Laboratory Report of Digital Signal Processing

Lab I. Fourier Analysis

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Score: _____________

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Contents

Introduction

This report is consisted of 2 tasks, both concern about the continuous time (CT) signal. It's easy to analyze the amplitude in time domain, but when it comes to frequency, we have to do the Fourier transform (FT) in order to analyze frequency and modulus.

In task 1, we analyze a harmonic signal by applying CTFT and adding windows. When it comes to frequency and energy, CTFT and window really help a lot. By CTFT, we clarify the frequencies of the combined signal, and by rectangle and hamming windows we know the relationship between the energy of the signal and the width of the window.

In task 2, what we are faced with is a realistic electrical wave signal. By adding window, we separate the different frequencies in the real signal, and with the help of *fft* function, this work could be easier.

Task 1

A signal involving two harmonics is expressed as

$$x(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2)\,, \qquad (1),$$

where $A_1 = 2,\ f_1 = 20, \phi_1 = \pi / 4, A_2 = 1,\ f_2 = 19.5, \phi_2 = 0,\ \ f_s = 1000$..

1.1 Plot the signal in time [-1, 1].

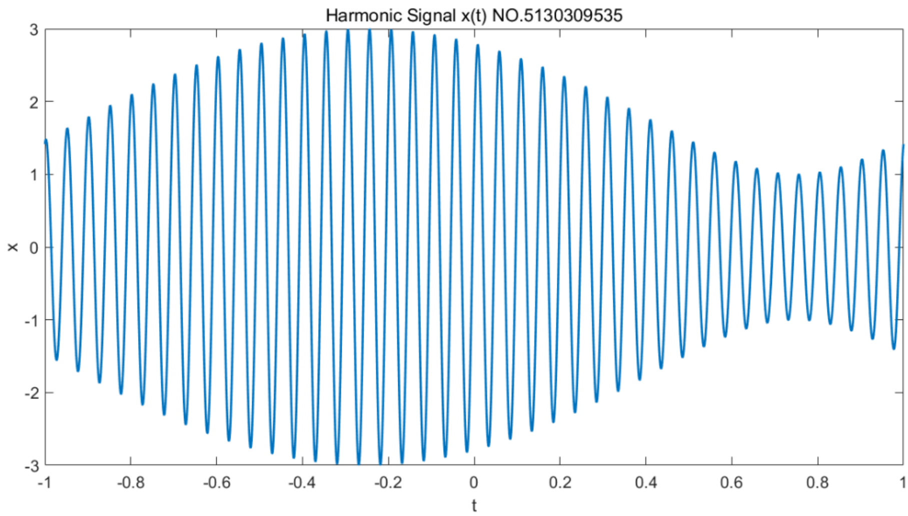

Figure 1: Harmonic signal *x(t)*

1.2 Calculate and plot the real and imagery parts of FS of the signal in frequencies [-50, 50].

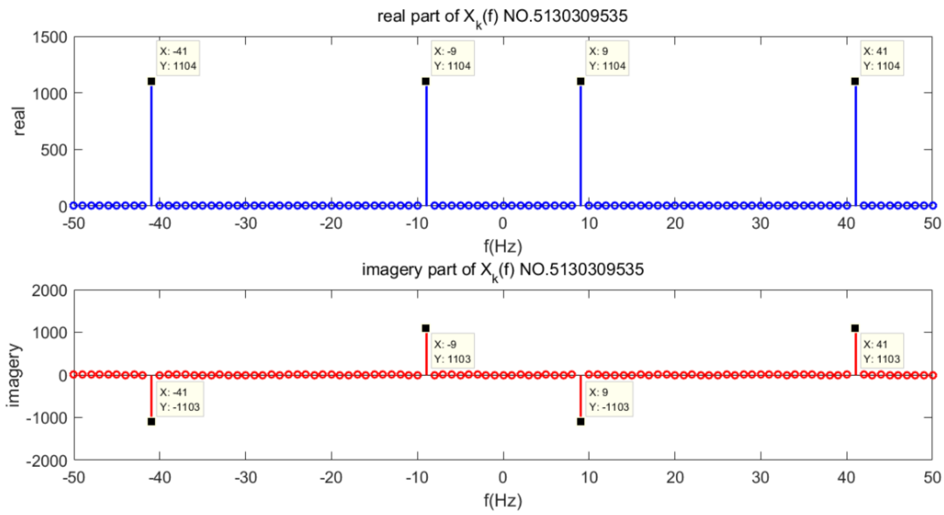

Figure 2: Real and imagery parts of FS of the signal *x(t)*

In order to calculate the CTFS of a combined signal of two parts with different frequencies, we have to get the current frequency first, which is the least common multiple of two parts. The new frequency is 780 Hz. There are 2 pulses in each side of Axis-Y, because *x(t)* is composed of two frequencies.

