Analysis of Water Flow Around Jet Using PIV Measurement

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This lab report covers the use of Particle Image Velocimetry (PIV) in order to analyze the near-field characteristics of turbulent and separating flow around a water jet. With the lab portion, we seek to demonstrate PIV principles in the lab and in the report, quantify the flow around the jet using the formulas and techniques provided by the professor, and identify normal and shear turbulence. The challenges in this lab will come from correctly following lab protocol to perform the experiment, as well as to properly interpret the data.

Nomenclature

$u(t)$	=	Instantaneous velocity(x-component)
$\bar{u}$	=	Time-averaged velocity (x-component)
$u'(t)$	=	Fluctuating velocity (x-component)
$v(t)$	=	Instantaneous velocity(y-component)
$\bar{v}$	=	Time-averaged velocity (y-component)
$v'(t)$	=	Fluctuating velocity (y-component)
$\overline{u'^2}$	=	Reynolds normal stress (x-directions)
$\overline{v'^2}$	=	Reynolds normal stress (y-directions)
$\overline{u'v'}$	=	Reynolds shear stress
ω_z	=	Vorticity
$\frac{\partial u}{\partial y}$	=	Velocity gradient
$\frac{\partial v}{\partial x}$	=	Velocity gradient
N	=	Sample count
x, y	=	Spatial coordinates
t	=	Time
T	=	Averaging time scale
ρ	=	Fluid density
μ	=	Dynamic viscosity
D	=	Nozzle exit diameter
U_e	=	Jet exit velocity

I. Introduction

THIS experiment is being conducted to learn about the velocity field and turbulence characteristics of a ffree round jet through the use of Particle Image Velocimetry (PIV). PIV is non-intrusive, which makes it ideal for hyper sensitive conditions where extremely small details need to be accurately measured. It utilized cameras taking high rate photos of particles moving within the flow, and processing that generates raw data for analysis on how the flow behavior over time.

This experiment was performed at the SDSU campus in the engineering department, at the setting designed for PIV observations. This was performed with a seeding process that applies tracer particles into the flow, and a laser light that records the movements of these particles. Through time-averaging and analysis of the raw data, key flow features will be calculated. This can include Reynolds stresses, vorticity, and mean velocity. Understanding these behaviors allows the student to understand mixing, momentum transport, and jet spreading behavior. Each of these will be discussed in length within this report.

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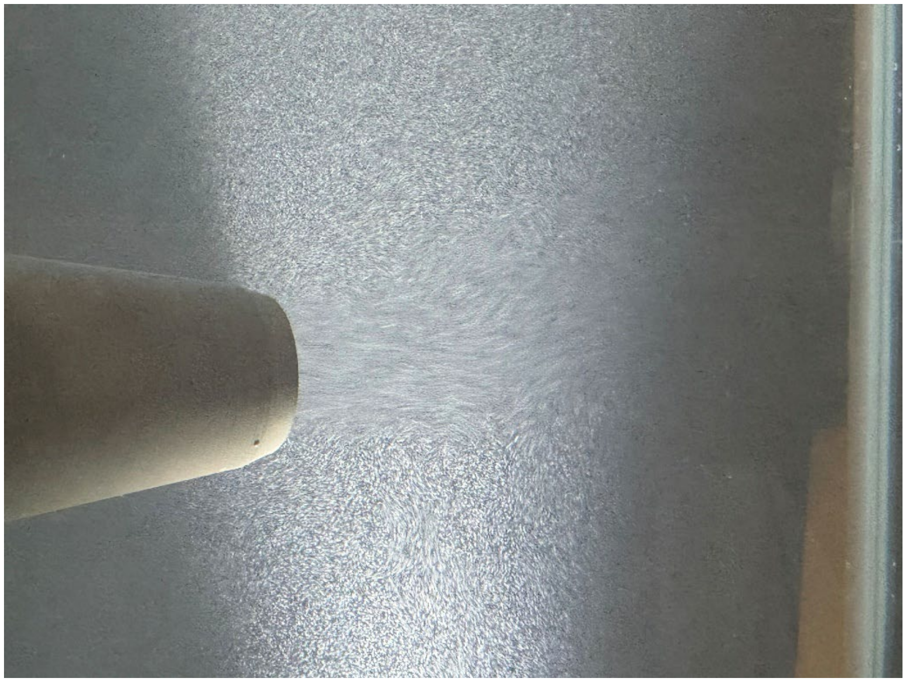

Fig 1: Particle flow in jet stream viewed with naked eye

I. Theory

The theory behind a lab report is useful for understanding the fundamental concepts being studied in the lab and in the lectures. This section aims to describe the equations used to perform this lab. To begin, much of what we are asked to consider relies on the time averaged results. This can be described in the following equation:

$$u(t) = \bar{u} + u'(t) \qquad \text{(equation 1)}$$

This formula, otherwise known as Reynolds Decomposition, allows for time averaged velocity and stresses in the fluid from the PIV observations. u(t) decomposes into a time averaged component $\bar{u}$, and the added fluctuating derivations from the average component u'(t). This process applies to both u and v components of velocity. The next equation is an expanded form from the previous, which gives the time-averaged velocity calculation. This works essentially as a basic average formula, with the specified time component.

$$\bar{u}(x, y) = \frac{1}{N}\sum_{k=1}^{N} u_k(x, y) \qquad \text{(equation 2)}$$

As mentioned, this behaves as a basic average equation, but it is important to note that this is a reoccurring formula for this lab, used to compute the fluctuating velocity as described in equation 1. These fluctuating velocity components will need to be simplified, which can be done by isolating turbulence for the Reynolds stress calculations. This comes from Reynolds decomposition.

