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T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**C**	F4 ________________

2018
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Summary Sheet

Summary

Based on Cellular Automata and Monte-Carlo method, we build a model to analysis the effects of self-driving cars on traffic flow.

First we break down the process of variance of traffic flow and establish SNED model for Time Headway, the model of P, D, V model to discrete the movement of traffic flow, the model of vehicles quantity change for the stability of quantity and the model of evaluation criteria to measure the degree of congestion.

Then we design rules to simulate different number of lanes and explore the effect it makes. Next we take the proportion of self-driving cars as the independent variable, we get the variance of the average speed of traffic flow when set up and do not set up special lanes, and then we get the tipping point (the percentage of self-driving car is 35%) to set up special lanes. We categorize the collaboration of self-driving car with other vehicles and the influence become greater as the density increases.

Then we use the classic Hydromechanics model in traffic flow to validate our CA model. Our simulation results and the validate model corroborate each other. Next, we changed the initial speed of simulation parameters and the Time Headway. Through the simulation, we find that the initial velocity has less effect on the model after a period of time and the Time Headway has a great influence on the simulation results.

Finally we analysis the strengths and weaknesses of our model and write a letter to the Governor’s office.

Keywords: **Cellular Automata, Monte-Carlo, Time Headway, Hydromechanics model**

Contents

1 Introduction

1.1 Literature review

Traffic flow modeling as the basis of traffic control, traffic design, traffic analysis, traffic simulation and traffic control decision making always is the the research focus in traffic engineering field. And it can be divided into macroscopic and microscopic models from the angle of study, mainly using the theory of fluid mechanics and the theory of car following[1]. Due to their accuracy and practicability, these two models play an important role in the development of traffic flow modeling[2].

The cellular automata (CA) is a kind of grid dynamical model in which time, space and state are discrete, and the space interaction and time causality are local. It has the ability to simulate the spatio-temporal evolution process of complex systems. The study of single lane traffic flow cellular automaton model began in 1986, the most famous of them is the NS model proposed by Nagel and Schreeckenberg in 1992[3]. However, this model has a great flaw, and has been gradually improved by follow-up scientists. Such as the T2 model[4] put forward by Takayasu and the VDR model[5] put forward by barlovic. Cellular automata is now always used to calculate traffic flow problems.

1.2 Restatement of the Problem

The drivers experience long delays during peak traffic hours, and self-driving cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads.

We need to build a model to measure the change when proportion of self-driving cars increases from 10% to 50% to 90% . First we should break down the process of variance of traffic flow. Then we should establish the sub model to discrete the movement of traffic flow, measure the degree of congestion and analysis the stability of quantity.

And then we should find ways to simulate the traffic flow. At the same time we need to design different rules to simulate the different number of lanes, categorize the collaboration of self-driving car with other vehicles and discuss the variance of the average speed of traffic flow about the set of special lanes.

Next we should try to validate our model and analysis the sensitivity of the model.

Finally, we should write a letter to the Governor's office.

2 Assumptions

(1) No overtaking. Because we mainly analyze the congestion in the road, overtaking is not supposed to happen in crowded situations.
(2) Road conditions and roadblocks are not considered. Because this part is not supported by data, we ignore this part.
(3) The distribution of vehicle flow at peak and ordinary times is uniform.
(4) The number of vehicles in each lane of the road is the same.

3 Symbols and Terminologies

3.1 Symbols

Table 1:Notation

Symbol	Meaning
C_i	the i th vehicle
t_i	the Time Headway between C_i and the front vehicle
T	the average Time Headway vector in a road
$F(t_i)$	the distribution function of t_i
S_i	the variance of the speed of all vehicles in the ith section of the road
V_i	the average speed of all vehicles in the section of the road
α_i	congestion evaluation index
n_i	the number of vehicles on a section of the road at a certain moment
t_{pi}	the length of the peak time
Q	the traffic flow vector
P	the position vector of all cars
D	the distance vector between adjacent cars
V	the speed vector of all cars
T	the Time Headway vector of all cars
K	the type vector of all cars

3.2 Terminologies

The symbol of matrix operation is basically referenced to MATLAB. **.*** means multiplying the corresponding positions in two matrices or vectors. For example

$$A. * B = [a_1 * b_1, a_2 * b_2 ..., a_n * b_n] \tag{3.1}$$

where A and B are vectors.