1.3 Indicate the frequencies, amplitudes and phases of the two harmonics in the plots.

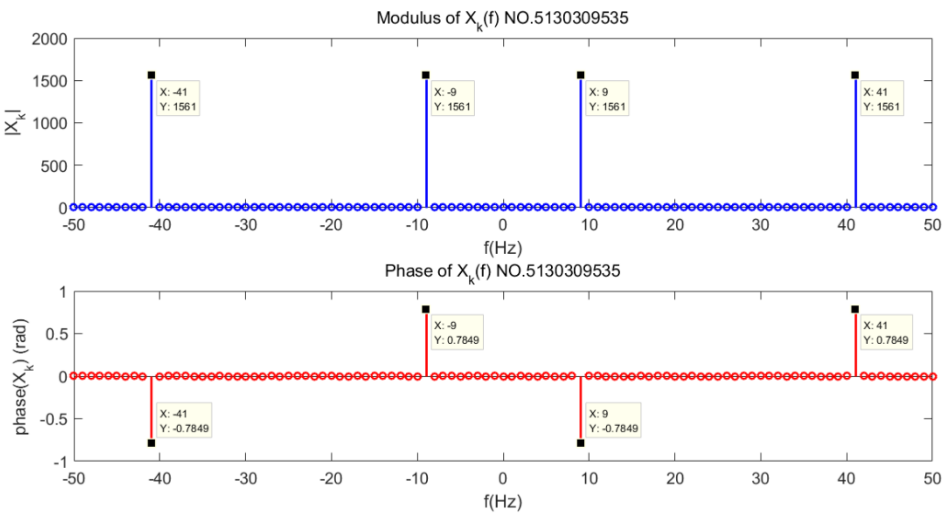

Figure 3: Frequencies, amplitudes and phases of the two harmonics

1.4 Use a rectangle window to truncate the signal in [-0.1, 0.1], plot the modulus of spectra in [$-f_s/2$, $f_s/2$].

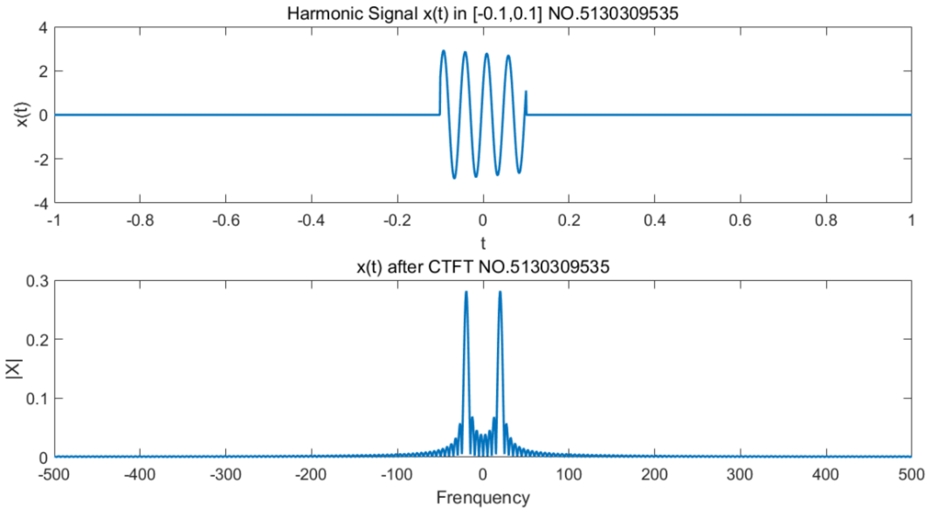

Figure 4: Rectangle windowed signal in [-0.1, 0.1] and modulus after CTFT

There is a rectangle window function *rectwin* in Matlab, alternatively I use the *mySqu* function which we created during a class. The frequency of the pulse is about 20 Hz. There should be two pulses on each side but there is only one. I think it's because two frequencies are two close to distinguish.

