离散化后 $\begin{cases} x(k+1) = A x_k + B u_k \\ y_k = C x_k \end{cases}$ Exp: y_r

N

$$J = \sum_{i=0}^{N-1} \| y_{k+i} - y_r \|_Q^2 + \underbrace{\sum_{i=0}^{N-1} \| u_{k+i} \|_R^2}_{J_u} + \| y_{k+N} - y_r \|_F^2$$

$$= J_u + J_y$$

$$J_u = \sum_{i=0}^{N-1} \| u_{k+i} \|_R^2 = u_k^T R u_k + u_{k+1}^T R u_{k+1} + \cdots + u_{k+N-1}^T R u_{k+N-1}$$

$$= \underbrace{\begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix}^T}_{U^T} \underbrace{\begin{bmatrix} R & & & \\ & R & & \\ & & \ddots & \\ & & & R \end{bmatrix}}_{\bar{R}} \underbrace{\begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix}}_{U}$$

$$J_u = U^T \bar{R} U$$

$$J_y = \sum_{i=0}^{N-1} \| y_{k+i} - y_r \|_Q^2 + \| y_{k+N} - y_r \|_F^2$$

$$= (y_k - y_r)^T Q (y_k - y_r) + (y_{k+1} - y_r)^T Q (y_{k+1} - y_r) + \cdots$$

$$+ (y_{k+N-1} - y_r)^T Q (y_{k+N-1} - y_r) + \underline{(y_{k+N} - y_r)^T F (y_{k+N} - y_r)}$$

同理

$$= \begin{bmatrix} y_k - y_r \\ y_{k+1} - y_r \\ \vdots \\ y_{k+N-1} - y_r \\ y_{k+N} - y_r \end{bmatrix}^T \underbrace{\begin{bmatrix} Q & & & & \\ & Q & & & \\ & & \ddots & & \\ & & & Q & \\ & & & & F \end{bmatrix}}_{\bar{Q}} \begin{bmatrix} y_k - y_r \\ y_{k+1} - y_r \\ \vdots \\ y_{k+N-1} - y_r \\ y_{k+N} - y_r \end{bmatrix}$$

令 $\begin{bmatrix} y_k - y_r \\ y_{k+1} - y_r \\ \vdots \\ y_{k+N-1} - y_r \\ y_{k+N} - y_r \end{bmatrix} = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+N-1} \\ y_{k+N} \end{bmatrix} - \begin{bmatrix} y_r \\ \vdots \\ y_r \end{bmatrix} = Y - Y_r$

则 $J_y = (Y - Y_r)^T \bar{Q} (Y - Y_r)$

$$= \underbrace{Y^T \bar{Q} Y}_{\text{推理}} \underbrace{- Y_r^T \bar{Q} Y - Y^T \bar{Q} Y_r}_{= -2 Y_r^T \bar{Q} Y} + \underbrace{Y_r^T \bar{Q} Y_r}_{\text{常數}}$$

$y_k = C x_k$

$y_{k+1} = C x_{k+1} = C(A x_k + B u_k) = CA x_k + CB u_k$

$y_{k+2} = C x_{k+2} = C(A x_{k+1} + B u_{k+1}) = CA^2 x_k + CAB u_k + CB u_{k+1}$

$$\vdots$$

$$y_{k+N} = CA^N x_k + CA^{N-1}B u_k + CA^{N-2}B u_{k+1} + \cdots + CB u_{k+1}$$

则
$$Y = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+N} \end{bmatrix} = \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^N \end{bmatrix}}_{\Phi} x_k + \underbrace{\begin{bmatrix} 0 & 0 & \cdots & \\ CB & 0 & & \\ CAB & CB & \ddots & \\ \vdots & & & \\ CA^{N-1}B, & CA^{N-2}B & \cdots & CB \end{bmatrix}}_{\Gamma} \underbrace{\begin{bmatrix} u_k \\ u_{k+1} \\ u_{k+2} \\ \vdots \\ u_{k+N-1} \end{bmatrix}}_{U}$$

$$Y = \Phi x_k + \Gamma U$$

$$Y = \Phi x_k + \Gamma U$$

$$\begin{aligned}
Y^T \bar{Q} Y &= (\Phi x_k + \Gamma U)^T \bar{Q} (\Phi x_k + \Gamma U) \\
&= \underbrace{x_k^T \Phi^T \bar{Q} \Phi x_k}_{\text{常数}} + \underbrace{x_k^T \Phi^T \bar{Q} \Gamma U + U^T \Gamma^T \bar{Q} \Phi x_k}_{= 2 x_k^T \Phi^T \bar{Q} \Gamma U} + U^T \Gamma^T \bar{Q} \Gamma U \\
&= U^T \Gamma^T \bar{Q} \Gamma U + 2 x_k^T \Phi^T \bar{Q} \Gamma U + \underbrace{x_k^T \Phi^T \bar{Q} \Phi x_k}_{\text{常数}}
\end{aligned}$$

$$\begin{aligned}
Y_r^T \bar{Q} Y &= Y_r^T \bar{Q} (\Phi x_k + \Gamma U) \\
&= \underbrace{Y_r^T \bar{Q} \Phi x_k}_{\text{常数}} + Y_r^T \bar{Q} \Gamma U
\end{aligned}$$

$$J_Y = Y^T\bar{Q}Y - Y_r^T\bar{Q}Y - Y^T\bar{Q}Y_r + Y_r^T\bar{Q}Y_r$$

$$= U^TP^T\bar{Q}PU + 2x_k^T\Phi^T\bar{Q}PU - 2Y_r\bar{Q}PU + \text{常值}$$

$$J = J_u + J_Y$$

$$= \underline{U^T\bar{R}U + U^TP^T\bar{Q}PU} + 2x_k^T\Phi^T\bar{Q}PU - 2Y_r\bar{Q}PU + \text{常值}$$

$$U^T(\underbrace{\bar{R} + P^T\bar{Q}P}_{H})U + 2\underbrace{(x_k^T\Phi^T - Y_r)\bar{Q}P}_{f}U + \text{常值}$$

$$= U^THU + 2fU + \text{常值}$$

增量MPC