$$u'(x, y, t) = u(x, y, t) - \bar{u}(x, y) \qquad \text{(equation 3)}$$

This equation subtracts the mean velocity from instantaneous PIV data. Moving away from Reynolds decomposition, we can calculate the Reynolds normal stresses observed across the fluid:

$$\overline{u'^2}(x, y) = \frac{1}{N}\sum_{k=1}^{N} [u'_k(x, y)]^2 \qquad \text{(equation 4)}$$

$$\overline{v'^2}(x, y) = \frac{1}{N}\sum_{k=1}^{N} [v'_k(x, y)]^2 \qquad \text{(equation 5)}$$

Both of these equations operate the same, one for each component of the velocity in the 2D plane. This equations squares fluctuating velocities found in equation 3. With this, we are able to quantify the turbulence intensity in both directions. This is major for analyzing the spreading and mixing of the fluid in critical points. The Reynolds Shear Stress is found as an extension from the previous two equations:

$$\overline{u'v'}(x, y) = \frac{1}{N}\sum_{k=1}^{N} [u'_k(x, y) \cdot v'_k(x, y)] \qquad \text{(equation 6)}$$

This equation is very useful, as it can identify the shear stress from the fluctuations and averages of the component velocities. This measures the momentum change due to the turbulence in the flow, and is critical to understanding turbulent shear layers, since this can sperate it down into 2D, something which would normally be far more complex to understand in 3D. The final equation is the Vorticity Calculation.

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad \text{(equation 7)}$$

This equation is computed from the special derivatives in each of the component directions, it requires partial differentiation, and is extremely important to understanding the flow from this experiment. The purpose is to identify the vortices, as well as the turbulence dynamics, in the fluid.

This section covers the sample calculations, which is very important to understanding how these are applied. For the purposes of keeping this report simple, the data used will be a set of 5 data lines from the 250th photo capture, the raw data is viewable from the appendix section A.

$$\bar{u}(x,y) = \frac{1}{N}\sum_{k=1}^{N} u_k(x,y)$$

$$= \frac{1}{5}\sum \frac{-0.002120 - 0.001830 - 0.001988 + 0.003514 + 0.00009440}{5} = -\,0.0004661\,\frac{m}{s}$$

$$u'(x,y,t) = u(x,y,t) - \bar{u}(x,y) =$$

Index	$\boldsymbol{u}$	$\boldsymbol{u' = u - \bar{u}}$
1	-0.00212	-0.00165
2	-0.00183	-0.00136
3	-0.00199	-0.00152
4	0.003514	0.00398
5	9.44E-05	0.000561

Table 1: Resulting data from raw input and equation 3.

$$\overline{u'^2}(x,y) = \frac{1}{N}\sum_{k=1}^{N}[u'_k(x,y)]^2$$

$$= \frac{1}{5}\sum \frac{(-0.001654)^2 + (-0.001364)^2 + (-0.001522)^2 + (0.003980)^2 + (0.0005605)^2}{5}$$

$$= 4.638 \cdot 10^{-6}\,\frac{m^2}{s^2}$$

$$\overline{u'v'}(x,y) = \frac{1}{N}\sum_{k=1}^{N}[u'_k(x,y) \cdot v'_k(x,y)]$$

$$= \frac{1}{5}\sum \frac{(0.001343 + 0.001362 + 0.001304 - 0.002657 + 0.0007833)}{5} = 0.0004271\,\frac{m}{s}$$

Index	vv	v'v'	u'v'u'v'
1	0.001343	0.000916	-1.515×10^{-6}
2	0.001362	0.000935	-1.276×10^{-6}
3	0.001304	0.000877	-1.334×10^{-6}
4	-0.00266	-0.00308	-1.228×10^{-5}

5	0.000783	0.000356	1.996×10^{-7}

Table 2: Resulting data from raw input and equation 1.

II. Setup

There are only a few pieces of equipment used in this lab, however it is important that we define each of them in order to be sure that readers fully understand the process that goes into performing these techniques. To start, the flow generation used in the lab is executed using a nozzle with an exit diameter of 5 centimeters, which the fluid will flow from smoothly into the water tank. The water tank is 80x35x40 (cm), and has a maximum capacity of 112 liters. It is an open top tank, since the water will not intrude in the interior fluids in this experiment. The pump and controller used to adjust the water flow speed is an EcoDrift 8.1 Aqua Medic,. This has an adjustable flow rate ranging between 1600 to 8000 L/h, and consumes 8-20 Watts.

The seeding for this lab is done through the use of Polyamide particles, supplied from Danec Dynamics. This material has a mean diameter of 50 picometers, and has a density of 1.03 g/cm^3. The use of this density is to be near that of water, which allows for near-neutral buoyancy in the water. When injected into the tank, it is mixed with rubbing alcohol in order to reduce surface tension and allow for proper and equal spreading of particles in the tank. In order to have proper viewing of the particles, proper light setup is critical. This lab uses EduPIV LED lights to supply high intensity illumination, which is optimal for particle tracking. Along this there are light sheet optics which act as a light guide to focus the rod lens, and a backdrop to separate the foreground from the background.