1.5 Broaden the window to [-0.2, 0.2] and compare the spectra with the result of 1.4.

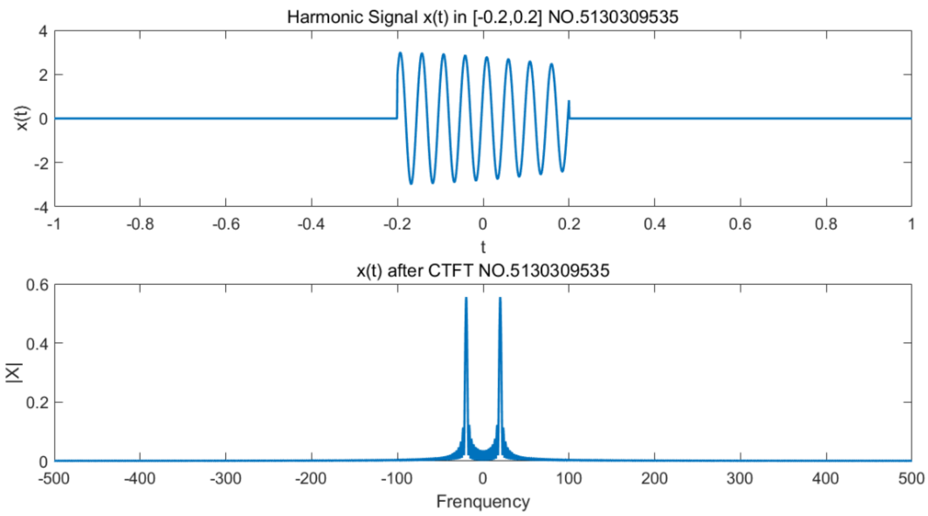

Figure 5: Rectangle windowed signal in [-0.2, 0.2] and modulus after CTFT

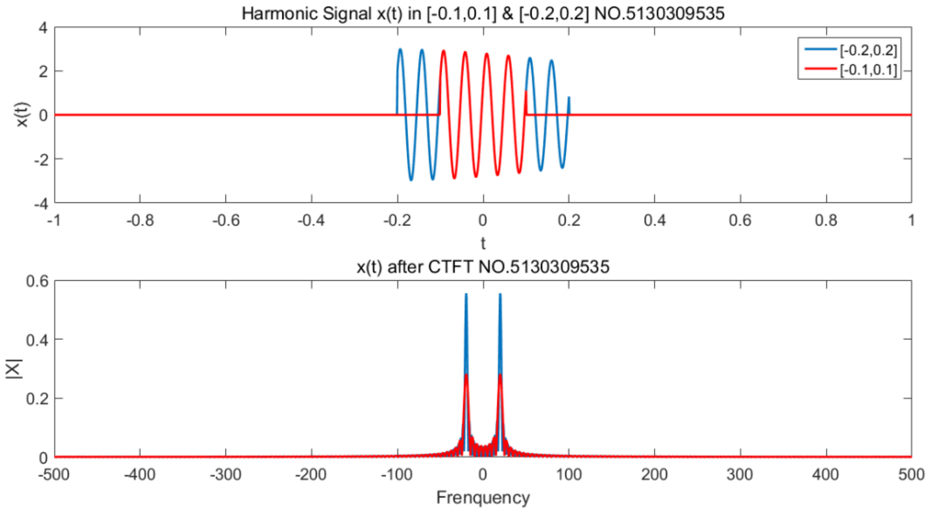

Figure 6: Comparison between two signals with different rectangle windows

In figure 6, it clearly shows that the pulse of narrow windowed signal is half of the wide one. So we can know that the width of the window determines the pulse height of the signal.

The following figures are signal *x(t)* directly plotted in the rectangle sections, as we can see, they are totally equal with the signals multiplied by gate functions.

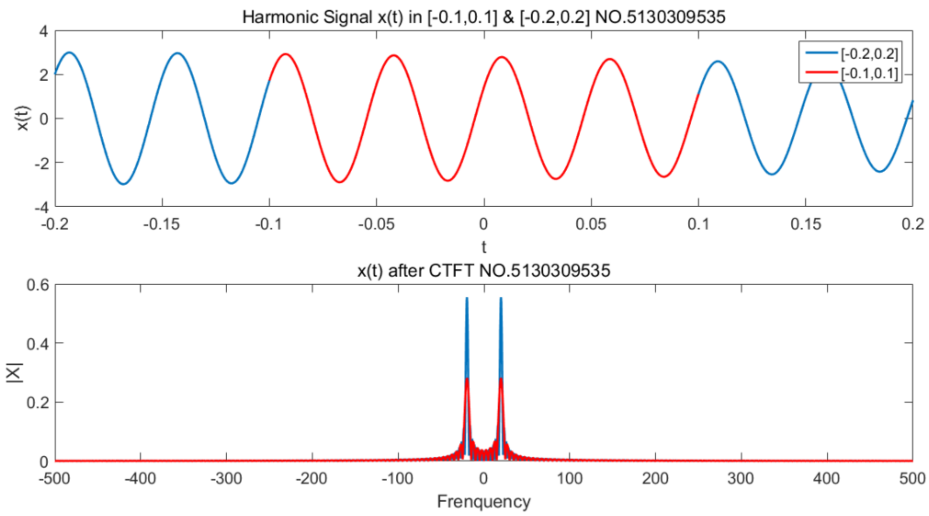

Figure 7: Signals directly plotted in the rectangle sections and their CTFT

1.6 Use Hamming windows to do the same work with the truncating section [-0.1, 0.1] and [-0.2, 0.2], respectively. Compare the modulus of spectra in one figure.

