作业 1:判别下列集合对所定义的运算是否构成 R 上的线性空间，并说明原因。

(1)n 阶实对称矩阵的集合，对于矩阵的加法和实数与矩阵的乘法。

(2)平面上全体向量的集合，对于通常的加法和如下定义的数乘运算 kx=0.

Q_1：有 $A^T = A \quad B^T = B$

证 1： $\Rightarrow (A+B)^T = A+B$ (封闭)

$1^\circ \ (A+B)^T = A^T + B^T = A+B$

或 $2^\circ \ (A+B)_{ij} = a_{ij} + b_{ij} = a_{ji} + b_{ji} = (A+B)_{ji} = (A+B)^T$

证 2：$\forall k \in R \quad (kA)^T = kA$ (乘法唯一封闭)

$kA = k \cdot a_{ij} = k \cdot a_{ji} = k \cdot A^T = (kA)^T$

八性质：(1) $1^\circ \ A+B = B+A \qquad 2^\circ \ A+(B+C) = (A+B)+C$

$3^\circ \ A+O = A \qquad 4^\circ \ A+(-A) = 0$

(2). $1^\circ \ E \cdot A = A \qquad 2^\circ \ k \cdot (l \cdot A) = k \cdot l(A)$

$3^\circ \ (k+l) \cdot A = kA + lA \qquad 4^\circ \ k(A+B) = kA + kB$

(也是相等矩阵一种，一定成立)

Q_2. 否. 举反例：$1 \cdot x = (k=1) \cdot x = 0 \neq x$. 违反条件

作业 2(提高题选做)设数域为 R，集合为 VV $= \{\alpha | \alpha = ($…

判断 V 是否构成 R 上的线性空间。

1. 矩阵分析.　　构式、名制 证明符合　　Date.

Q1：加法：设 $\alpha=(x_1, y_1)$, $\beta=(x_2, y_2)$, $\gamma=(x_3, y_3)$

(1) $\alpha \oplus \beta = (x_1+x_2,\ y_1+y_2+x_1x_2)$

$= (x_2+x_1,\ y_2+y_1+x_2x_1) = \beta \oplus \alpha$

(2) 1° $(\alpha \oplus \beta) \oplus \gamma = (x_1+x_2,\ y_1+y_2+x_1x_2) \oplus \gamma$

$= (x_1+x_2+x_3,\ y_1+y_2+y_3+x_1x_2+x_3(x_1+x_2))$　"="

2° $\alpha \oplus (\beta \oplus \gamma) = \alpha \oplus (x_2+x_3,\ y_2+y_3+x_2x_3)$

$= (x_1+x_2+x_3,\ y_1+y_2+y_3+x_2x_3+x_1(x_2+x_3))$

(3) $0 = (0, 0)$　$0+\alpha = \alpha$

(4) $-\alpha = (-x_1, -y_1)$　$\alpha + (-\alpha) = 0$

乘法：(1) $1 = (1, 1)$　$1\cdot\alpha = (1\cdot x_1, 1\cdot y_1) = \alpha$

(2). 1°. $k\cdot(t\cdot\alpha) = k\cdot[t\cdot x_1,\ t\cdot y_1 + \frac{1}{2}t(t-1)\cdot x_1^2]$

$= \{k t\cdot x_1,\ k\cdot[t\cdot y_1 + \frac{1}{2}t(t-1)x_1^2]$

$+ \frac{1}{2}k\cdot(k-1)\cdot(t\cdot x_1)^2]$

$= k\cdot t\cdot y_1 + \frac{kt}{2}(t-1)x_1^2 + \frac{kt^2}{2}(k-1)x_1^2$

$= kty_1 + \frac{kt x_1^2}{2}(tk-1)$

2°. $(k\cdot t)\cdot\alpha = kty_1 + \frac{kt x_1^2}{2}(tk-1)$

(3) $k \odot (\alpha \oplus \beta) = k \odot \alpha \oplus k \odot \beta$

1° $k \odot (\alpha \oplus \beta) = k \odot (x_1+x_2,\ y_1+y_2+x_1x_2)$

$= \{k(x_1+x_2),\ k(y_1+y_2+x_1x_2) + \frac{1}{2}k(k-1)(x_1+x_2)^2\}$

$ky_1+ky_2+kx_1x_2+\frac{1}{2}k(k-1)x_1^2+k(k-1)x_1x_2+\frac{1}{2}k(k-1)x_2^2$

$= k(y_1+y_2) + k^2x_1x_2 + \frac{1}{2}k(k-1)(x_1^2+x_2^2)$　"="