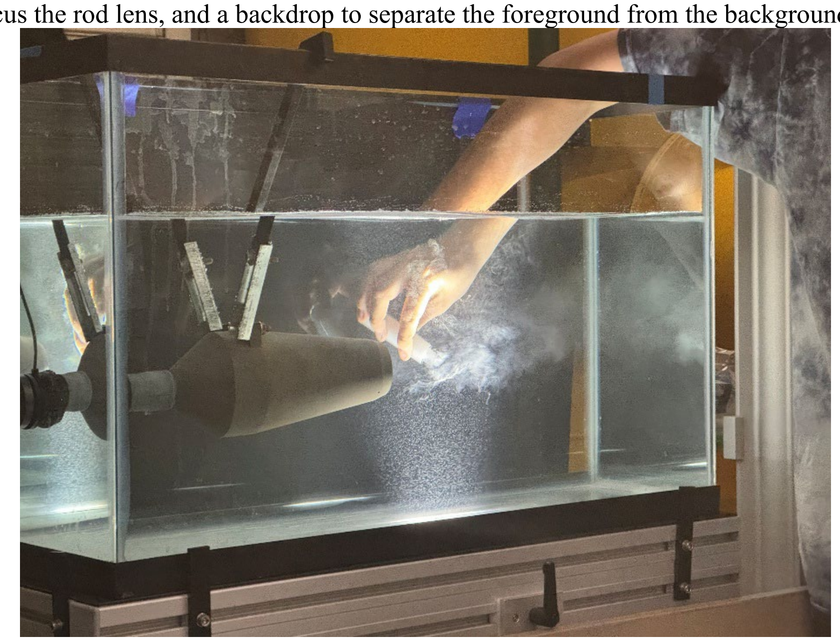

Fig 2: The dispensing of the tracers in the jet stream after mixing with alcohol

The imaging system used for this experiment is a FlowSense USB 2M-165 camera from Dantec Dynamics. This camera has a resolution of 1920 x 1200 pixels, and can run up to 160 fps. For this experiment, we are using 150 Hz. The lens is a 35 mm low distortion lens which will lock onto the aperture. A calibration target is useful to ensure accuracy when running the experiment. A ruler placed in the laser sheet plane is used to calibrate the device, by having a set standard to work off of and normalize distances.

Using a DynamicsStudio software from Dantec, the researchers are able to perform 2D PIV analysis, and adaptive window deformation. This analysis is done, then the data is saved to excel sheets which will be parsed in MATLAB for the purposes of this lab. To ensure the most optimal working conditions, it is required to have lights turned off in the room, the tank should be perfectly aligned with the camera, and the pump should be given 10 minutes to warm up in order to minimize error.

III. Procedure

1. Fill the tank to its near capacity with water
2. Connect the pump to the nozzle, which should be placed in the tank and secured
    a. Nozzle should be horizontal, and secured using a magnet
3. Plug the pump into a power source and ensure it operates properly.
4. Connect the LED light source to power and turn it on
5. Mix seeding particles with water and rubbing alcohol
    a. After mixing, add contents into the tank to be spread in the water flow
6. Set up the camera and attach the 35 mm lens
    a. Mount the camera to the rail, approximately 19 cm away from the tank
    b. Connect the camera to the laptop
7. Configure Dynamic Studio Software and ensure settings are correct
8. Turn off room lights, adjust LED light, place backdrop behind tank
9. Adjust camera settings to 750 picoseconds, and a trigger rate of 150 Hz
    a. Click preview to check the partial visibility to the camera
10. Set pump to maximum speed and allow 10 minutes for stable flow
11. Place calibration ruler in cameras view
    a. Adjust camera settings to be focused and lined up to the camera
12. Acquire a minimum of 300 images in a ~2 second frame of data collection
    a. Click "Acquire" in the software to record the images
    b. Safe collected data
13. Upload data to MATLAB and parse files
    a. With acquired data, perform the requirements for the lab report

IV. Results

The following are all of the required graphics and explanations for the completion of this lab. After analyzing, it should be clear that the lab procedure was followed properly and accurately, and the end result is a correctly interpretated set of data, with specific explanations on the theory, interpretation, and source of issues.

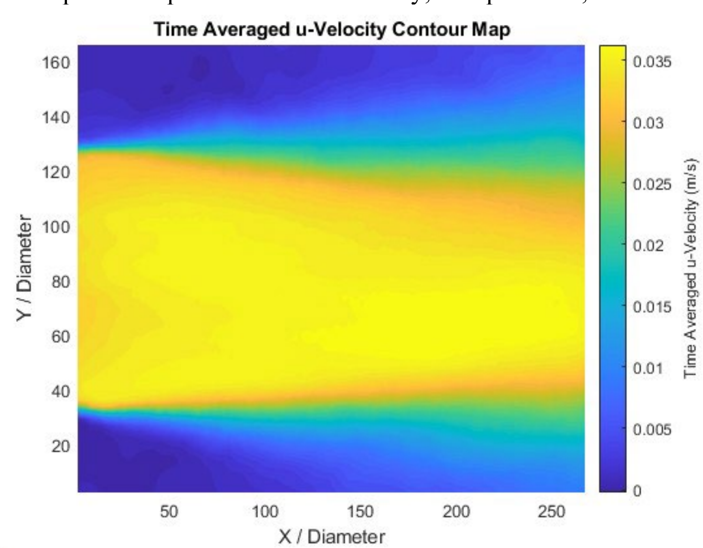

Fig 3: Time Averaged u-Velocity Contour Map

For this first image, we have the time averaged contour map of velocity in the u direction. This graphic makes sense for what you would expect from jet flow, where the center holds to greatest speed concentrations. The higher velocities when considering the x direction will also be seen near the nozzle exit where X/D=0. This is consistent with gradual decay due to momentum diffusion, friction, and other sources of entropy in the environment. This is also observable further to the right of the image, where areas that were of a higher velocity (yellow) on the left transfer to a lower velocity (in green) on the right.

This is conducive to diffusion of momentum, and matches expected theoretical data, since any jet should expect a drop off in energy as the distance from it to the source increases. The only source of potential error to be seen comes from the slight offsets in symmetry. This will be apparent in all contour figures and will not be addressed again until the conclusion.