Similarly, ./ means the division of the corresponding position of the elements. And $A(j:k)$ means the slicing operation of a vector or matrix. For example

$$A(i:j) = [a_i, a_i + 1 ..., a_j] \tag{3.2}$$

4 Models

4.1 Data preprocessing

Data preprocessing is mainly to extract the data from the Excel and calculate the traffic flow of a single lane. And the daily traffic flow in each lane is

$$N = Counts/(Decr + Incr) \tag{4.1}$$

where N is average daily traffic counts on each lane. $Counts$ is average daily traffic counts year_2015, $Decr$ is the Number of Lanes DECR MP direction and $Incr$ is the

Number of Lanes Incr MP direction. And the influence of the number of lanes will be discussed in 5.1.1.

4.2 Design of Cellular Automata

Large quantities of former traffic simulations based on Cellular Automata (CA) indicate that CA model is a feasible and effective method to emulate traffic flow. Space, time and status are all discrete in Cellular Automata.

For example, The model abstracts the car into a particle, and time is divided into small units. This feature predigests the simulation process significantly. Besides, the status of a cell is controlled by its neighboring cells following a set of rules, which is much similar to real-life traffic where a car's movement largely depends on its neighboring cars' movements. Therefore, it is rational for us to apply Cellular Automata in solving our problem.

4.2.1 The data structure of cellular automata

We set 5 attributes for each car, including the position (p), the distance from the front car (d), the speed (v), the driver's head time (t), the type of the car (k).

The property vector of each car can be expressed as

$$car_i = [p_i, d_i, v_i, t_i, k_i] \tag{4.2}$$

The properties of all the cars can be superimposed and combined into a two-dimensional matrix, the matrix is as follows

$$Cars = \begin{bmatrix} car_1 \\ car_2 \\ \cdots \\ car_n \end{bmatrix} = \begin{bmatrix} p_1, d_1, v_1, t_1, k_1 \\ p_2, d_2, v_2, t_2, k_2 \\ \cdots \\ p_n, d_n, v_n, t_n, k_n \end{bmatrix} = [P, D, V, T, K] \tag{4.3}$$

where P, D, V, T, K are column vectors. For example

$$V = [v_1, v_2, ..., v_{n-1}, v_n]^T \tag{4.4}$$

P is a column vector for all vehicle locations, which unit is kilometer. D is the distance between each car and the previous one. D can be calculated as follows

$$D = [P^T(2:n) - P^T(1:n-1), 0]^T \tag{4.5}$$

V is a column vector for all vehicle speed, which unit is km/h. The initial value of V is a random vector which obeys normal distribution. T is the headway of all drivers or self-driving systems. And K is all vehicles' types. The meaning and value of the headway will be given in 4.3. There are two kinds of values in k_i, 0 means self-driving car and 1 means manual-driving car.

4.2.2 The Integral Structure of Cellular Automata

The model of Cellular Automata is mainly based the following aspects

Speed change mechanism
The change of speed is based on the Time Headway, the distance from the front car, the speed of the front car, and the type of the vehicle. The model will be given in 4.5.

The calculation of position and vehicle distance
The calculation of position and vehicle distance is based on the speed (V) and the position of the last moment. The model will be given in 4.5.

Changes in the number of cars
The inflow of vehicles in a section of the road consists of two parts. The first part is the inflow of the starting position of the road. The second part is the flow of cars from auxiliary road to the main road.
The outflow of vehicles include the outflow of the end position of the road and the flow from the main road to auxiliary road.
And the changes in the number of cars will be analyzed in the detail in 4.7.

The integral structure of single step of the simulation
Cellular automata simplifies the problem by discretization of the model. Figure 1 shows the single step of the simulation.

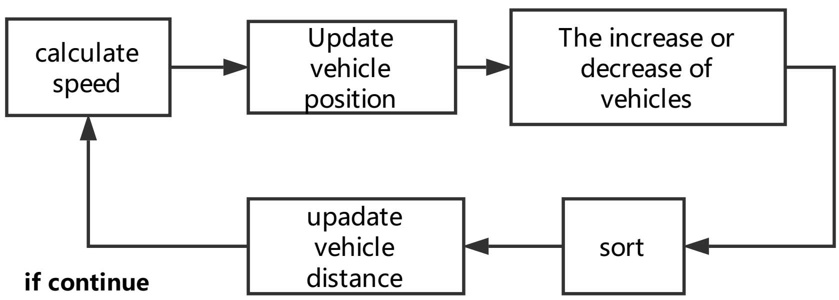


Figure 1: Single step of the simulation

We set the step length of the simulation to be 1 second. And the simulation results are analyzed and verified in the later paper.

