Wyatt Whiting

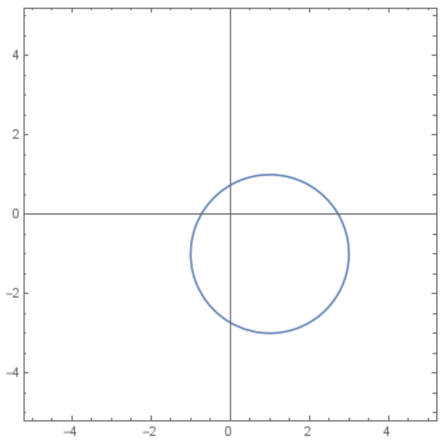
1. Write the following complex numbers in standard form $a+ib$
   (a) $(2+i)(3+4i) = 2+11i$.
   (b) $(1+2i)^4 = -7-24i$.
   (c) $\frac{1+i}{2-3i} = -\frac{1}{13} + \frac{5}{13}i$
   (d) $\frac{i+a}{i-a} = \frac{i^2+2ai+a^2}{i^2-a^2} = \frac{(a^2-1)+2ai}{-a^2-1} = \frac{a^2-1}{-a^2-1} + \frac{2a}{-a^2-1}i$
2. Write in polar form:
   (a) $2i = 2\text{cis}(\frac{\pi}{2})$
   (b) $1+i = \sqrt{2}\text{cis}(\frac{\pi}{4})$
   (c) $-3+\sqrt{3}i = \sqrt{12}\text{cis}(\frac{5\pi}{6})$
   (d) $-i = \text{cis}(\frac{-\pi}{2})$
   (e) $(2-i)^2 = 5\text{cis}(-\arctan(\frac{4}{3}))$
   (f) $|3-4i| = 5\text{cis}(0)$
   (g) $\sqrt{5}-i = \sqrt{6}\text{cis}(-\arctan(\frac{1}{\sqrt{5}}))$
   (h) $(\frac{1-i}{\sqrt{3}})^4 = -\frac{4}{9}\text{cis}(0)$
3. Write in rectangular form:
   (a) $\sqrt{2}e^{i\frac{3\pi}{4}} = -1+i$
   (b) $34e^{i\frac{\pi}{2}} = 0+34i$
   (c) $-e^{i250\pi} = -1+0i$
   (d) $2e^{4\pi i} = 2+0i$
4. Find all complex solutions (written in standard form) of the following equations.
   (a) $2z^2+2z+5=0 \implies z_1 = -\frac{1}{2}+\frac{3}{2}i, z_2 = -\frac{1}{2}-\frac{3}{2}i$
   (b) $5z^2+4z+1=0 \implies z_1 = -\frac{2}{5}+\frac{1}{5}i, z_2 = -\frac{2}{5}-\frac{1}{5}i$
   (c) $z^2+2z+1-i=0 \implies z_1 = -1, z_2 = -\frac{1+i}{\sqrt{2}}-1$
   (d) $z^4 = z \implies z^4-z=0 \implies z_1 = 0, z_2 = 1, z_3 = -\frac{1}{2}+\frac{\sqrt{3}}{2}i, z_4 = -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(e) $z^4 - z^2 + 4 = 0 \implies z_1 = \frac{5}{2\sqrt{5}} + \frac{\sqrt{3}}{2}i, z_2 = -\frac{5}{2\sqrt{5}} - \frac{\sqrt{3}}{2}i, z_3 = \frac{5}{2\sqrt{5}} - \frac{\sqrt{3}}{2}i, z_4 = -\frac{5}{2\sqrt{5}} + \frac{\sqrt{3}}{2}i$

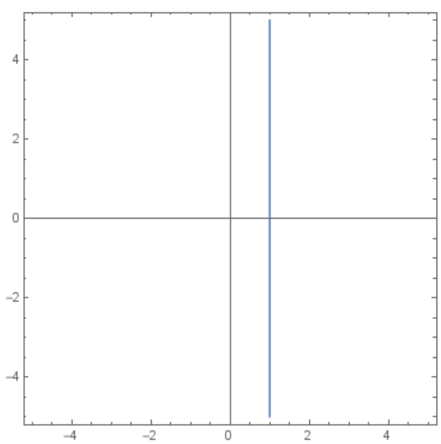
(f) $z^6 - z^3 - 2 = 0 \implies z_1 = -1 + 0i, z_2 = 2^{1/3} + 0i, z_3 = -\frac{1}{2^{2/3}} - \frac{\sqrt{3}}{2^{2/3}}i, z_4 = \frac{1}{2} + \frac{\sqrt{3}}{2}i, z_5 = \frac{1}{2} - \frac{\sqrt{3}}{2}i, z_6 = -\frac{1}{2^{2/3}} + \frac{\sqrt{3}}{2^{2/3}}i$

