

# Supplementary Material for “From Individual-Level Learning to Dimensional-Level Learning: Dimension Match Learning Multitask Optimization”

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TABLE S.I THE CEC17 EXPERIMENTAL RESULTS OF DMLMTO AND FIXED DLP

Problem	DMLMTO	$dlp = 0.0$	$dlp = 0.2$	$dlp = 0.3$	$dlp = 0.4$	$dlp = 0.5$	$dlp = 0.6$	$dlp = 0.7$	$dlp = 0.8$	$dlp = 0.9$	$dlp = 1.0$
CIHS-T1	<b>0</b>	2.49E-4(+)	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	7.17E-4(≈)	<b>0(≈)</b>	2.95E-4(≈)
CIHS-T2	<b>0</b>	2.06E+0(+)	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	1.14E-14(+)	2.08E-14(+)	5.12E-14(+)	8.21E-1(+)	9.28E-14(+)	3.64E-1(+)
CIMS-T1	<b>2.18E-11</b>	2.52E-6(+)	6.76E-11(+)	7.15E-10(+)	1.62E-7(+)	4.23E-5(+)	1.10E-3(+)	1.27E-2(+)	3.67E-2(+)	8.57E-2(+)	3.36E-1(+)
CIMS-T2	<b>0</b>	8.99E-9(+)	<b>0(≈)</b>	<b>0(≈)</b>	3.43E-11(+)	1.98E-6(+)	1.52E-3(+)	1.84E-1(+)	9.34E-1(+)	4.24E+0(+)	3.14E+1(+)
CILS-T1	2.07E+1	2.06E+1(≈)	2.09E+1(≈)	<b>2.04E+1(≈)</b>	2.11E+1(+)	2.12E+1(+)	2.05E+1(+)	2.12E+1(+)	2.07E+1(+)	2.12E+1(+)	2.12E+1(+)
CILS-T2	3.00E+3	<b>2.88E+3(≈)</b>	3.05E+3(≈)	3.24E+3(+)	3.32E+3(+)	3.59E+3(+)	3.77E+3(+)	5.33E+3(+)	7.32E+3(+)	1.19E+4(+)	1.64E+4(+)
PIHS-T1	1.03E+2	9.92E+1(≈)	9.94E+1(≈)	<b>9.46E+1(≈)</b>	1.08E+2(≈)	1.08E+2(≈)	1.08E+2(≈)	1.21E+2(+)	1.93E+2(+)	4.13E+2(+)	5.06E+2(+)
PIHS-T2	1.19E-9	<b>3.19E-10(-)</b>	1.85E-8(+)	2.31E-7(+)	3.43E-6(+)	7.05E-5(+)	1.65E-3(+)	4.66E-2(+)	1.00E+0(+)	5.56E+1(+)	4.75E+3(+)
