Columbia University
MATHG4074: Programming for Quant and Comp Finance

Case Study 2 (tentative due date: Thursday March 3, 2016)

Ali Hirsa

February 11, 2016

Pricing a European put under a pure jump process – Let $w(x,\tau)$ be the value of a derivative security that satisfies the following PIDE

$$\begin{aligned}&\frac{\partial w}{\partial \tau}(x,\tau) - (r-q)\frac{\partial w}{\partial x}(x,\tau) + rw(x,\tau)\\ -\int_{-\infty}^{\infty}&\left[w(x+y,\tau) - w(x,\tau) - \frac{\partial w}{\partial x}(x,\tau)(e^y - 1)\right]k(y)dy = 0\end{aligned}$$

where

$$k(y) = \frac{e^{-\lambda_p y}}{\nu y}\mathbf{1}_{y>0} + \frac{e^{-\lambda_n |y|}}{\nu |y|}\mathbf{1}_{y<0},$$

with

$$\begin{aligned}\lambda_p &= \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{\frac{1}{2}} - \frac{\theta}{\sigma^2},\\ \lambda_n &= \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{\frac{1}{2}} + \frac{\theta}{\sigma^2}.\end{aligned}$$

and $x = \ln(S)$ and $\tau = T - t$. For a Eurpean put option premium, this PIDE must be solved subject to the initial condition

$$w(x,0) = (K - e^x)^+,$$

and boundary conditions

$$\begin{aligned}w(x_0,\tau) &= (K - \exp(x_0))^+ \quad \forall\tau,\\ w(x_N,\tau) &= 0 \quad \forall\tau.\end{aligned}$$

Use explicit-implicit finite difference scheme **covered during the lecture** to solve the PIDE.

Calculate the option premium in this framework for the following parameters: spot price, $S_0 = \$1800$; strike price $K = 1650$; risk-free interest rate, $r = 0.5\%$; dividend rate, $q = 1.35\%$; maturity, $T = 0.5$ year; $\sigma = 20\%$, $\nu = 0.17$ and $\theta = -0.15$ (sanity check – option premium for this parameter set is around \$45.58 depending on the time- and spatial-steps).

We can evaluate $\sigma^2(\Delta x)$ and $\Omega(\Delta x)$ in terms of exponential integral function

$$\begin{aligned}\sigma^2(\Delta x) &= \int_{|y|<\Delta x} y^2 k(y) dy \\ &= \frac{1}{\nu\lambda_p^2}(1-(1+\lambda_p\Delta x)e^{-\lambda_p\Delta x}) \\ &+ \frac{1}{\nu\lambda_n^2}(1-(1+\lambda_n\Delta x)e^{-\lambda_n\Delta x})\end{aligned}$$

and

$$\begin{aligned}\omega(\Delta x) &= \int_{|y|>\Delta x} (1-e^y) k(y) dy \\ &= \frac{1}{\nu}(\mathrm{expint}(\lambda_p\Delta x) - \mathrm{expint}((\lambda_p-1)\Delta x))) \\ &+ \frac{1}{\nu}(\mathrm{expint}(\lambda_n\Delta x) - \mathrm{expint}((\lambda_n+1)\Delta x)))\end{aligned}$$