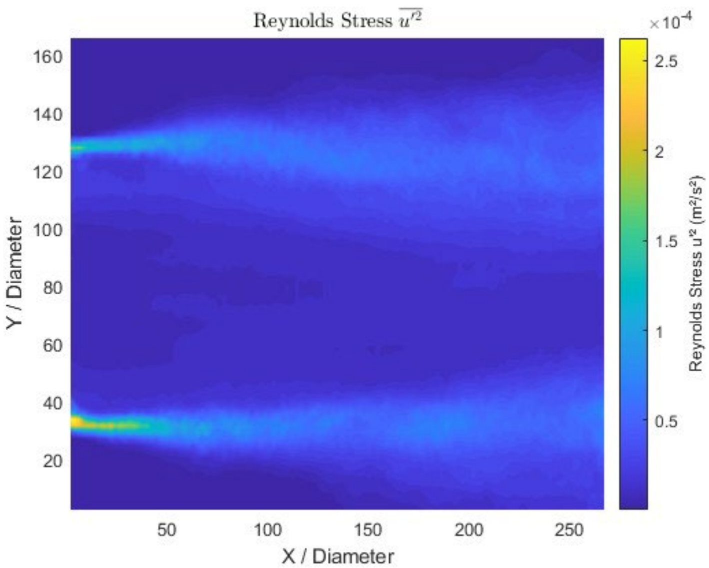

Fig 4: u-Velocity Reynolds Normal Stress

This plot shows the turbulence on the outer edges of the flow, which represents the shear layer of the jet flow, where the velocity gradients and turbulence are at their highest. As opposed to figure 3, the value decreases as you approach the centerline of the jet flow, and head upstream. This is expected, however some data may have been mixed in the calculations, as the stress is smaller than what would typically be considered optimal. This will be covered in the conclusion.

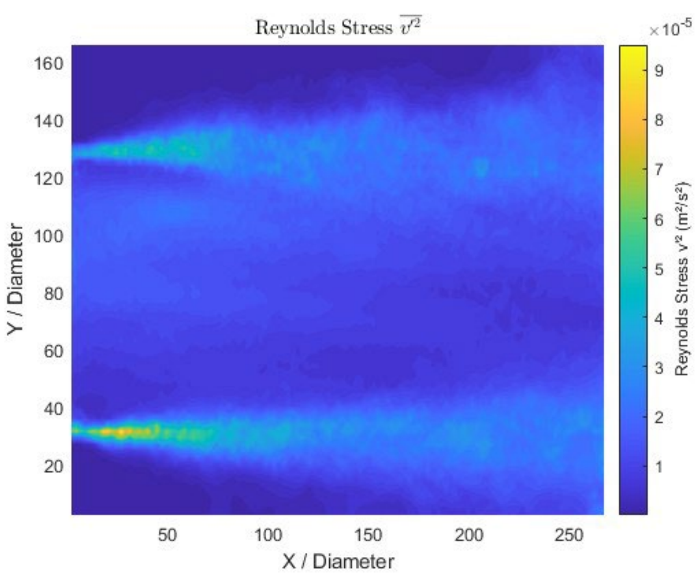

Fig 5: v-Velocity Reynolds Normal Stress

This figure is based on the same as that in figure 4, and behaves similarly in nearly all areas. One key difference is how the centerline is slightly brighter colored than that of the previous figure. This could likely be noted as asymmetry. To add to the previous mention of unit issues, it is likely that there isa simple unit error of x10^-3, which, if taken away from the calculations, would yield far more realistic results. Luckily, the behavior of the graph is correct, and it is more likely this is a small error than anything to be concerned over.

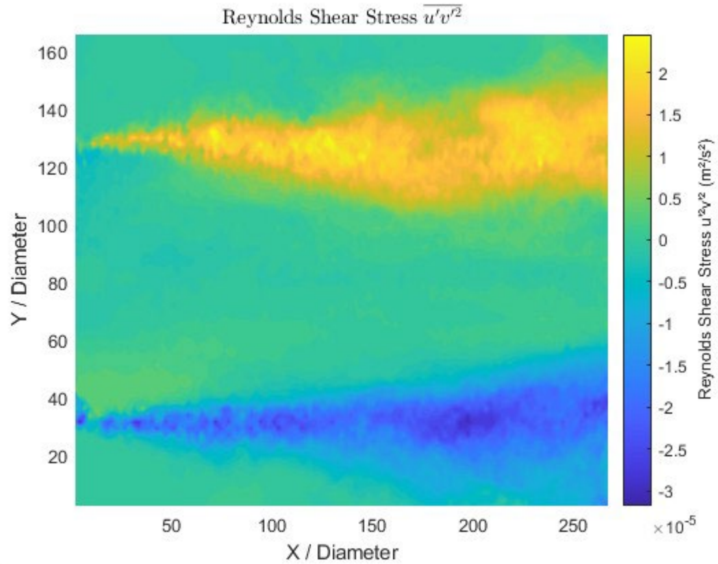

Fig 6: Reynolds Shear Stress

This shear stress figure represents the momentum transport due to turbulence in the system. Ideally, it should be opposing the centerline, with both positive and negative sections which represent the turn of fluid around the jet. Once again, the data is subject to potential magnitude error, however the observation from the plot shows that the trend is accurate, as you have both positive and negative highlights of shear stress that oppose the centerline trends. The flow grows bigger further to the right, away from the exit, since the turbulence in the system increases.