PIMS-T1	<b>1.11E-5</b>	3.74E-5(+)	1.79E-5(+)	6.82E-5(+)	1.89E-1(+)	3.81E-1(+)	2.67E-1(+)	8.67E-1(+)	1.26E+0(+)	1.75E+0(+)	3.30E+0(+)
PIMS-T2	5.60E+1	7.08E+1(+)	5.44E+1(≈)	5.19E+1(≈)	<b>4.30E+1(≈)</b>	5.13E+1(≈)	4.75E+1(-)	4.94E+1(≈)	5.87E+1(+)	9.36E+1(+)	1.67E+2(+)
PILS-T1	2.51E-4	2.64E-3(+)	2.89E-5(-)	2.19E-4(-)	<b>1.60E-5(-)</b>	4.12E-5(-)	1.03E-4(≈)	9.91E-5(≈)	3.56E-4(≈)	1.89E-5(-)	8.83E-2(+)
PILS-T2	2.43E-3	3.33E-1(+)	<b>6.81E-4(-)</b>	6.89E-4(-)	8.84E-4(-)	1.73E-3(≈)	2.58E-3(≈)	4.39E-3(+)	8.48E-3(+)	1.69E-3(≈)	2.98E-1(+)
NIHS-T1	4.78E+1	9.02E+1(+)	4.77E+1(+)	<b>4.68E+1(+)</b>	4.80E+1(+)	4.82E+1(+)	4.81E+1(+)	4.86E+1(+)	4.88E+1(+)	5.24E+1(+)	6.11E+1(+)
NIHS-T2	1.86E+0	4.41E+0(+)	1.03E+0(≈)	3.42E+0(+)	5.31E-1(+)	<b>3.10E-6(+)</b>	1.16E+0(+)	2.93E+0(+)	1.09E+0(+)	1.02E+1(+)	1.67E+2(+)
NIMS-T1	<b>3.63E-7</b>	4.93E-4(≈)	3.30E-4(+)	5.59E-6(+)	2.90E-4(+)	2.18E-4(+)	1.94E-3(+)	1.18E-2(+)	8.91E-2(+)	6.03E-1(+)	1.30E+0(+)
NIMS-T2	9.83E-1	2.99E+0(+)	<b>7.07E-1(≈)</b>	8.74E-1(≈)	1.06E+0(≈)	1.09E+0(≈)	1.75E+0(+)	2.57E+0(+)	4.32E+0(+)	8.21E+0(+)	1.69E+1(+)
NILS-T1	1.24E+2	1.15E+2(≈)	1.24E+2(≈)	<b>1.14E+2(≈)</b>	1.19E+2(≈)	1.16E+2(≈)	1.21E+2(≈)	2.01E+2(+)	4.05E+2(+)	5.21E+2(+)	1.30E+3(+)
NILS-T2	<b>2.37E+3</b>	2.55E+3(≈)	2.61E+3(+)	2.77E+3(+)	2.72E+3(+)	3.09E+3(+)	3.30E+3(+)	3.64E+3(+)	4.48E+3(+)	7.93E+3(+)	1.67E+4(+)
Number of + / ≈ / -		11 / 6 / 1	6 / 10 / 2	8 / 8 / 2	10 / 6 / 2	11 / 6 / 1	12 / 5 / 1	15 / 3 / 0	16 / 2 / 0	15 / 2 / 1	17 / 1 / 0