There is a *hamming* function in Matlab, so it's easy to apply the Hamming window in the base of figure 7. After CTFT, we can get a smoother spectra compared to rectangle windowed spectra. The law we get is the same with the rectangle windowed one, which is spectra that two times higher is two times wider in time domain than the other one.

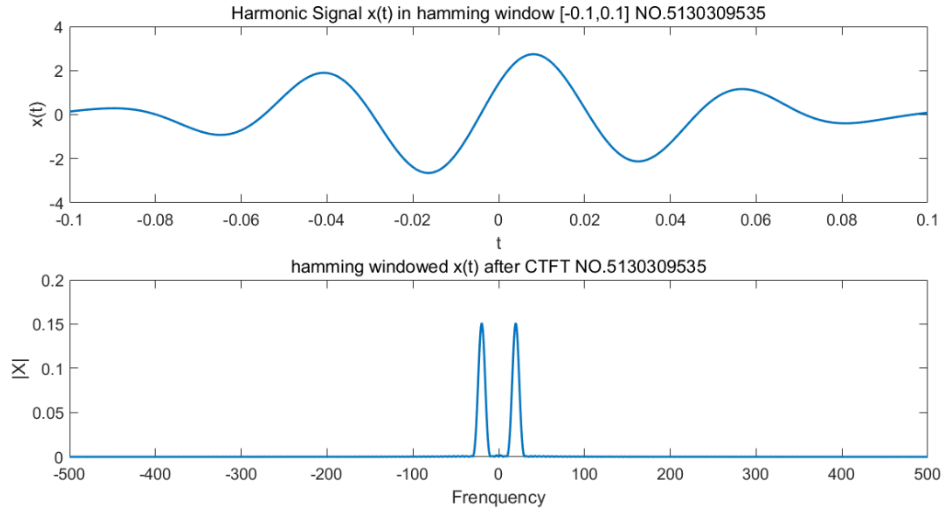

Figure 8: Signal *x(t)* truncated by hamming windows

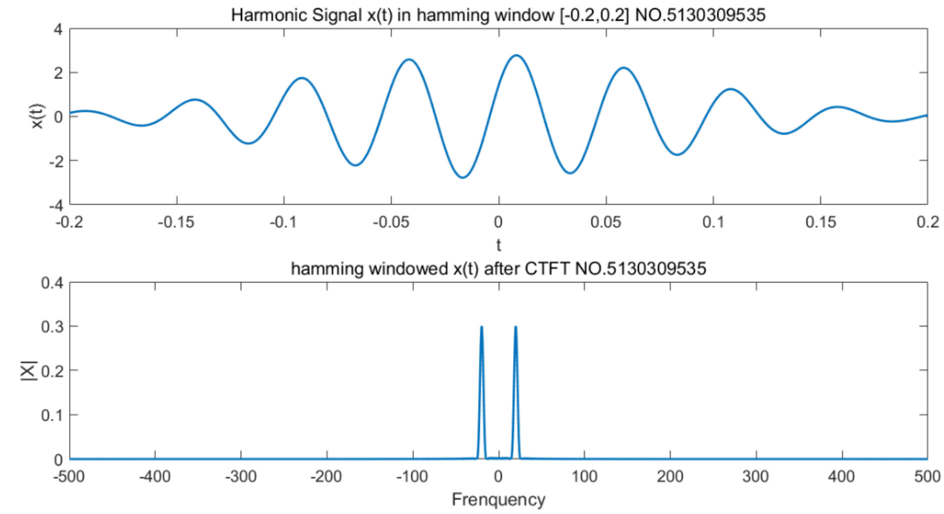

Figure 9: Signal *x(t)* truncated by a wider hamming windows

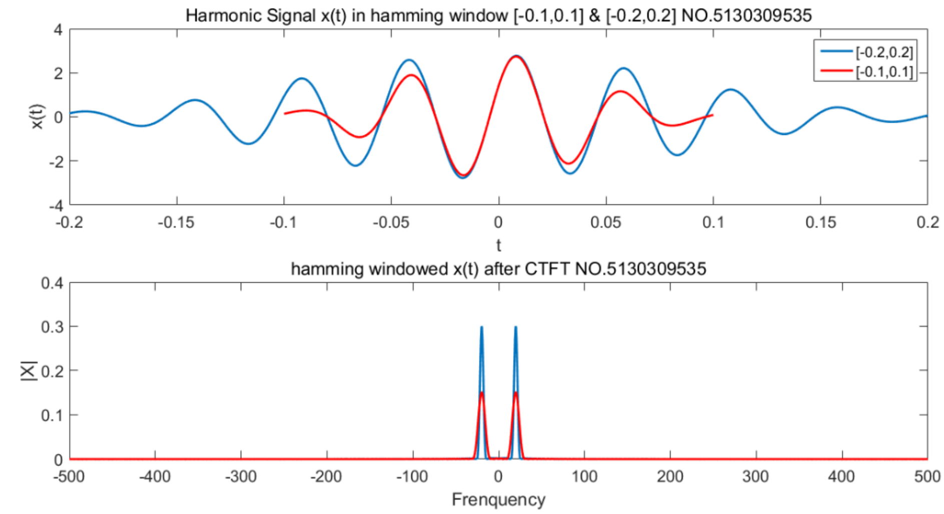

Figure 10: Comparison between signals between hamming windows with different widths

Task 2