4.3 Time Headway

4.3.1 The Set of Time Headway

Time Headway (TH) is the time difference between the two adjacent vehicles passing through the same location. In general, the TH is a random value. But when people drive a car, the front car has an emergency brake or deceleration in a very short time, the following car driver can respond only after a certain reaction time. It is easy to cause traffic accidents if the distance between the two cars is two small. So the TH between the two cars needs to be greater than a minimum safety TH. The safety TH is depended on the reaction time of person.

The TH of two vehicles is determined by the subjective judgment of the driver. But generally speaking, when the rear car is far away from the front car, the rear car will speed up and make the TH of two cars smaller. When the driver of rear is aware that the TH is too small, as the TH of reality is less than the TH he expected, the driver will slow down and increase the TH between two cars.

Then the ratio distribution of the headway of the car needs to be determined. We can know from the previous research[7], when there is a minimum safety TH and crowding, the shift negative exponential distribution(SNED) can well reflect the distribution of the headway of a large number of vehicles.

So the random variable follows the SNED model. It means,

$$F(t) = 1 - e^{-\frac{t_i - \tau}{T - \tau}} \tag{4.6}$$

where T is the average Time Headway in a road.

In traffic flow theory, traffic flow density

$$\begin{aligned} K &= \frac{Q}{V} \\ D_a &= \frac{1}{K} = \frac{V}{Q} \end{aligned} \tag{4.7}$$

where K is the traffic flow density, Q is the traffic flow, V is the average velocity of vehicles and D_a is the average distance of vehicles.

Because the average Time Headway is the ratio of the average distance D_a to the average velocity V,

$$T = \frac{D_a}{V} = \frac{1}{Q} \tag{4.8}$$

It means the average distance is the reciprocal of the traffic flow.

4.3.2 The generation of Time Headway following SNED model

When the parameters T, τ in $F(t)$ are obtained, we need to produce a large number of data that follows the SNED.

MATLAB can only produce a group of data following the uniform distribution. To produce a group of data following $F(t_i) = 1 - e^{-\frac{t_i - \tau}{T - \tau}}, t_i > \tau$, we consider a random variable,

$$t_i = F^{-1}(t_i) = G(u) = \tau - (T - \tau) \cdot ln(1 - u) \tag{4.9}$$

$G(u)$ is the inverse function of $F(t)$.
When $u \sim U(0,1)$, for some constant number $t_0 > \tau$,

$$\begin{aligned} P\{t_i \leqslant t_0\} &= P\{\tau \leqslant t_i \leqslant t_0\} \\ &= P\{G^{-1}(\tau) \leqslant u \leqslant G^{-1}(t_0)\} \\ &= \int_{G^{-1}(\tau)}^{G^{-1}(t_0)} f(u)du \\ &= \int_{F(\tau)}^{F(t_0)} f(u)du \\ &= \int_{F(\tau)}^{F(t_0)} 1du = F(t_0) - F(\tau) \end{aligned} \tag{4.10}$$

Because the minimum Time Headway is τ, $t_i$must be larger than τ ,so

$$F(\tau) = Pt_i < \tau = 0 \tag{4.11}$$

$$P\{t_i \leqslant t_0\} = F(t_0) \tag{4.12}$$

It means t follows the SNED $F(t_0)$.Therefore, the conclusion is obtained that when $u \sim U(0,1)$, function type random variable follows SNED with parameters τ, T.

Vector U is input, of which the element follows uniform distribution. And the output vector $T = \tau - (T - \tau) \cdot \ln(1 - U)$ follows SNED distribution.

4.4 The model of calculating P and D

4.4.1 Calculation of the initial P

P is a column vector including the location of all the vehicles (the unit of P is km). With the process of simulation, P is a vector that varies orderly. So we only need to consider how to determine the initial value of P.

In our CA model, vehicles are distributed across the whole road from the beginning of simulation. And we set the difference between every startMilepost and endMilepost as the section. From the given data sheet, we know that the traffic flow is different in different section of the road. So the distribution of vehicles is not uniform.

At the same time, the conclusion is obtained that the length of each section of the road is about 1 to 2 kilometers, and every road has about dozens of sections. So in one section of the road, the distribution has little influence on the whole distribution of vehicles. Therefore, we suppose that in every one section of road, the distribution of the vehicles is uniform.

From the given data sheet, we can get the start and end, The proportion of traffic flow in the peak period accounts for 8% of the whole and Q_i in every section of the road. By calculating, we can get the length of every section l_i and the number of vehicles n_i at some moment.