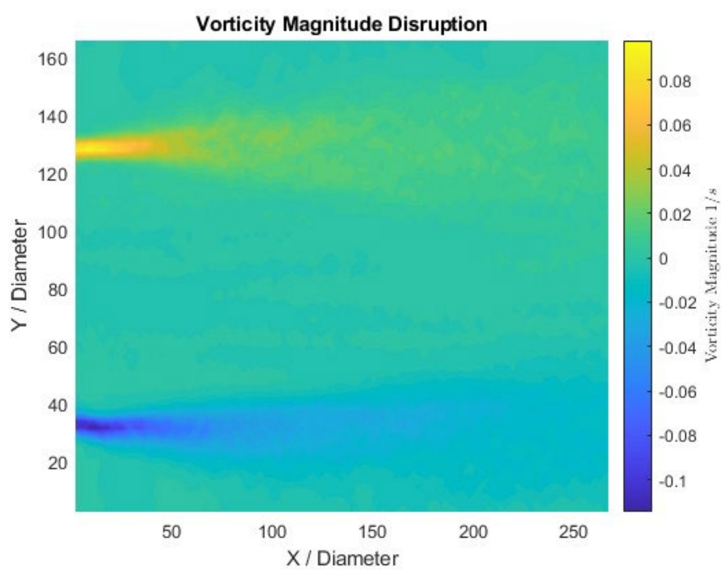

Fig 7: Vorticity Magnitude Disruption

This figure displays the vorticity in the system, and the rotational structures that form within the shear layer. This is caused by a sync of turbulence when velocities are at their highest. Again, there may be error in the data from some unknown error of magnitude. Without that concern, this is a simple graph that makes sense conceptually, as it would match the shear stress in both location and direction, and vorticity would be smaller than the shear stress since it takes place within it.

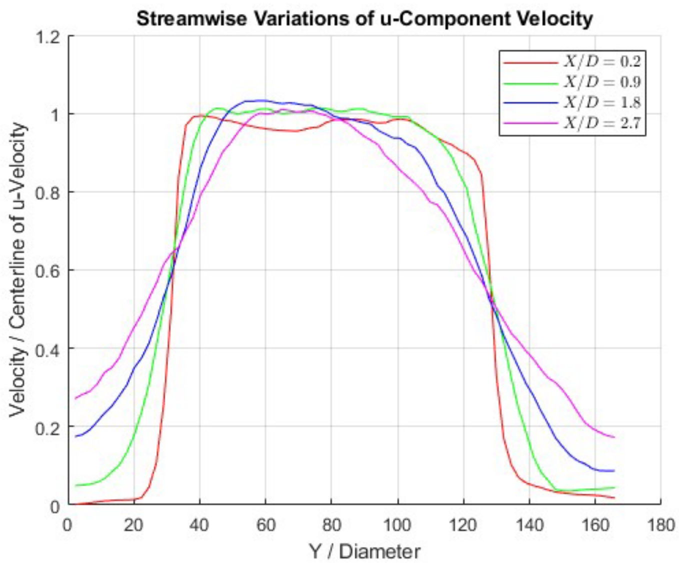

Fig 8: Streamwise Variations of u-Component Velocity

Beyond contour maps, we can observe the streamwise variations of the mean velocity in each component. At each profile, there is jet spreading along the centerline and decaying as the flow goes downstream. This is clear to notice in this figure, where the closer to the exit the observation is, the more dramatic the change in average the velocity is, and that as you move from the exit to the flow, you can observe smaller speeds, and less dramatic changes in states.

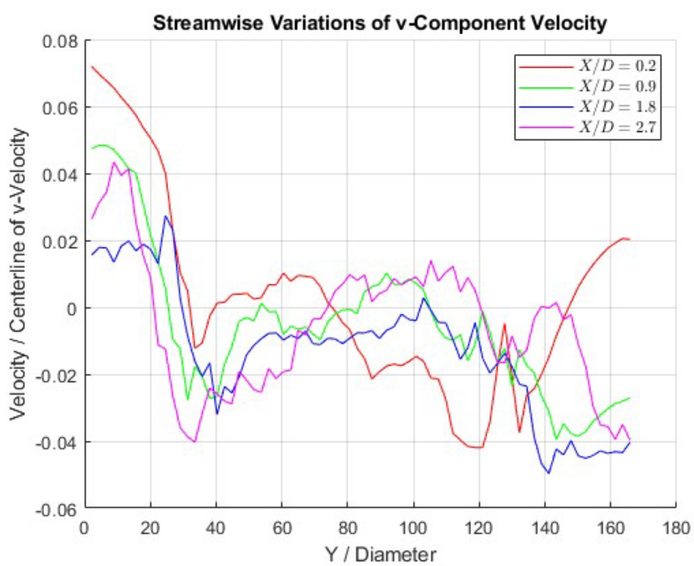

Fig 9: Streamwise Variations of v-Component Velocity

The trend from figure 8 can also be observed in figure 9. It is easy to note that the closer to the exit, the more dramatic the changes in average velocity, and the further from the exit, the less capable to absorb the energy being imparted from the jet. This figure is more prone to noise it seems, which could have to do with the direction of the flow versus the measured direction. Meaning, that since the flow moves horizontally and this displays vertical speeds, it makes sense that the range in numbers is far smaller than that of figure 8, and in addition to that, it would have substantially more noise.