An electricity wave, whose nominal frequency and amplitude are 50 Hz and 220 V, was sampled at 1000 Hz and saved in a file named 'data.mat'. The signal may be polluted by some frequency components.

2.1 Import the data to the MATLAB workspace and plot the signal.

As we can see, the amplitude of the signal is 220V.

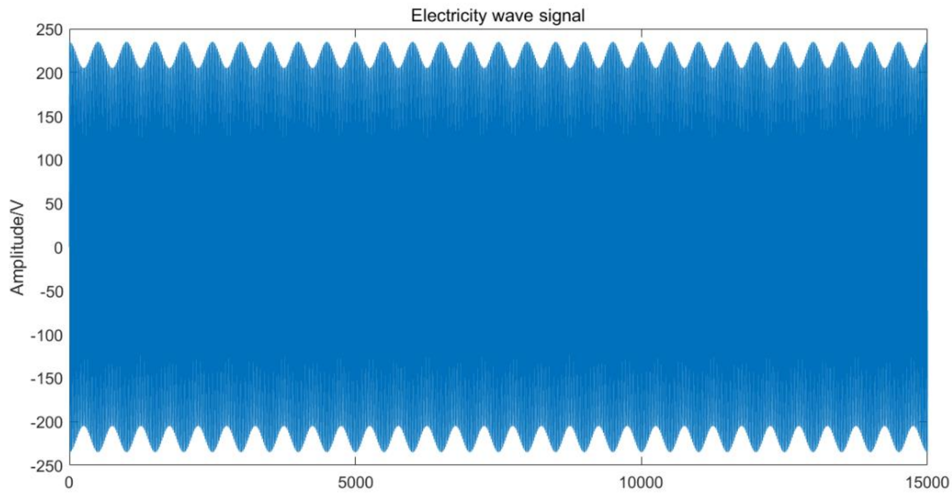

Figure 11: Electricity wave signal

2.2 Choose a proper window as short as possible to calculate and plot the CTFT of the signal. Find all the frequency components and list their amplitudes and phases. Write your window type and length.

After calculating and trying, I choose the hamming window and set the width of 2500. Its amplitudes and phases are as follows. So it's polluted by 52-Hz signal.