TABLE S.II THE CEC17 EXPERIMENTAL RESULTS OF DMLMTO AND FIXED EOP

Problem	DMLMTO	$eop = 0.0$	$eop = 0.1$	$eop = 0.2$	$eop = 0.3$	$eop = 0.4$	$eop = 0.5$	$eop = 0.6$	$eop = 0.7$	$eop = 0.9$	$eop = 1.0$
CIHS-T1	<b>0</b>	1.08E-1(+)	3.67E-2(+)	8.58E-3(+)	6.76E-4(+)	1.33E-16(≈)	3.36E-10(≈)	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>
CIHS-T2	<b>0</b>	7.76E+1(+)	4.14E+1(+)	1.73E+1(+)	2.39E+0(+)	1.88E-13(+)	4.28E-7(≈)	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>
CIMS-T1	<b>2.18E-11</b>	7.83E-1(+)	2.73E-1(+)	1.76E-1(+)	7.21E-2(+)	1.37E-8(+)	4.45E-10(+)	1.73E-10(+)	4.69E-11(+)	2.26E-11(≈)	1.20E-10(+)
CIMS-T2	<b>0</b>	4.69E+1(+)	1.70E+1(+)	8.13E+0(+)	5.67E+0(+)	3.28E-12(≈)	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>	<b>0(≈)</b>
CILS-T1	2.07E+1	2.03E+1(≈)	2.01E+1(-)	2.01E+1(-)	2.00E+1(-)	2.01E+1(-)	<b>1.94E+1(-)</b>	2.02E+1(-)	2.02E+1(-)	2.11E+1(+)	2.12E+1(+)
CILS-T2	3.00E+3	4.50E+3(+)	3.89E+3(+)	3.81E+3(+)	3.64E+3(+)	3.39E+3(+)	3.42E+3(+)	3.24E+3(≈)	3.22E+3(≈)	<b>2.89E+3(≈)</b>	1.10E+4(+)
PIHS-T1	1.03E+2	3.20E+2(+)	2.33E+2(+)	2.11E+2(+)	1.68E+2(+)	1.52E+2(+)	1.35E+2(+)	1.32E+2(+)	1.12E+2(≈)	<b>9.60E+1(≈)</b>	2.56E+2(+)
PIHS-T2	1.19E-9	1.92E+1(+)	1.38E+0(+)	3.01E-1(+)	5.18E-2(+)	1.31E-2(+)	2.40E-4(+)	9.82E-5(-)	<b>4.55E-10(-)</b>	1.57E-8(+)	4.30E-6(+)
PIMS-T1	<b>1.11E-5</b>	2.35E+0(+)	1.95E+0(+)	1.68E+0(+)	9.74E-1(+)	5.20E-1(+)	2.17E-1(+)	2.94E-2(+)	4.96E-5(+)	1.67E-5(+)	6.74E-5(+)
PIMS-T2	5.60E+1	1.24E+2(+)	9.46E+1(+)	7.59E+1(+)	5.23E+1(≈)	5.80E+1(≈)	5.15E+1(≈)	5.19E+1(≈)	5.47E+1(≈)	4.67E+1(≈)	<b>4.65E+1(-)</b>
PILS-T1	2.51E-4	3.02E-1(+)	3.02E-1(+)	3.48E-1(+)	1.73E-1(+)	6.11E-2(+)	1.28E-1(+)	7.21E-2(+)	1.10E-2(+)	<b>1.14E-4(+)</b>	1.07E-3(+)
PILS-T2	2.43E-3	7.07E-1(+)	6.27E-1(+)	6.46E-1(+)	4.09E-1(+)	2.08E-1(+)	1.73E-1(+)	9.24E-2(+)	2.17E-2(+)	2.14E-3(-)	<b>1.05E-3(≈)</b>
NIHS-T1	4.78E+1	1.07E+2(+)	8.27E+1(+)	5.26E+1(+)	6.32E+1(+)	5.00E+1(+)	4.87E+1(+)	5.14E+1(≈)	5.03E+1(≈)	4.79E+1(≈)	<b>4.77E+1(-)</b>
NIHS-T2	1.86E+0	5.38E+1(+)	2.57E+1(+)	1.25E+1(+)	1.44E+1(+)	1.05E+1(+)	5.90E+0(+)	7.10E+0(+)	4.54E+0(+)	<b>5.84E-9(+)</b>	6.08E-7(+)
NIMS-T1	<b>3.63E-7</b>	4.83E-1(+)	2.09E-1(+)	1.25E-1(+)	5.60E-2(+)	2.75E-2(+)	7.23E-3(+)	1.55E-3(+)	5.85E-4(+)	5.99E-7(+)	1.98E-5(+)
NIMS-T2	9.83E-1	1.46E+1(+)	1.33E+1(+)	1.18E+1(+)	8.75E+0(+)	6.78E+0(+)	5.14E+0(+)	3.10E+0(+)	1.77E+0(+)	2.98E-1(-)	<b>2.23E-1(-)</b>
NILS-T1	1.24E+2	7.36E+2(+)	5.14E+2(+)	3.86E+2(+)	3.38E+2(+)	2.79E+2(+)	2.17E+2(+)	1.83E+2(+)	1.57E+2(+)	<b>8.92E+1(-)</b>	3.35E+2(+)
NILS-T2	2.37E+3	4.45E+3(+)	3.85E+3(+)	3.71E+3(+)	3.50E+3(+)	3.63E+3(+)	3.34E+3(+)	3.05E+3(+)	2.79E+3(+)	2.13E+3(≈)	<b>2.11E+3(-)</b>
Number of + / ≈ / -		17 / 1 / 0	17 / 0 / 1	17 / 0 / 1	16 / 1 / 1	14 / 3 / 1	13 / 4 / 1	10 / 6 / 2	9 / 7 / 2	6 / 9 / 3	10 / 4 / 4