2° $k \odot \alpha = (k\cdot x_1,\ ky_1 + \frac{1}{2}k(k-1)\cdot x_1^2)$

$\Rightarrow k \odot \alpha \oplus k \odot \beta = \{kx_1 + kx_2,\ [ky_1 + ky_2 + \frac{1}{2}k(k-1)x_1^2$

$+ \frac{1}{2}k(k-1)x_2^2 + kx_1\cdot kx_2$

$$\begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$

$$2 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = 6$$

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(4). $(k+t) \odot \alpha = k \odot \alpha \oplus t \odot \alpha$

1°. $(k+t) \odot \alpha = [(k+t) \cdot x_1, (k+t) \cdot y_1 + \frac{1}{2}(k+t)(k+t-1) \cdot x_1^2]$

2° $k \odot \alpha = (k x_1, k y_1 + \frac{1}{2}k(k-1) \cdot x_1^2)$

$t \odot \alpha = (t x_1, t y_1 + \frac{1}{2} t(t-1) \cdot x_1^2)$

$\Rightarrow [k x_1 + t x_1, k y_1 + \frac{1}{2}k(k-1) x_1^2 + t y_1 + \frac{1}{2} t(t-1) x_1^2 + k x_1 \cdot t x_1$

$(k+t) \cdot y_1 + \frac{1}{2} k(k-1) + t(t-1) \cdot x_1^2 + k t x_1^2$

$= \frac{1}{2}[k(k-1) + t(t-1)] + kt$

$= \frac{1}{2} k^2 t + t^2 - t + 2kt$

$= \frac{1}{2}[(k+t)^2 - (k+t)] = \frac{1}{2}(k+t)(k+t-1)$

作业 1:设四维线性空间 V 的基(I)x1,x2,x3,x4 和(II)y1,y2,y3,y4 满足…

Class 2.

作业 1：(1) 题. 将 y 用 x 表示

$y_3 = x_1 + 2x_2$

$y_4 = x_2 + 2x_3$

$y_2 = x_4 - 2y_3 = x_4 - 2(x_1 + 2x_2) = x_4 - 2x_1 - 4x_2$

$y_1 = x_3 - 2y_2 = x_3 - 2x_4 + 4x_1 + 8x_2$

则 $C = \begin{bmatrix} 4 & -2 & 1 & 0 \\ 8 & -4 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ -2 & 1 & 0 & 0 \end{bmatrix}$

(2) $2y_1 = \begin{matrix} 8 \\ 16 \\ 2 \\ -4 \end{matrix}$ $\quad -y_2 = \begin{matrix} 2 \\ 4 \\ 0 \\ 2 \end{matrix}$ $\quad y_3 = \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$ $\quad y_4 = \begin{matrix} 0 \\ 1 \\ 2 \\ 0 \end{matrix}$