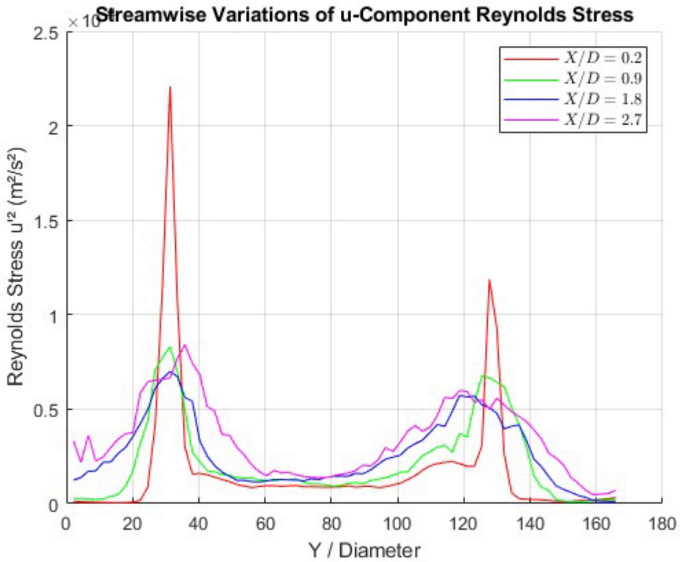

Fig 10: Streamwise Variations of u-Component Reynolds Stress

The turbulence intensity can be observed from the graphing of Reynolds stress. In this case, there are two peaks in turbulence. On either end of the nozzle, there is a clearly observable increase in the stress in the flow. This is a sign of the separation and turbulence being generated around the nozzle where the slow, static fluid meets the high-speed jet fluids. Beyond those two peaks, the flow dissipates, which should be reasonable as the flow is not exposed to the turbulence of the flow. It also would explain how the smaller X/D values have higher peaks, since there is more stress when the flow is at its fastest, but would also quickly drop off, since the barrier between the speeds is smaller from the lack of turbulence.

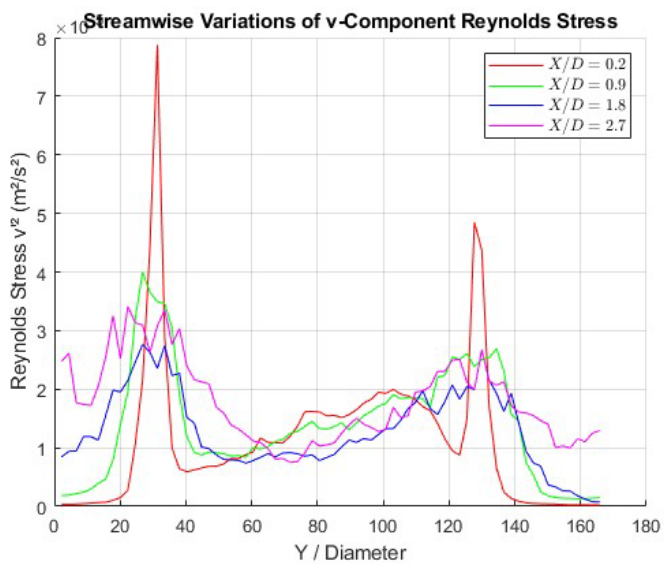

Fig 11: Streamwise Variations of v-Component Reynolds Stress

This plot also relays all of the same observations found in figure 10. Similarly, there is more noise and a smaller range. This is similar to my description on figure 9 about how the smaller range from v component graphing leads to greater noise. This noise will also be covered within the conclusion, but as far as this plot goes it covers the same concepts as figure 10, only slightly less clearly.

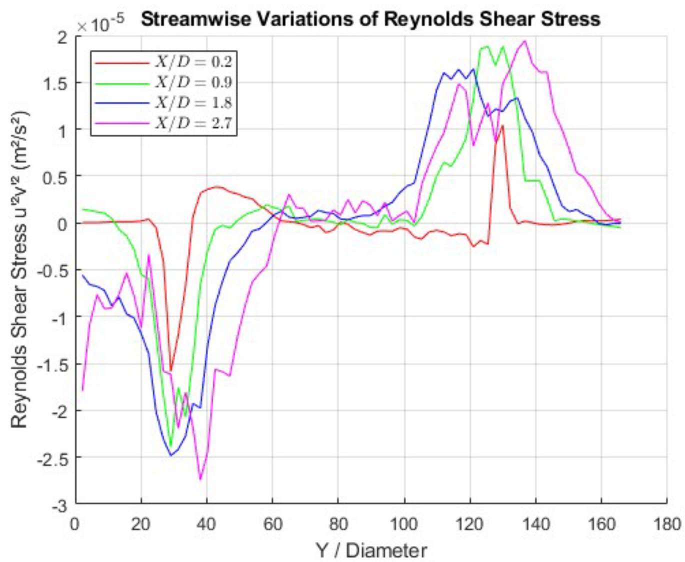

Fig 12: Streamwise Variations of Reynolds Shear Stress

From the figure we observe a drop in shear stress, a leveling out, a raise in shear stress, and future normalization. This pairs with the previous figures, where there are noticeable changes in the behavior at the outer sides of the flow. The drop compared to the rise in shear stress is the same behavior observed in figure 6, where there was negative and positive shear stress based on the direction the movement was based on. This is another of the last few figures being alternative depictions of the figures in the beginning of the theory section. This has been helpful in seeing how the flow changes as it gets further from the exit nozzle.

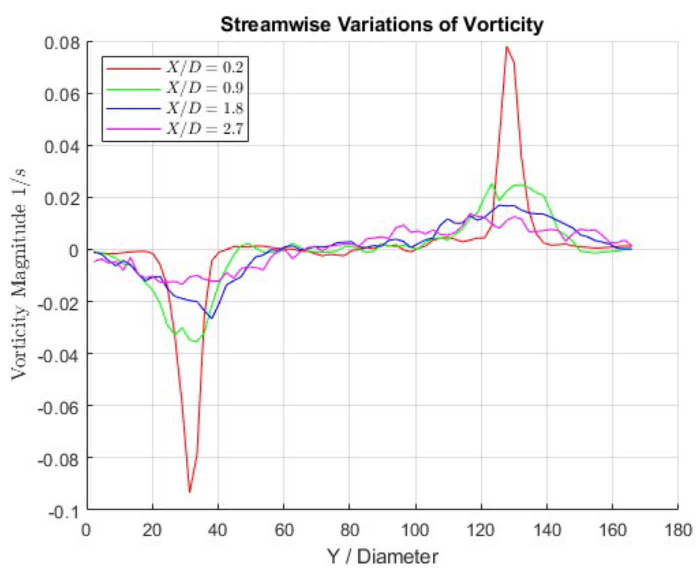

Fig 13: Streamwise Variations of Vorticity

From the final figure we can see a similar pattern to that of figure 12, and equally to figure 7. In both of these, there is a positive and negative vorticity based on the direction into and out of the 2D plane. This shows that there is a direction of flow around the higher speed funnel coming from the nozzle.

