Determines the distance form a point P to a segment AB

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Question: Gave a 2D point P , a 2D segment AB , found the closest point Q that from P to AB , as shown in the figure:

1 Solution:

1. Calculate t1:

$$Q = A + (B - A) * t1 \tag{1}$$

$$A + (B - A) * t1 = P + N * d \tag{2}$$
$$N^T * (B - A) = 0 \tag{3}$$

BA

$$N = \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} \tag{4}$$

(3) substitution (1):

$$\begin{bmatrix} A_x \\ A_y \end{bmatrix} + \begin{bmatrix} B_x - A_x \\ B_y - A_y \end{bmatrix} * t1 = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} * d \tag{5}$$
$$\begin{bmatrix} A_x - P_x \\ A_y - P_y \end{bmatrix} + \begin{bmatrix} B_x - A_x \\ B_y - A_y \end{bmatrix} * t1 = \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} * d \tag{6}$$

Derive:

$$(A_x - P_x) + (B_x - A_x) * t1 = (B_y - A_y) * d \tag{7}$$
$$(A_y - P_y) + (B_y - A_y) * t1 = (A_x - B_x) * d \tag{8}$$

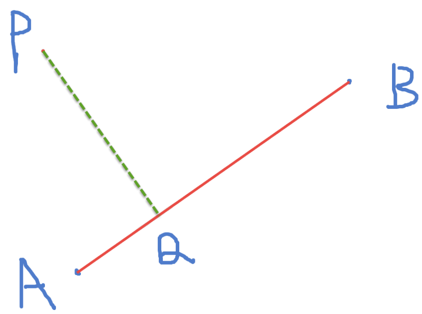

Figure 1:

Determine t1:

$$t1 = \frac{(A_y - P_y) * (B_y - A_y) - (A_x - P_x) * (A_x - B_x)}{(B_x - A_x) * (A_x - B_x) - (B_y - A_y) * (B_y - A_y)} \tag{9}$$

That is :

$$t1 = \frac{(A_x - P_x) * (A_x - B_x) + (A_y - P_y) * (A_y - B_y)}{(A_x - B_x)^2 + (A_y - B_y)^2} \tag{10}$$

2. Calculate t2:

$$Q = B + (A - B) * t2 \tag{11}$$

Similarly:

$$t2 = \frac{(P_x - B_x) * (A_x - B_x) + (P_y - B_y) * (A_y - B_y)}{(A_x - B_x)^2 + (A_y - B_y)^2} \tag{12}$$

3. Conclusion: If $t1 <= 0$, the closed point from P to the segement AB is point A . So the Q = A ;

If $t2 <= 0$, the closed point from P to the segement AB is point B . So the Q = B ;
Other, The Q within the segement AB,

$$Q = A + (B - A) * t \tag{13}$$

Set:

$$a_1 = (A_x - P_x) * (A_x - B_x) + (A_y - P_y) * (A_y - B_y) \tag{14}$$
$$a_2 = (P_x - B_x) * (A_x - B_x) + (P_y - B_y) * (A_y - B_y) \tag{15}$$

Follow :

$$\vec{Q} = \vec{A} + \frac{a_1 * (\vec{B} - \vec{A})}{a_1 + a_2} \tag{16}$$