

# Determines the distance form a point P to a segment AB

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Question: Gave a 2D point P , a 2D segment AB , found the closest point Q that from P to AB , as shown in the figure:

## 1 Solution:

1. Calculate t1:

$$Q = A + (B - A) * t1 \quad (1)$$

$$A + (B - A) * t1 = P + N * d \quad (2)$$

$$N^T * (B - A) = 0 \quad (3)$$

BA

$$N = \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} \quad (4)$$

(3) substitution (1):

$$\begin{bmatrix} A_x \\ A_y \end{bmatrix} + \begin{bmatrix} B_x - A_x \\ B_y - A_y \end{bmatrix} * t1 = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} * d \quad (5)$$

$$\begin{bmatrix} A_x - P_x \\ A_y - P_y \end{bmatrix} + \begin{bmatrix} B_x - A_x \\ B_y - A_y \end{bmatrix} * t1 = \begin{bmatrix} B_y - A_y \\ A_x - B_x \end{bmatrix} * d \quad (6)$$

Derive:

$$(A_x - P_x) + (B_x - A_x) * t1 = (B_y - A_y) * d \quad (7)$$

$$(A_y - P_y) + (B_y - A_y) * t1 = (A_x - B_x) * d \quad (8)$$

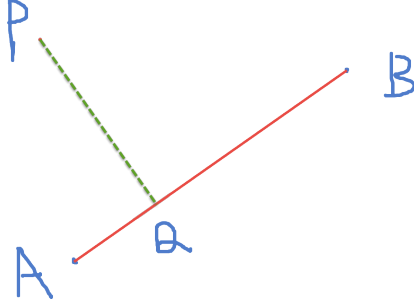


Figure 1:

Determine  $t1$ :

$$t1 = \frac{(A_y - P_y) * (B_y - A_y) - (A_x - P_x) * (A_x - B_x)}{(B_x - A_x) * (A_x - B_x) - (B_y - A_y) * (B_y - A_y)} \quad (9)$$

That is :

$$t1 = \frac{(A_x - P_x) * (A_x - B_x) + (A_y - P_y) * (A_y - B_y)}{(A_x - B_x)^2 + (A_y - B_y)^2} \quad (10)$$

2. Calculate  $t2$ :

$$Q = B + (A - B) * t2 \quad (11)$$

Similarly:

$$t2 = \frac{(P_x - B_x) * (A_x - B_x) + (P_y - B_y) * (A_y - B_y)}{(A_x - B_x)^2 + (A_y - B_y)^2} \quad (12)$$

3. Conclusion: If  $t1 \leq 0$ , the closed point from P to the segment AB is point A . So the  $Q = A$  ;

If  $t_2 \leq 0$  , the closed point from P to the segment AB is point B . So the Q = B ;

Other, The Q within the segment AB,

$$Q = A + (B - A) * t \quad (13)$$

Set:

$$a_1 = (A_x - P_x) * (A_x - B_x) + (A_y - P_y) * (A_y - B_y) \quad (14)$$

$$a_2 = (P_x - B_x) * (A_x - B_x) + (P_y - B_y) * (A_y - B_y) \quad (15)$$

Follow :

$$\vec{Q} = \vec{A} + \frac{a_1 * (\vec{B} - \vec{A})}{a_1 + a_2} \quad (16)$$