We discuss the value of n_i in two situation between the perus hour and non-peak time of the vehicle.

$$n_i = \begin{cases} \frac{0.08Q}{3600t_1} \cdot \frac{l_i}{V_i} & \text{in the peak time} \\ \frac{0.92Q}{3600(24-t_1)} \cdot \frac{l_i}{V_i} & \text{in the non-peak time} \end{cases} \tag{4.13}$$

While v_i means the average speed of cars and t_i is the length of the rush hour.

$$p_i = (i-1) \cdot \frac{l_n}{n_i} + p_{start} \qquad i = 1, 2, 3, n \tag{4.14}$$

where p_{start} is the start mile post.

The p_i refers to the distance between the ith car and the starting point of the road.

4.4.2 Calculation of D

After we get the position vector P of all vehicles, we can update the vehicle distance vector D. We can update all the data by the operation of the vector.The calculation method is as follows

$$\begin{gathered} d_i = p_{i+1} - p_i \\ D = [P(2:n) - P(1:n-1); 0] \end{gathered} \tag{4.15}$$

4.5 The model of calculating V

4.5.1 ΔV of Manual-driving car

Because people want to keep their time headway as stable as possible, the first part of speed change is based on time headway. The difference between actual and expected Time Headway is

$$\Delta t_i = t_i - \frac{d_i}{v_i} \tag{4.16}$$

We first directly put Δt as the coefficient of vehicle acceleration. But we find the range of T is very large. So we use tanh function which is widely used in Neural network to normalize it.

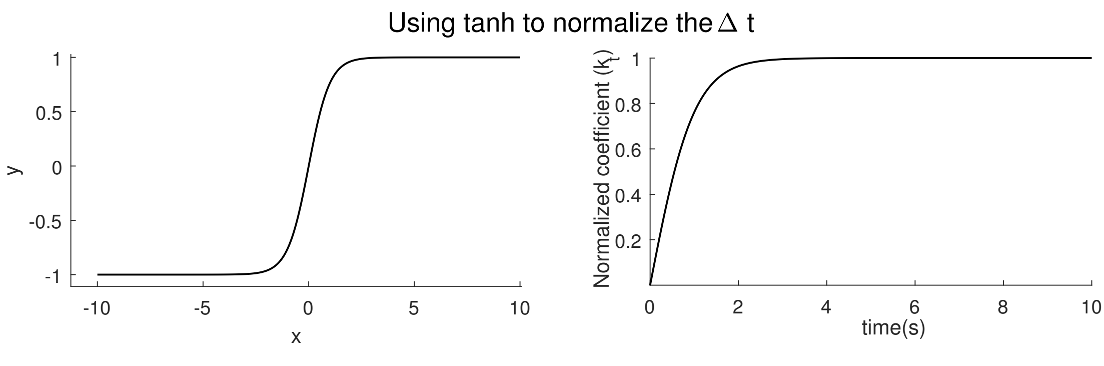


Figure 2: Normalization of Δt_i by tanh function

In the form of vector, we can get the following

$$\begin{gathered} \Delta T = T - D./V \\ K_t = tanh(\Delta T) \end{gathered} \tag{4.17}$$

where $t_n = 0$.

And the velocity variation ΔV_1 caused by ΔT is

$$\Delta V_1 = K_t. * A \tag{4.18}$$

where A is a vector of random a_i, and a_i is the maximum acceleration of the i_{th} vehicle with uniform distribution. The unit of a_i is m/s^2 and $a_i \sim U(1.1, 2.1)$.

When the speed of the rear car is higher than that of the front car, the acceleration of the rear car will be reduced to prevent pileup. The velocity variation ΔV_2 caused by reducing acceleration in advance.

$$\begin{aligned} \Delta v_2(i) &= k(v(i) - v(i+1)) \\ \Delta V_2 &= k(V - [V(2:n); 0]) \end{aligned} \tag{4.19}$$

where $v_2(n) = 0$.

Through the experiment, we set k to 0.15. And the amount of change in the speed of a single simulation step is

$$\Delta V = \Delta V_1 - \Delta V_2 \tag{4.20}$$

4.5.2 ΔV of Self-driving car

The analysis of velocity variation ΔV_1 caused by ΔT is basically the same with the Manual-driving car. But the precision of autopilot is much higher. The Time Headway of an automatic driving vehicle can be shorter than that of a man. So we can set the Time Headway of automatic to be a constant. If $k_i = 0$,$t_i = 2$.

In addition to having less TH time, autopilot can have a more stable speed. We discuss the speed according to the type of car and it's front car.

Cooperation between self-driving cars

If the types of car and it's front car are both self-driving car, we can assume that when the distance between the two cars is in a certain range, the rear car can keep the same speed with the front car.