Frequency	50	52
Amplitude	1.484e+05	1.012e+04
Phase	2.4 π	2.4 π

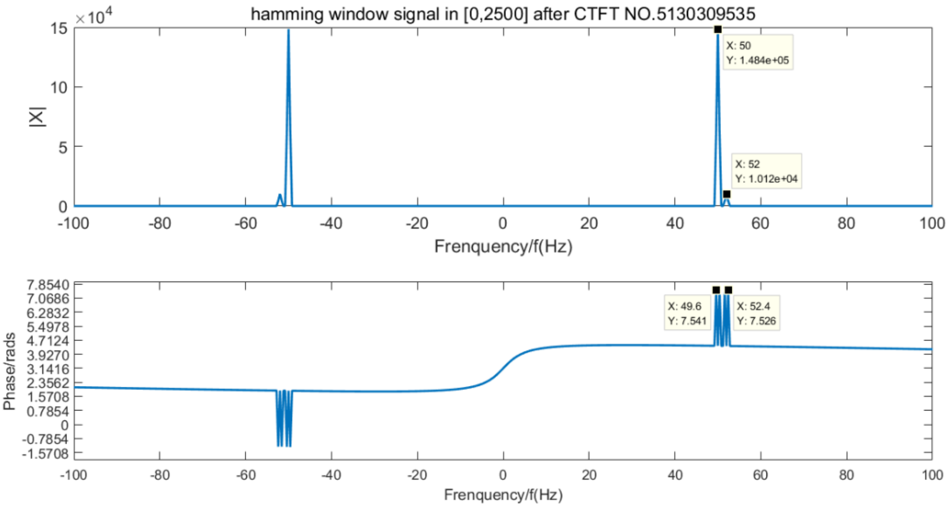

Figure 11: Amplitudes and phases of the frequencies after hamming window

2.3 Use MATLAB function fft to do the same work as 2.2 and compare with the result of CTFT.

This time we use *fft()* and *fftshift()* to do the transform. The modulus of two methods are exactly the same, and the phases are also can be considered the same.

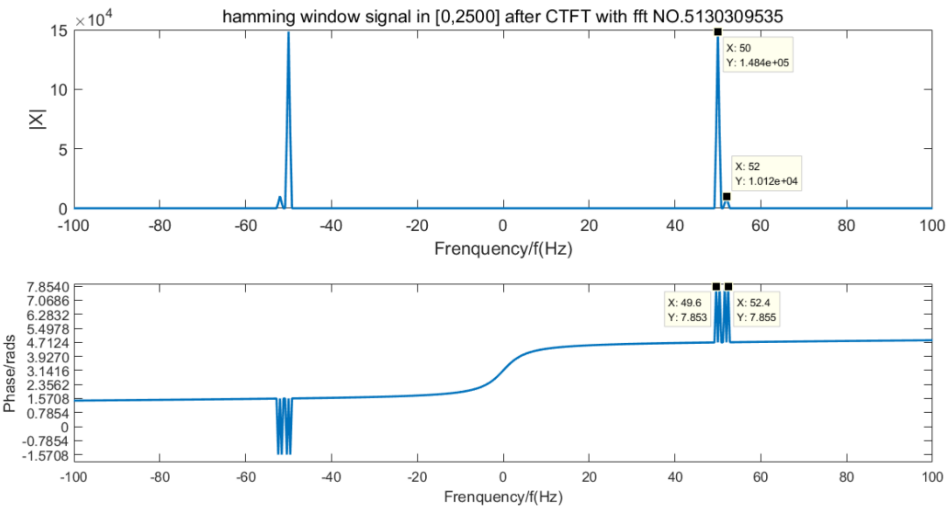

Figure 12: Using fft method

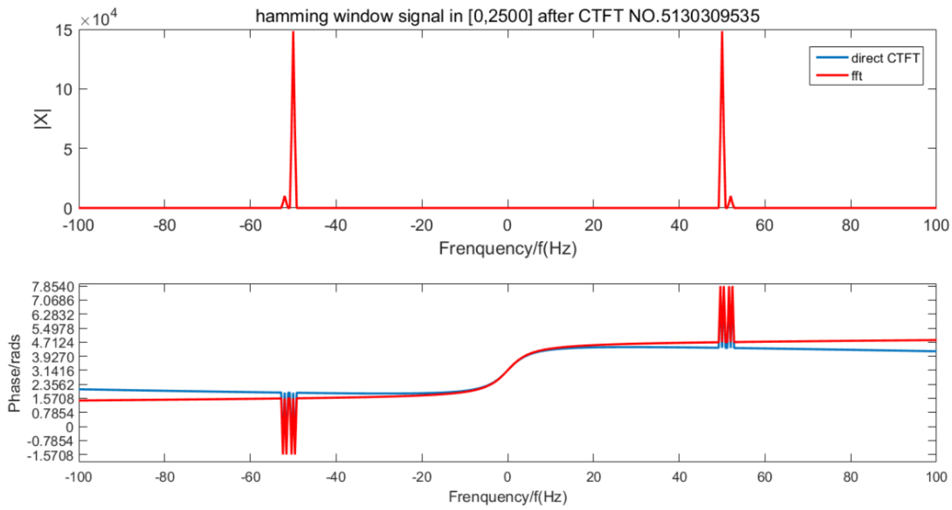

Figure 13: Difference between direct CTFT and fft method

Summary

After more than 10 hours' work, I finally work out all these tasks. At first, it was not easy to apply the hamming to the signal because the difference between of the lengths of hamming window signal and the target signal, then with the help of assistants I was aware that I could cut the signal by directly plotting the signal in the expected range, as figure 6 and 7 showed. In task 2 the official document of Matlab helped a lot, and I learnt the usages of *fft* and *hamming*.

I learnt a lot by doing this report. I get a deep understand of Fourier transform and signal in frequency domain. Matlab is an important tool and I get more familiar with it. What's more, I learnt some tips in making an academic report, which will help a lot in my future academic reports.