The near field region is described in figures 8-13, and analyzes the flow for small X/D values. To start, the mean velocity for near field regions remain nearly constant at both the centerline and the outer diameter of the flow, because there is not enough time to resist the flow and cause a major difference in velocity, i.e. there is a lack of momentum diffusion from the shear layer to majorly disrupt the flow. There is a spreading rate to the shear layer, as the flow spreads out due to the diffusion. This spreading rate is linear with the distance from the nozzle, and can be observed in figure 3.

The normal and shear stresses have maximum and minimum locations at ± 0.5 y/D from the center of the nozzle, which lines up with the shear layer. The flow tapers off in both sets of figures, and this is due to the decay from the diffusion as the turbulence moves along the centerline, meaning the stresses get smaller as the difference in speeds converge to zero.

V. Conclusion

This paper studies the use of PIV technology with a turbulent round jet in water. From this, we learned about flow development, and how velocity, shear and normal stresses, as well as vorticity change down the line of flow in a jet stream. This also identified the turbulence characteristics from the figures displayed in the theory section. This paper revealed the points of peak stresses of $\pm$0.5 y/D from the center of the nozzle, and how the stresses diminish as the distance from the exit decreases and the speed diminishes, as well has how the vorticity roll-up occurs, which drives the momentum. Some of the issues that arose from this lab that resulted in non-optimal data would be the unsymmetric seeding witnessed in the figures, resulting in imperfect comparisons between either side of the nozzle, where the upper end would seem to have higher velocity, vorticity, and forces compared to the lower side. In addition, there may have been constraints in the number of samples taken, where more seconds would have resulted in more reliable data. Also, there was the issue with being offset by a few magnitudes, due to some post processing calculation errors, which resulted in the figures being off. The data agrees with the Navier-Stokes equations supplied by the professor for the theory behind the lab.

Design Recommendations

There does not appear to be too much that can be done about error, as all of the substantial sources of error would come from the computer program which identifies the movement of particles. If there were to be some changes that could help minimize the error, It might be better to increase the number of samples in order to have a more accurate average. Multiple cameras running the program separately would be useful to check for small sources of error. In addition, having different angles recorded can avoid possible changes in the data from the geometry of the tank. Potentially having more easily noticed particles by the camera could be handy as well. Overall, I would say there is very little actual error, and what could be done about it would be extremely complicated.

Appendix

A. Raw Data

Index	x (mm)	y (mm)	U (m/s)	V (m/s)
1	1.229	1.229	-0.00212	0.001343
2	1.341	1.229	-0.00183	0.001362
3	1.454	1.229	-0.00199	0.001304
4	1.566	1.229	0.003514	-0.00266
5	1.678	1.229	9.44E-05	0.000783

Table 3: Raw data used in sample calculations.