$\Rightarrow x = \begin{bmatrix} 11 \\ 23 \\ 4 \\ -2 \end{bmatrix}$

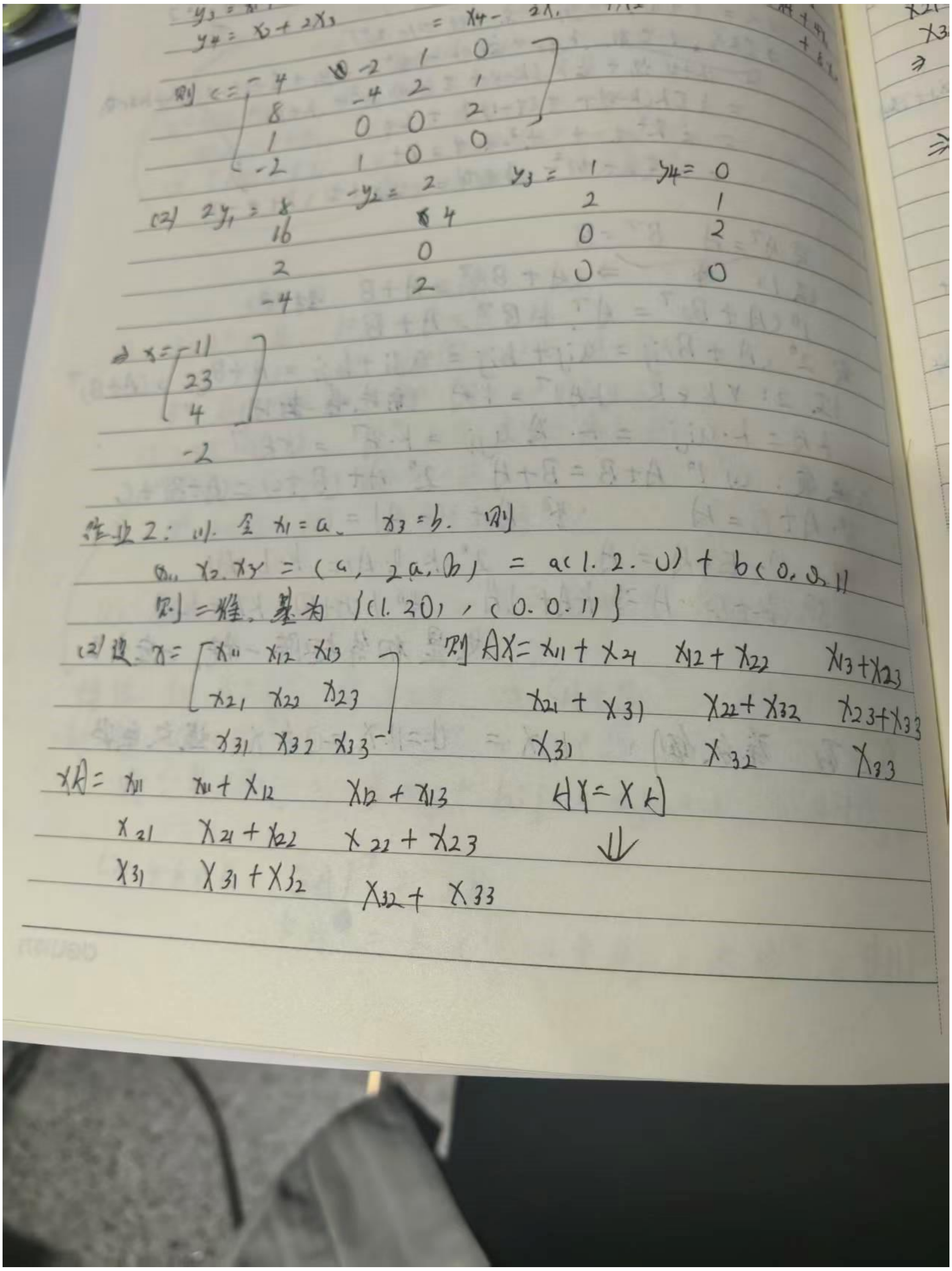
作业 2：(1). 令 $x_1 = a$, $x_3 = b$. 则

$x_1, x_2, x_3 = ($ [illegible]

作业 2:

(1) $V_1=\{(x_1,x_2,x_3)|2x_1-x_2=0\}$, 求 V_1 的基与维数。

(2)(提高题选做) $V_2=\{x|AX=XA,X\in R^{3\times3}\}$，其中 A=⋯



$x_{11} + x_{21} = x_{11}$

$x_{11} + x_{12} = x_{12} + x_{22}$ $\quad$ $x_{13} + x_{23} = x_{12} + x_{13}$

$x_{21} = x_{21} + x_{31}$ $\quad$ $x_{22} + x_{32} = x_{21} + x_{22}$ $\quad$ $x_{23} + x_{33} = x_{22} + x_{23}$