5. Problem 1.23, (a), (c), (d), (h)

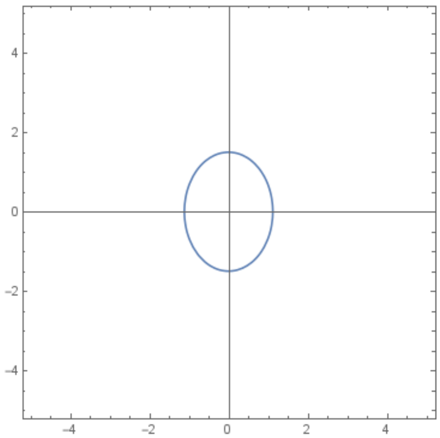
(a) $\{z \in \mathbb{C} : |z - 1 + i| = 2\}$



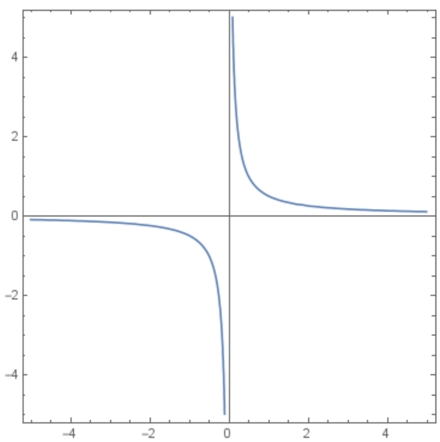
(c) $\{z \in \mathbb{C} : \text{Re}(z + 2 - 2i) = 3\}$



(d) $\{z \in \mathbb{C} : |z - i| + |z + i| = 3\}$

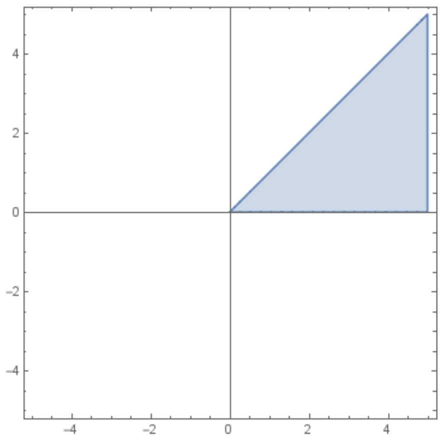


(h) $\{z \in \mathbb{C} : \text{Im}(z^2) = 1\}$

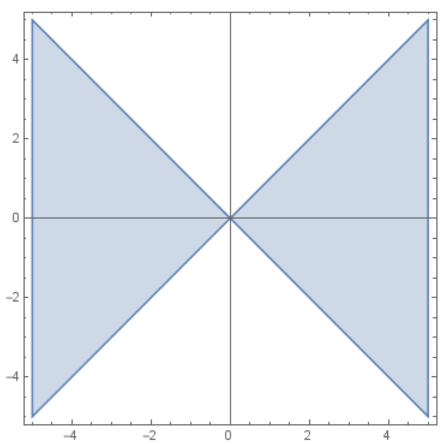


6. Sketch the following sets on the complex plane.

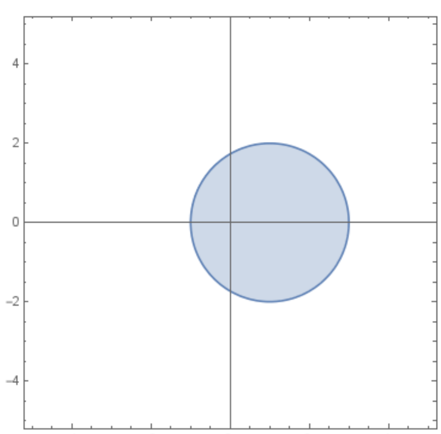
   (a) $0 \leq \arg z \leq \frac{\pi}{4}$



(b) $\mathrm{Re}(z^2) > 0$



(c) $0 < |z - 1| < 2$



(d) $|z| \leq |z-4|$