Columbia University
MATHG4074: Programming for Quant and Comp Finance

Case Study 3 (Monday March 21, 2016)

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Pricing a European put under a pure jump process via simulation

Calculate a European put premium assuming the stock price process follows a Variance Gamma process for the following parameters: spot price, $S_0 = \$1800$; strike price $K = 1650$; risk-free interest rate, $r = 0.50\%$; dividend rate, $q = 1.35\%$; maturity, $T = 0.5$ year; VG parameters of $\sigma = 20\%$, $\nu = 0.17$ and $\theta = -0.15$ (option premium for this parameter set is around \$45.58 depending on the time steps and number of simulation paths).

Implement the following pseudo-code covered during the lecture to obtain the premium and compare it with your result from **Case Study 2**. Naturally they are supposed to match.

payoff $= 0$

FOR $j = 1, \ldots,$ nSimulationPaths

FOR $i = 1, \ldots,$ nTimeSteps

$z \sim \mathcal{N}(0, 1)$

$g \sim \text{gamrand}(h/\nu, \nu)$

$x = \theta g + \sigma \sqrt{g} z$

$\log S_i = \log S_{i-1} + (r - q)h + \omega h + x$

END

if $S_T < K$ then payoff $=$ payoff $+ (K - S_T)$

END

where $\omega = \frac{1}{\nu} \log(1 - \theta\nu - \sigma^2\nu/2)$ and $h = T/\text{nTimeSteps}$. The discounted average payoff would be the premium.

Examine various different scenarios for number of time intervals and number of simulated paths. In one trial, fix number of time intervals (e.g. 40 or 50) and keep increasing the number of simulated paths (e.g. 100,000, 200,000, 500,000, 1000,000) and see how the premium approaches the true value. In another trial fix number of simulated paths (e.g. $500,000$) and keep increasing the number of time intervals (e.g. 10, 20, 50, 100) and see whether or not the premium approaches the true value[1].

[1] For seed number you can use any long integer you wish. Just examine a couple of different ones to see the impact on the premium.