$x_{31} = x_{31}$ $\quad$ $x_{32} = x_{31} + x_{32}$ $\quad$ $x_{33} = x_{22} + x_{33}$

$\Rightarrow x_{21} = 0$ $\quad$ $x_{11} = x_{22}$ $\quad$ $x_{12} = x_{23}$

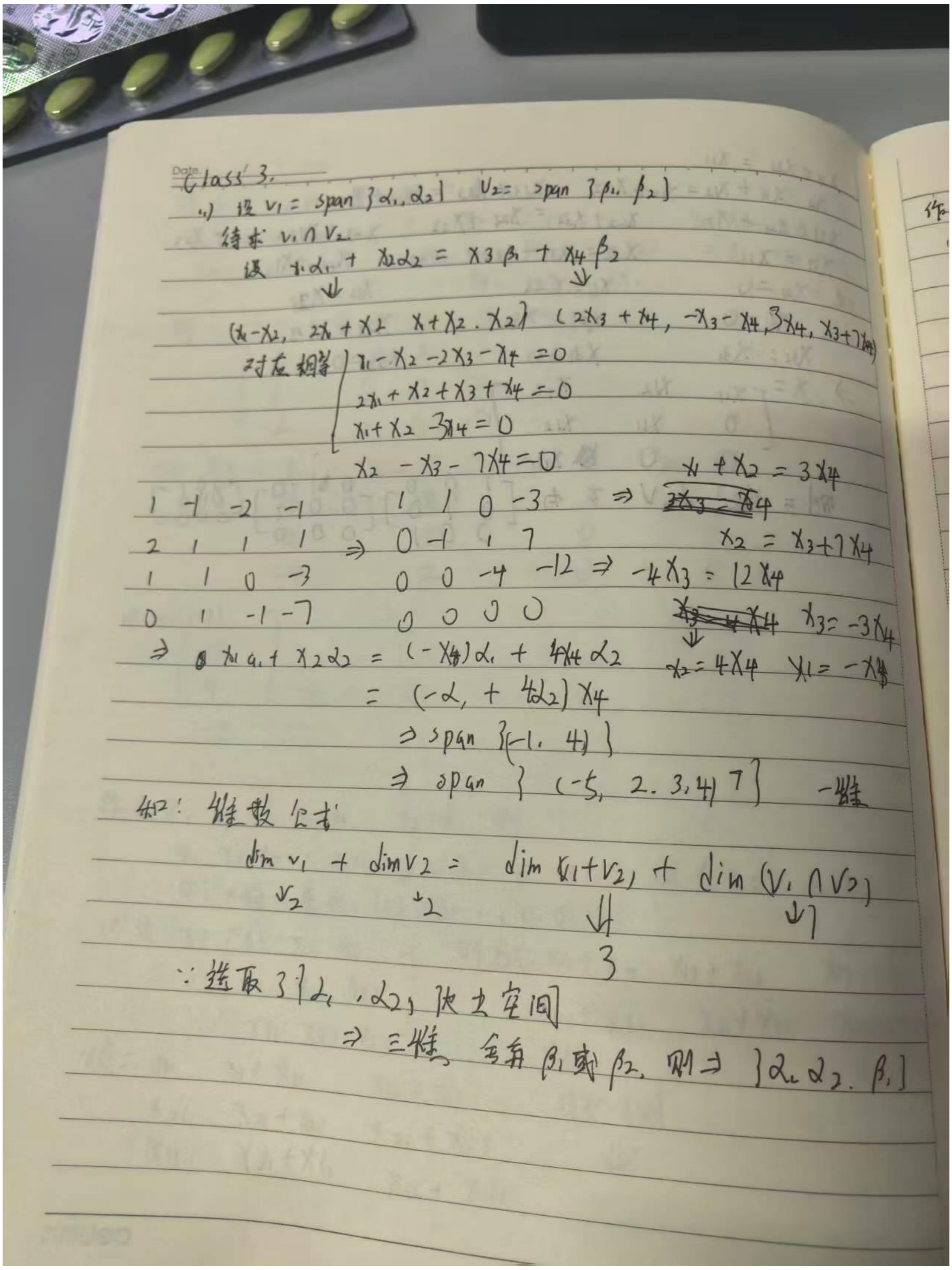
$x_{31} = 0$ $\quad$ $x_{21} = x_{32}$ $\quad$ $x_{22} = x_{33}$