Special thanks to teaching assistants and my fellow classmates, for helping me with some ticklish problem.

Appendix

Matlab code of Task 1

```
clc; clear;

close all;


%%


f1 = 20;

A1 = 2;

pha1 = pi/4;

f2 = 19.5;

A2 = 1;

pha2 = 0;

fs = 1000;

dt = 1/fs;

t = -1:dt:1;

f = 780;     % Least Common Multiple of x1 and x2


%%

%The combined signal

x1 = mySin(f1,A1,pha1,t,fs);

x2 = mySin(f2,A2,pha2,t,fs);
```

```
x = x1+x2;


%%

%Task1_1

figure

plot(t,x,'LineWidth',2)

xlabel('t')

ylabel('x')

title(['Harmonic Signal x(t)' ' NO.5130309535'])

xlim([min(t) max(t)])

set(gca,'Fontsize',15)


%%

%Task1_2

%CTFS


T = 1/f;

k = -50:1:50;

Xk = CTFS(x,t,T,k,f);

%

figure

subplot(211)
```

```
stem(k,real(Xk),'b','LineWidth',2)

xlabel('f(Hz)')

ylabel('real')

title(['real part of X_k(f)' ' NO.5130309535'])

xlim([min(k) max(k)])

set(gca,'Fontsize',15)

subplot(212)

stem(k,imag(Xk),'r','LineWidth',2)

xlabel('f(Hz)')

ylabel('imagery')

title(['imagery part of X_k(f)' ' NO.5130309535'])

xlim([min(k) max(k)])

set(gca,'Fontsize',15)


%%

%Task1_3


figure

subplot(211)

stem(k,abs(Xk),'b','LineWidth',2)

xlabel('f(Hz)')

ylabel('|X_k|')
```

```
title(['Modulus of X_k(f)' ' NO.5130309535'])

set(gca,'Fontsize',15)

subplot(212)

stem(k,angle(Xk),'r','LineWidth',2)

xlabel('f(Hz)')

ylabel('phase(X_k) (rad)')

title(['Phase of X_k(f)' ' NO.5130309535'])

set(gca,'Fontsize',15)


%%

%Task1_4

%Rectangle window


A3 = 1;

D = 0.2;

x3 = mySqu(A3,D,fs,t);

x4 = x3.*x;

dw = 0.01;

fLim = 500;     %limit of frequency

w = -fLim*2*pi : dw : fLim*2*pi;

X = CTFT(x4,t,w);

%
```

```
figure

subplot(211)

plot(t,x4,'LineWidth',2)

xlabel('t')

ylabel('x(t)')

title(['Harmonic Signal x(t) in [-0.1,0.1]' ' NO.5130309535'])

set(gca,'Fontsize',15)

subplot(212)

plot(w/2/pi,abs(X),'LineWidth',2)

xlabel('Frenquency')

ylabel('|X|')

xlim([-fs/2 fs/2])

title(['x(t) after CTFT' ' NO.5130309535'])

set(gca,'Fontsize',15)


%%

%Task1_5

%Rectangle window


x5 = mySqu(A3,0.4,fs,t);

x6 = x5.*x;

X1 = CTFT(x6,t,w);
```

```
figure

subplot(211)

plot(t,x6,'LineWidth',2)

xlabel('t')

ylabel('x(t)')

title(['Harmonic Signal x(t) in [-0.2,0.2]' ' NO.5130309535'])

set(gca,'Fontsize',15)

subplot(212)

plot(w/2/pi,abs(X1),'LineWidth',2)

xlabel('Frenquency')

ylabel('|X|')

xlim([-fs/2 fs/2])

title(['x(t) after CTFT' ' NO.5130309535'])

set(gca,'Fontsize',15)


%%

%Task1_6

%hamming window


t1 = -0.1 : dt/10 : 0.1;

t2 = -0.2 : dt/5 : 0.2;

L1 = length(t1);
```

```
h1 = hamming(L1);

r = rectwin(L1);

L2 = length(t2);

h2 = hamming(L2);

%

x11 = mySin(f1,A1,pha1,t1,fs);

x21 = mySin(f2,A2,pha2,t1,fs);

x01 = x11+x21;

x12 = mySin(f1,A1,pha1,t2,fs);

x22 = mySin(f2,A2,pha2,t2,fs);

x02 = x12+x22;

xhh1 = x01.*h1.';

xhh2 = x02.*h2.';

X01 = CTFT(xhh1,t1,w);

X02 = CTFT(xhh2,t2,w);


%[-0.1,0.1]

figure

subplot(211)

plot(t1,xhh1,'LineWidth',2)

xlabel('t')

ylabel('x(t)')
```

```
title(['Harmonic Signal x(t) in hamming window [-0.1,0.1]' '
NO.5130309535'])
set(gca,'Fontsize',15)
subplot(212)
plot(w/2/pi,abs(X01),'LineWidth',2)
xlabel('Frenquency')
ylabel('|X|')
xlim([-fs/2 fs/2])
title(['hamming windowed x(t) after CTFT' ' NO.5130309535'])
set(gca,'Fontsize',15)

%[-0.2,0.2]
figure
subplot(211)
plot(t2,xhh2,'LineWidth',2)
xlabel('t')
ylabel('x(t)')
title(['Harmonic Signal x(t) in hamming window [-0.2,0.2]' '
NO.5130309535'])
set(gca,'Fontsize',15)
subplot(212)
plot(w/2/pi,abs(X02),'LineWidth',2)
```

```
xlabel('Frenquency')

ylabel('|X|')

xlim([-fs/2 fs/2])

title(['hamming windowed x(t) after CTFT' ' NO.5130309535'])

set(gca,'Fontsize',15)


%%

%Task1_4 different form


X41 = CTFT(x01,t1,w);

%[-0.1,0.1]

figure

subplot(211)

plot(t1,x01,'LineWidth',2)

xlabel('t')

ylabel('x(t)')

title(['Harmonic Signal x(t) in [-0.1,0.1]' ' NO.5130309535'])

set(gca,'Fontsize',15)

subplot(212)

plot(w/2/pi,abs(X41),'LineWidth',2)

xlabel('Frenquency')

ylabel('|X|')
```

```
xlim([-fs/2 fs/2])
title(['x(t) after CTFT' ' NO.5130309535'])
set(gca,'Fontsize',15)


%[-0.2,0.2]
X42 = CTFT(x02,t2,w);
figure
subplot(211)
plot(t2,x02,'LineWidth',2)
xlabel('t')
ylabel('x(t)')
title(['Harmonic Signal x(t) in [-0.2,0.2]' ' NO.5130309535'])
set(gca,'Fontsize',15)
subplot(212)
plot(w/2/pi , abs(X42) , 'LineWidth',2)
xlabel('Frenquency')
ylabel('|X|')
xlim([-fs/2 fs/2])
title(['x(t) after CTFT' ' NO.5130309535'])
set(gca,'Fontsize',15)
```

