Columbia University

MATHG4074: Programming for Quantitative and Computational Finance

Case Study 4 (Due by midnight of April 28, 2016)

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April 12, 2016

Calibration

Let $b(t;\theta,\sigma) \equiv \theta t + \sigma W(t)$ be a Brownian motion with constant drift rate θ and volatility σ, where $W(t)$ is a standard Brownian motion. Denote by $\gamma(t;\nu)$, the gamma process with independent gamma increments of mean h and variance νh over non-overlapping intervals of length h. The three parameter *VG process* $X(t;\sigma,\theta,\nu)$ is defined by

$$X(t;\sigma,\theta,\nu) = b(\gamma(t;\nu),\theta,\sigma)$$

We see that the process $X(t)$ is a Brownian motion with drift evaluated at a gamma time change. The characteristic function for the time t level of the VG process as shown on Page 23 is given by

$$\phi_{X(t)}(u) = \mathbb{E}(e^{iuX(t)}) = \left(\frac{1}{1 - iu\theta\nu + \sigma^2 u^2 \nu/2}\right)^{\frac{t}{\nu}}$$

The VG dynamics of the stock price mirrors that of geometric Brownian motion for a stock paying a continuous dividend yield of q in an economy with a constant continuously compounded interest rate of r. The risk neutral drift rate for the stock price is $r-q$ and the forward stock price is modeled as the exponential of a VG process normalized by its expectation. Let $S(t)$ be the stock price at time t. The VG risk neutral process for the stock price is given by

$$S(t) = S(0)e^{(r-q)t + X(t) + \omega t} \tag{0.1}$$

where the normalization factor $e^{\omega t}$ ensures that $\mathbb{E}_0[S(t)] = S(0)e^{(r-q)t}$. It follows from the characteristic function evaluated at $-i$ that

$$\omega = \frac{1}{\nu}\ln(1 - \sigma^2\nu/2 - \theta\nu)$$

Therefore the characteristic function for the log of stock price process is given by

$$\begin{aligned}
\Phi(u) &= \mathbb{E}(e^{iu\ln S_t}) \\
&= \exp\left(iu(\ln S_0 + (r - q + \frac{1}{\nu}\ln(1 - \sigma^2\nu/2 - \theta\nu))t)\right) \\
&\times \left(\frac{1}{1 - iu\theta\nu + \sigma^2 u^2\nu/2}\right)^{\frac{t}{\nu}}
\end{aligned}$$

Assume a stock price process follows a variance gamma model. For spot price $S_0 = \$100$; risk-free interest rate $r = 0.50\%$, dividend rate $q = 1.25\%$ we have the following premiums for European call options for various strike prices and maturities.

Maturity	Strike	Premium
0.25	100	7.4329
	104	5.1797
	108	3.2578
	110	2.4397
	113	1.4172
0.50	100	9.3535
	105	6.7238
	109	4.9094
	111	4.1056
	115	2.7162
0.75	100	9.8771
	106	6.8642
	110	5.1683
	115	3.4252
	120	2.1199

Obtain the VG parameter set, σ, ν, θ that was used to tabulate the following table via calibration (for each maturity separately). As discussed during the lecture first use a grid search to find a suitable starting point for the optimizer. You might assume the following ranges for your search: $[0.05 \;\; 0.50]$, $[0.05 \;\; 0.75]$, and $[-0.9 \quad -0.05]$ for σ, ν, and θ respectively.