B. Lab Code

```
%% Read Data
if ~exist('data','var') % Check if data read already

    winsize = [119 74]; % Size of vector grid
    L = winsize(1)*winsize(2); % Get size of vector field
```

```
    N = 300; % Number of files (time steps)

    data = zeros(L,11,N); % Pre-allocate

    for i = 1:N
        fprintf('Now Retrieving File %d of 300\n',i);
        file = sprintf('EduPIV_lab.62tbxosb.%06d.csv',i-1);
        data(:,:,i) = readmatrix(file); % Save each file to data
    end
end

%% Parse Data
s = 1;
t = 1;
u = zeros(119,74,300); v = u;
u_raw = u; u_pix = u;
v_raw = v; v_pix = v; % Pre-Allocate
x = u; y = v;
N = 300;
for i = 1:N
    fprintf('Now Parsing Set %d of 300\n',i); % Status display
    for q = 1:8806
        if data(q,11,i) ~= 0
            u(s,t,i) = nan; % Replace flagged vectors with nan
            v(s,t,i) = nan;
            u_pix(s,t,i) = nan;
            v_pix(s,t,i) = nan;
        else
            u(s,t,i) = data(q,9,i); % Parse valid vectors
            v(s,t,i) = data(q,10,i);
            u_pix(s,t,i) = data(q,7,i);
            v_pix(s,t,i) = data(q,8,i);
        end

        u_raw(s,t,i) = data(q,9,i);
        v_raw(s,t,i) = data(q,10,i);

        x(s,t,i) = data(q,5,i); % Record grid
        y(s,t,i) = data(q,6,i);

        s = s + 1; % Move to next row

        if mod(q,winsize(1)) == 0 % Detect is done with column
            t = t + 1; % Move to next vector column
            s = 1; % Reset to first row
        end
        if q == L
            t = 1; % Finished with frame, reset t
%             figure(1) % Update only figure 1
%             contourf(x(:,:,i),y(:,:,i),sqrt(u(:,:,i).^2 +
v(:,:,i).^2),20,'LineStyle','none')
%             axis equal % Update cool animation of the flow
%             xlabel('x [cm]')
```

```
%              ylabel('y [cm]')
%              drawnow
        end
    end
end

X = x(:,:,1); % Grid doesn't change, so we just need a 2D matrix
Y = y(:,:,1);

clear file i q s t x y L

%% Data Processing
% Calculation
D = 0.05; % m
mu = 0.0010016;
rho = 998.23;
Xndm = X / D;
Yndm = Y / D;

uAvg = mean(u, 3, 'omitnan');
vAvg = mean(v, 3, 'omitnan');

uDif = u - uAvg;
vDif = v - vAvg;

uStr = mean(uDif .^ 2, 3, 'omitnan');
vStr = mean(vDif .^ 2, 3, 'omitnan');

uvStr = mean(uDif .* vDif, 3, 'omitnan');

[dudy, dudx] = gradient(uAvg, Y(1,:), X(:,1));
[dvdy, dvdx] = gradient(vAvg, Y(1,:), X(:,1));

vort = dvdx - dudy;

uex = uAvg(1, 12:59);
vex = vAvg(1, 12:59);

Ucenter = mean([uAvg(:,35), uAvg(:,36)], 'all');
Vcenter = mean([vAvg(:,35), vAvg(:,36)], 'all');

Uex = mean(sqrt(uex .^ 2 + vex .^ 2));

Re = rho * Uex * D / mu;

Ycenter = Y / D - (max(Y / D) - min(Y / D)) / 2; % Centered

%% Plots

figure(1)
hold on, grid on
contourf(Xndm, Yndm, uAvg, 50, 'LineColor', 'none')

colorbar;
```

```
ylabel(colorbar, "Time Averaged u-Velocity (m/s)")
xlabel('X / Diameter')
ylabel('Y / Diameter')
title('Time Averaged u-Velocity Contour Map')



figure(2)
hold on, grid on
contourf(Xndm, Yndm, uStr, 50, 'LineColor', 'none')

colorbar;
ylabel(colorbar, "Reynolds Stress u'² (m²/s²)")
xlabel('X / Diameter')
ylabel('Y / Diameter')
title('Reynolds Stress $\overline{u''^2}$', 'Interpreter', 'latex')



figure(3)
hold on, grid on
contourf(Xndm, Yndm, vStr, 50, 'LineColor', 'none')

colorbar;
ylabel(colorbar, "Reynolds Stress v'² (m²/s²)")
xlabel('X / Diameter')
ylabel('Y / Diameter')
title('Reynolds Stress $\overline{v''^2}$', 'Interpreter', 'latex')



figure(4)
hold on, grid on
contourf(Xndm, Yndm, uvStr, 50, 'LineColor', 'none')
ylabel(colorbar, "Reynolds Stress u'²v'² (m²/s²)")

colorbar;
ylabel(colorbar, "Reynolds Shear Stress u'²v'² (m²/s²)")
xlabel('X / Diameter')
ylabel('Y / Diameter')
title('Reynolds Shear Stress $\overline{u''v''^2}$', 'Interpreter',
'latex')



figure(5)
hold on, grid on
contourf(Xndm, Yndm, vort, 50, 'LineColor', 'none')

colorbar;
ylabel(colorbar, "Vorticity Magnitude $1/s$", 'Interpreter', 'latex')
xlabel('X / Diameter')
ylabel('Y / Diameter')
title('Vorticity Magnitude Disruption')
```

```
figure(6)
hold on, grid on
plot(Ycenter(1,:), uAvg(10,:) / Ucenter, 'r')
plot(Ycenter(1,:), uAvg(40,:) / Ucenter, 'g')
plot(Ycenter(1,:), uAvg(80,:) / Ucenter, 'b')
plot(Ycenter(1,:), uAvg(119,:) / Ucenter, 'm')

xlabel("Y / Diameter")
ylabel("Velocity / Centerline of u-Velocity")
title("Streamwise Variations of u-Component Velocity")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex");



figure(7)
hold on, grid on
plot(Ycenter(1,:), vAvg(10,:) / Ucenter, 'r')
plot(Ycenter(1,:), vAvg(40,:) / Ucenter, 'g')
plot(Ycenter(1,:), vAvg(80,:) / Ucenter, 'b')
plot(Ycenter(1,:), vAvg(119,:) / Ucenter, 'm')

xlabel("Y / Diameter")
ylabel("Velocity / Centerline of v-Velocity")
title("Streamwise Variations of v-Component Velocity")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex");



figure(8)
hold on, grid on
plot(Ycenter(1,:), uStr(10,:), 'r')
plot(Ycenter(1,:), uStr(40,:), 'g')
plot(Ycenter(1,:), uStr(80,:), 'b')
plot(Ycenter(1,:), uStr(119,:), 'm')

xlabel("Y / Diameter")
ylabel("Reynolds Stress u'² (m²/s²)")
title("Streamwise Variations of u-Component Reynolds Stress")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex");



figure(9)
hold on, grid on
plot(Ycenter(1,:), vStr(10,:), 'r')
plot(Ycenter(1,:), vStr(40,:), 'g')
plot(Ycenter(1,:), vStr(80,:), 'b')
plot(Ycenter(1,:), vStr(119,:), 'm')
```

```
xlabel("Y / Diameter")
ylabel("Reynolds Stress v'² (m²/s²)")
title("Streamwise Variations of v-Component Reynolds Stress")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex");



figure(10)
hold on, grid on
plot(Ycenter(1,:), uvStr(10,:), 'r')
plot(Ycenter(1,:), uvStr(40,:), 'g')
plot(Ycenter(1,:), uvStr(80,:), 'b')
plot(Ycenter(1,:), uvStr(119,:), 'm')

xlabel("Y / Diameter")
ylabel("Reynolds Shear Stress u'²v'² (m²/s²)")
title("Streamwise Variations of Reynolds Shear Stress")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex", 'Location', 'northwest');



figure(11)
hold on, grid on
plot(Ycenter(1,:), vort(10,:), 'r')
plot(Ycenter(1,:), vort(40,:), 'g')
plot(Ycenter(1,:), vort(80,:), 'b')
plot(Ycenter(1,:), vort(119,:), 'm')

xlabel("Y / Diameter")
ylabel("Vorticity Magnitude $1/s$", 'Interpreter', 'latex')
title("Streamwise Variations of Vorticity")
legend("$X/D = 0.2$", "$X/D = 0.9$", "$X/D = 1.8$", "$X/D = 2.7$",
"Interpreter", "latex", 'Location', 'northwest');
```