$x_{31} = x_{31}$ $\quad$ $x_{31} = 0$ $\quad$ $x_{22} = 0$

$$\Rightarrow X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{11} & x_{12} \\ 0 & 0 & x_{11} \end{bmatrix}$$

则三维 $\dim V = 3$ 为 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

作业 3：已知…求$span(\alpha_1, \alpha_2)$和$span(\beta_1, \beta_2)$的和与交的基于维数



Class 3.

(1) 设 $V_1 = span\{\alpha_1, \alpha_2\}$ $V_2 = span\{\beta_1, \beta_2\}$

待求 $V_1 \cap V_2$

设 $x_1\alpha_1 + x_2\alpha_2 = x_3\beta_1 + x_4\beta_2$

$(x_1 - x_2, 2x_1 + x_2, x_1 + x_2, x_2)$ $(2x_3 + x_4, -x_3 - x_4, 3x_4, x_3 + 7x_4)$

对应相等
$$\begin{cases} x_1 - x_2 - 2x_3 - x_4 = 0 \\ 2x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_4 = 0 \\ x_2 - x_3 - 7x_4 = 0 \end{cases}$$

$$\begin{matrix} 1 & -1 & -2 & -1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 0 & -3 \\ 0 & 1 & -1 & -7 \end{matrix} \Rightarrow \begin{matrix} 1 & 1 & 0 & -3 \\ 0 & -1 & 1 & 7 \\ 0 & 0 & -4 & -12 \\ 0 & 0 & 0 & 0 \end{matrix} \Rightarrow$$

