Using simple geometry, the shockwave half-angle is estimated to be 33.6° from the horizontal. Utilizing the Compressible Aerodynamic Calculator provided in the materials for this lab, the following data can be derived from the Mach number the flow was running at, as well as the Cone Half Angle, displayed in the table below:

Mach	2.0	Cone Angle	15.0°
Wave Angle	33.93°	Shock Turn Angle	4.596°
$\frac{P_2}{P_1}$	1.287	$\frac{P_{0,2}}{P_{0,1}}$	0.9984
$\frac{P_c}{P_1}$	1.568	$\frac{P_{0,c}}{P_{0,1}}$	0.9984
$\frac{\rho_2}{\rho_1}$	1.197	$\frac{\rho_c}{\rho_1}$	1.378
$\frac{T_2}{T_1}$	1.075	$\frac{T_c}{T_1}$	1.138

Table 2: Compressible Aerodynamic Inputs for Conical Shock Relations, given $\gamma = 1.4$

From Table 2, we see the wave angle is equal to 33.93°, which is 0.33 degrees from experimental, giving an error of $< 1\%$. This is evidence of the reliability of this experiment, and its collected data. Thanks to Schlieren imaging, we can clearly observe the effective results of shockwaves and other traits of high speed flow in a wind tunnel. Beyond the shock wave, it is also evident that there is an expansion fan generated in the trailing end of the cone. This is due to the convex edge around the trailing edge, which allows space for flow to expand into it, resulting in a sudden change in density that is viewable through Schlieren imaging.

The table is very useful for a number of calculations, and generating information about the flow without needing sensors to probe inside, which can be expensive and difficult to accurately measure. As an example, given ambient air temperatre in the lab at 20°, the local static temperature in the test section can be calculated as follows:

$$\frac{\boldsymbol{T_{cone}}}{\boldsymbol{T_1}} = 1.138 \implies T_{cone} = T_1 \cdot 1.138 = 20°C \cdot 1.138 = \mathbf{22.76°C} \tag{1}$$

This result follows theory, as across a shockwave temperature is expected to increase with pressure and density, since the shockwave compresses the air, converting kinetic energy into heat. Moisture needs to be removed from the system before every run for a few reasons, including the possibility of disturbing flow, as well as causing minor issues with Schlieren observations. Primarily, however, any water in the system can lead to condensation when the temperature drops rapidly as the air speeds up. Not only can this cause condensation, in some cases water crystals may form, which could damage equipment.

To maintain constant air pressure, a control valve is implemented, which regulates airflow from the compressor and storage tanks. The computer also monitors the pressure in the pressure chamber, and calculates how the control valve should open or close to maintain a constant pressure as the storage pressures fluxuate throughout a test.

If we were to estimate the pressure required in the settling chamber to create M=2 flow after the nozzle, we could estimate based on a figure from the test.

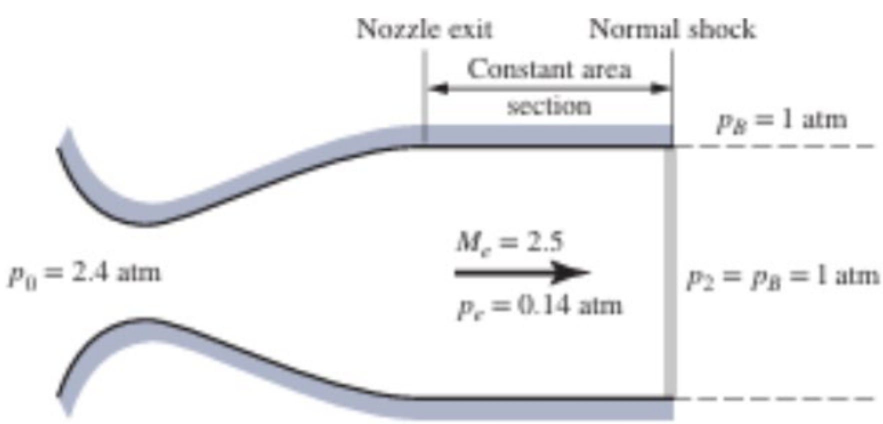

Figure 2: Nozzle exhausting into constant area duct

For Mach 2 flow, we can estimate the ratio of pressure before the shock to be $\frac{P_2}{P_1} = 4.5$, and assuming $P_2 = atm$, we calculate that $P_1 = 0.2222\ atm$, which is the pressure before the shock. Then following isentropic tables, $\frac{P_0}{P_1} = 7.02 \Longrightarrow P_0 = 0.2222 \cdot 7.02 = \mathbf{1.56}\ \boldsymbol{atm}$, which is the estimated stagnation pressure needed to produce mach 2 speeds.

When designing a supersonic wind tunnel, the cross-sectional area of the second throat but be greater than that of the first throat. The reason for this is that if the second throat is too small, or smaller than the first, flow cannot adjust to the shock, and diffusion occurs, or there is less control and flow separation occurs. This can be demonstrated through equation 2:

$$\frac{A_{t,2}}{A_{t,1}} = \frac{P_{0,1}}{P_{0,2}} \tag{2}$$

If $A_{t,1} > A_{t,2}$, then the pressure ratio would suggest an increase in pressure after the shock, which does not match theory, since pressure increases across a shock. Therefore, it must be that the second throat area is greater than the first throat to handle the reduced total pressure in the second chamber when the shockwave occurs.

The knife edge in the Schlieren system must be placed at the focal point of the mirror, where light is brought to its greatest focus. The purpose of this is to convert the lights deflections and refractions into intensity variations by shading the refractions with different levels of darkness. By doing so we can observe the density gradients with the naked eye.

VII. Conclusions

This lab had a very simple mission, yet taught a lot about supersonic flow and the Schlieren technique. In the theory section, we described the supersonic flow and the main principles of the Schlieren technique. In the results and discussion section, we analyze the resulting figure from the test, and compare it to the theoretical results. After, we estimate the local static temperature in the test section using the gathered data, and answer discussion questions. In all cases, and allowing for slight experimental data, the result sof this lab made sense and matched up to expected theoretical standards.

The lab itself had some sections that could only be described as ‘janky’ the beam used to stabilize the camera and mirror for the Schlieren technique was unstable and could easily lead to unfocused results, and with the knife edge even slightly offplaced, it could give a bad image and result in inaccurate flow angle measurements. The solution to this would be to manufacture a more sturdy setup, including a way to properly strap the mirror to the beam to avoid knocking.