Matlab code of Task 2

```
%% Task 2_1

t = 0 : 1 : 15000-1;

L = length(t);

figure

plot(t,y)

set(gca,'Fontsize',15)

ylabel('Amplitude/V')

title('Electricity wave signal')


%% Task 2_2

clc; clear;

close all;

%

fs = 1000;

load data.mat;

n = 2500;

h = hamming(n);

x = zeros(1,n);

for ii = 1:n

     x(ii) = h(ii)*y(ii);

end

%
```

```
w = -pi:2*pi/n:pi;

N = 1:n;

X = CTFT(x,N,w);

f = -fs/2:fs/n:fs/2;

figure(1)

subplot(211)

plot(f,abs(X),'LineWidth',2)

xlabel('Frenquency/f(Hz)')

ylabel('|X|')

title(['hamming window signal in [0,2500] after CTFT' ' NO.5130309535'])

xlim([-100 100])

set(gca,'Fontsize',15)

subplot(212)

plot(f,phase(X),'LineWidth',2)

xlabel('Frenquency/f(Hz)')

ylabel('Phase/rads')

set(gca,'YTick',-pi:pi/4:3*pi)

set(gca,'Fontsize',15)

xlim([-100 100])


%% Task2_3



clc; clear;
```

```
close all;

%

fs = 1000;

load data.mat;

n = 2500;

h = hamming(n);

x = zeros(1,n);

for ii = 1:n

     x(ii) = h(ii)*y(ii);

end

%

w = -pi:2*pi/n:pi;

X = fftshift(fft(x));

N = length(X);

f = linspace(-N/2,N/2 - 1, N) *fs/N;

figure(2)

subplot(211)

plot(f,abs(X),'LineWidth',2)

xlabel('Frenquency/f(Hz)')

ylabel('|X|')

title(['hamming window signal in [0,2500] after CTFT with fft' '
NO.5130309535'])
```

```
xlim([-100 100])

set(gca,'Fontsize',15)

subplot(212)

plot(f,phase(X),'LineWidth',2)

xlabel('Frenquency/f(Hz)')

ylabel('Phase/rads')

set(gca,'YTick',-pi:pi/4:3*pi)

set(gca,'Fontsize',13)

xlim([-100 100])
```