作业1. 求 $\lambda E-A$

$$A=\begin{pmatrix}4 & 6 & 0\\ -3 & -5 & 0\\ -3 & -6 & 1\end{pmatrix}\Rightarrow \lambda E-A=\begin{pmatrix}\lambda-4 & -6 & 0\\ 3 & \lambda+5 & 0\\ 3 & 6 & \lambda-1\end{pmatrix}$$

$\because x_{21}=3$、$x_{12}=-6$ 常数 $\Rightarrow$ $\therefore D_1(\lambda)=1$

$$D_3(\lambda)=\det(\lambda E-A)=\begin{vmatrix}\lambda-4 & -6 & 0\\ 3 & \lambda+5 & 0\\ 3 & 6 & \lambda-1\end{vmatrix}=(\lambda-1)\cdot\begin{vmatrix}\lambda-4 & -6\\ 3 & \lambda+5\end{vmatrix}$$

$$=(\lambda-1)^2(\lambda+2)$$

$D_2(\lambda)$ 1° $m_{33}=\begin{vmatrix}\lambda-4 & -6\\ 3 & \lambda+5\end{vmatrix}=(\lambda+2)(\lambda-1)$

$\Rightarrow D_2(\lambda)=(\lambda-1)$

2° $m_{13}=\begin{vmatrix}3 & \lambda+5\\ 3 & 6\end{vmatrix}=-3(\lambda-1)$

$\because d_k(\lambda)=D_k(\lambda)/D_{k-1}(\lambda)$ $\Rightarrow$ Smith $\begin{pmatrix}1 & 0 & 0\\ 0 & \lambda-1 & 0\\ 0 & 0 & (\lambda-1)(\lambda+2)\end{pmatrix}$

$\Rightarrow d_1(\lambda)=1$ $\quad d_3(\lambda)=(\lambda-1)(\lambda+2)$

$d_2(\lambda)=\lambda-1$

作业2: $A(\lambda)=\begin{bmatrix}1-\lambda & \lambda^2 & \lambda\\ \lambda & \lambda & -\lambda\\ 1+\lambda^2 & \lambda^2 & \lambda\end{bmatrix}\xrightarrow{r_3-r_1}\begin{bmatrix}1-\lambda & \lambda^2 & \lambda\\ \lambda & \lambda & -\lambda\\ \lambda^2+\lambda & 0 & 0\end{bmatrix}$

$\xrightarrow[r_1\leftrightarrow r_2]{r_2+r_1}\begin{bmatrix}1 & \lambda(\lambda+1) & 0\\ 1-\lambda & \lambda^2 & \lambda\\ \lambda(\lambda+1) & 0 & 0\end{bmatrix}\xrightarrow{r_2-(1-\lambda)r_1}\begin{bmatrix}1 & \lambda(\lambda+1) & 0\\ 0 & \lambda^3+\lambda^2-\lambda & \lambda\\ \lambda(\lambda+1) & 0 & 0\end{bmatrix}$

$\xrightarrow{r_3-\lambda(\lambda+1)r_1}\begin{bmatrix}1 & \lambda(\lambda+1) & 0\\ 0 & \lambda^3+\lambda^2-\lambda & \lambda\\ 0 & -\lambda^2(\lambda+1)^2 & 0\end{bmatrix}\xrightarrow{c_2-\lambda(\lambda+1)c_1}\begin{bmatrix}1 & 0 & 0\\ 0 & \lambda^3+\lambda^2-\lambda & \lambda\\ 0 & -\lambda^2(\lambda+1)^2 & 0\end{bmatrix}$

$\xrightarrow[c_3-(\lambda^2+\lambda-1)c_2]{c_2\leftrightarrow c_3}\begin{bmatrix}1 & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & -\lambda^2(\lambda+1)^2\end{bmatrix}$

$\Rightarrow d_1(\lambda)=1$

$d_2(\lambda)=\lambda$ $\quad\Rightarrow$ Smith $\begin{bmatrix}1 & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda^2(1+\lambda)^2\end{bmatrix}$

$d_3(\lambda)=\lambda^2(\lambda+1)^2$ 整除

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3. $A(\lambda)=\begin{bmatrix} \lambda(\lambda+1) & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & (\lambda+1)^2 \end{bmatrix}$ 则初等因子为 $\{\lambda, \lambda, \lambda+1, (\lambda+1)^2\}$

从 $d_3(\lambda)$ 开始，$d_3(\lambda)=\lambda(\lambda+1)^2$

$d_2(\lambda)=\lambda(\lambda+1)$

$d_1(\lambda)=1$

则 Smith：$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda(\lambda+1) & 0 \\ 0 & 0 & \lambda(\lambda+1)^2 \end{bmatrix}$

4. 初等因子共 $n=7$ 阶，待求 $d_1(\lambda)$ — $d_7(\lambda)$

初等因子：1° $\lambda^2, \lambda, \lambda$

2° $(\lambda-1)^2$

3° $(\lambda+1)$

$\Rightarrow d_7(\lambda)=\lambda^2(\lambda-1)^2(\lambda+1)$ $\quad d_6(\lambda)=\lambda$ $\quad d_5=\lambda$

$d_4=1=d_3=d_2=d_1$

$\Rightarrow$ 不变因子 $1, 1, 1, 1, \lambda, \lambda, \lambda^2(\lambda-1)^2(\lambda+1)$

Smith：

$\mathrm{diag}: (1, 1, 1, 1, \lambda, \lambda, \lambda^2(1-\lambda)^2(\lambda+1))$