$x_1 + x_2 = 3x_4$

~~$2x_3 = 7x_4$~~

$x_2 = x_3 + 7x_4$

$\Rightarrow -4x_3 = 12x_4$

~~$x_3 = 4x_4$~~ $x_3 = -3x_4$

$x_2 = 4x_4$ $x_1 = -x_4$

$\Rightarrow x_1\alpha_1 + x_2\alpha_2 = (-x_4)\alpha_1 + 4x_4\alpha_2$

$= (-\alpha_1 + 4\alpha_2)x_4$

$\Rightarrow span\{(-1, 4)\}$

$\Rightarrow span\{(-5, 2, 3, 4)^T\}$ 一维

和：维数公式

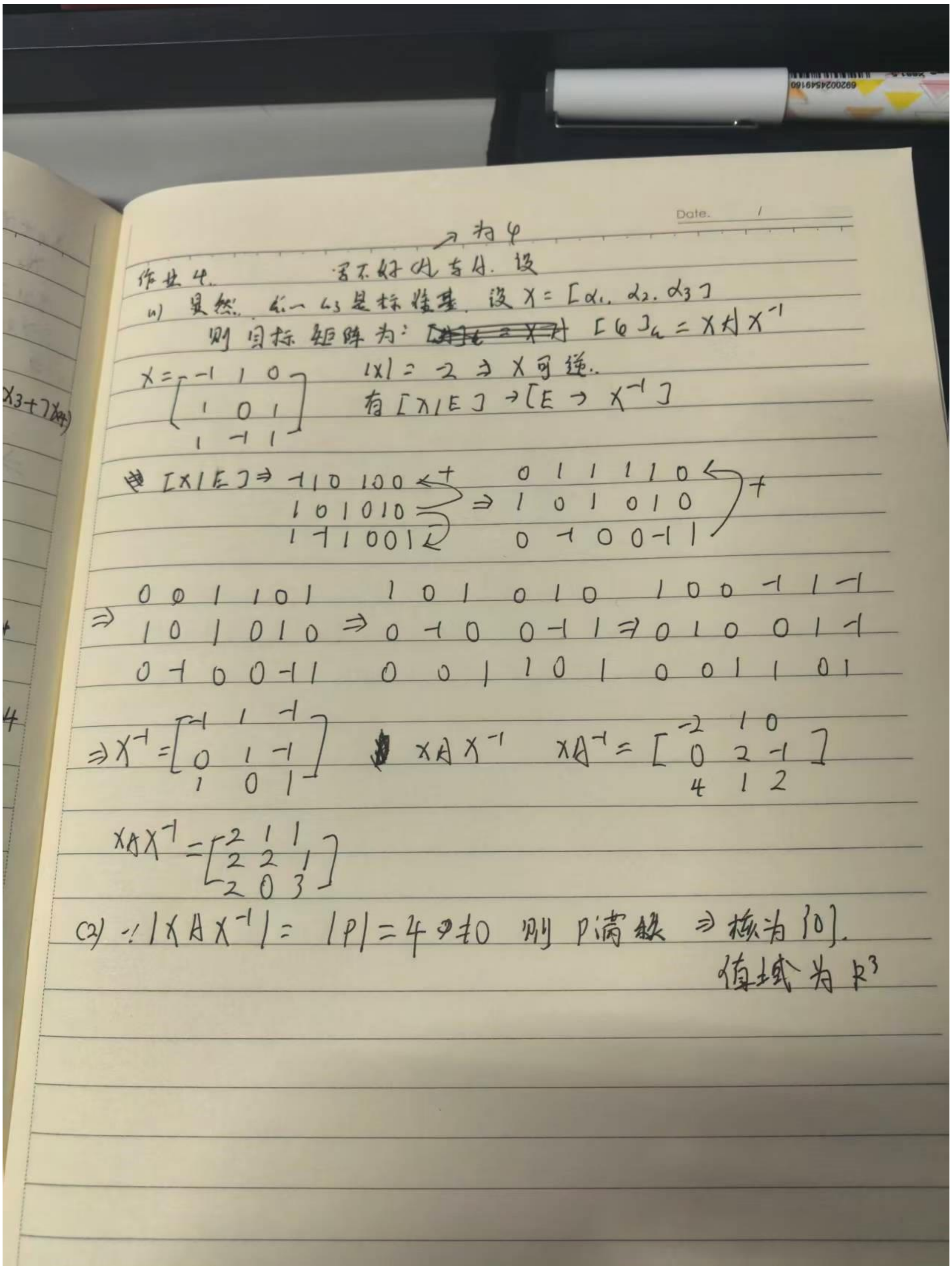
$\dim V_1 + \dim V_2 = \dim(V_1 + V_2) + \dim(V_1 \cap V_2)$

$\dim V_1 = 2$, $\dim V_2 = 2$, $\dim(V_1+V_2) = 3$, $\dim(V_1 \cap V_2) = 1$

∵ 选取 $\{\alpha_1, \alpha_2\}$ 作为空间

$\Rightarrow$ 三维，舍弃 β_1 或 β_2，则 $\Rightarrow \{\alpha_1, \alpha_2, \beta_1\}$

作业 4：…求 A 的值域与核



作业4. 写不好ψ为A. 设

(1) 显然 ε_1 — ε_3 是标准基. 设 $X=[\alpha_1, \alpha_2, \alpha_3]$

则对应矩阵为: ~~$[A]_\varepsilon = XA$~~ $[\varphi]_\alpha = XAX^{-1}$

$$X=\begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$|X| = -2 \Rightarrow X$ 可逆.

有 $[X|E] \to [E \to X^{-1}]$

$$[X|E] \Rightarrow \begin{matrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{matrix} \Rightarrow \begin{matrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{matrix}$$

$$\Rightarrow \begin{matrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{matrix} \Rightarrow \begin{matrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix} \Rightarrow \begin{matrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix}$$

$$\Rightarrow X^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

XAX^{-1} $XA^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 2 & -1 \\ 4 & 1 & 2 \end{bmatrix}$

$$XAX^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

(2) $\because |XAX^{-1}| = |P| = 4 \neq 0$ 则 P满秩 $\Rightarrow$ 核为 $\{0\}$.

值域为 R^3