The rules are described in the mathematical language as follows

$$\begin{cases} v_i = v_{i+1} & if \quad p_{i+1} - p_i < d_{min} \\ v_i = v_i + dv & else \end{cases} \tag{4.21}$$

Interaction between self-driving and manual-driving vehicles

When the front is manual-driving car, the rear car can not predict the change of speed well, so based on the above rules, we add random velocity perturbations. The rules are described in the mathematical language as follows

$$\begin{cases} v_i = v_{i+1} + r & if \quad p_{i+1} - p_i < d_{min} \\ v_i = v_i + dv & else \end{cases} \tag{4.22}$$

where r is a random variable, and dv is calculated the same as manual-driving car.

4.6 Model of evaluation criteria for the degree of congestion

4.6.1 Use V_i to evaluate the degree of congestion

- **When there is little congestion.** When the speed distribution of all the vehicles on the road is very uniform, or even the speed of each vehicle is almost the same, the vehicles on the whole section of road needn't speed up or speed down. Each vehicle can keep the same fast speed, and because every two vehicles can be regarded as relatively stationary, there will almost be no congestion. Because the speed distribution is very uniform, the deviation between the speed and the average speed of all the vehicles is small.

- **When there is some congestion.** When there is a congestion on a road, the average velocity of the vehicles must be slow. When there is a congestion only in one part of the road, the velocity in this part must be much lower than the average velocity V_i. So the speed distribution must be not uniform. Therefore, the deviation between the speed and the average speed of each vehicle is large.

- **How to evaluate the degree of congestion by V_i.** The conclusion is obtained that the degree of congestion must be related with the average speed of the speed of all vehicles in the section of the road and the deviation between the speed and average speed of all the vehicles.

4.6.2 Use α_i to evaluate the degree of congestion

For some section of the road, at some moment t, the velocity of every vehicle can be obtained. So we can use V_i as the first index to evaluate the degree of congestion. Therefore, the smaller V_i of a road is, the more serious the crowding the road is.

- **How to evaluate the degree of congestion by α_i.** And we use the standard deviation of the speed of all the vehicles S_i to evaluate the degree of congestion. However, the larger the average velocity is, the larger the absolute value of the deviation is. To represent the degree of the road more objectively, we use relative deviation $\alpha_i = \frac{S_i}{V_i}$ as the congestion evaluation index. The larger α_i is, the more serious the crowding the road is.

4.6.3 Comprehensive evaluation by V_i and α_i

Our evaluation criteria for the degree of congestion includes: V_i, α_i. More crowding road must have a smaller V_i and a larger $alpha_i$.

4.7 The model of changes in the number of cars

4.7.1 Monte Carlo Method

Next, we consider the Monte Carlo method to simulate the change in the number the vehicle.

Monte Carlo method is a stochastic simulation method based on probability and statistical theory. It is a method of using random numbers to solve many computational problems. In order to obtain the approximate solution of the problem, the problem is connected with a certain probability model.

4.7.2 The Establishment of the Model

From the known data, we can get the traffic inflow on each road section of the day. The value of net inflow or net outflows of cars is the difference between the traffic flow of two adjacent road sections for one day.

In the model of the cellular automata, the time to simulate one step is a second. So we should consider the net flow of cars at unit time as Δn. And the traffic conditions on the road divided into the peak hour and non peak hour. We can get

$$\Delta n = \begin{cases} (Q_i - Q_{i-1}) \cdot \frac{0.08}{t_i \cdot 3600} & \text{in the peak time} \\ (Q_i - Q_{i-1}) \cdot \frac{0.92}{(24-t_i) \cdot 3600} & \text{in the non-peak time} \end{cases} \tag{4.23}$$

Where i means the ith road section.

We use the matrix P_{change} to represent the probability distribution of all the road sections inflow and outflow. So the matrix can be expressed as

$$P_{change} = [\Delta n_1, \Delta n_2, \Delta n_3, \cdots, \Delta n_n] \tag{4.24}$$

By calculation, we found that Δn is a decimal less than 1. We can't use this value directly as the net flow of cars in each simulation step of the cellular automata. Because the number of vehicles added or reduced in each step must be an integer.

So we use the Monte Carlo Method to simulate.

We generate a random vector R

$$R_{random} = [r_1, r_2, r_3, \cdots, r_n] \tag{4.25}$$

the length of which is the quantity of road sections as n. And the range of the random number is 0 to 1.

The situation of vehicles flow is

$$\Delta n_i = \begin{cases} 0 & \text{if } P_{change}[i] < R_{random}[i] \\ 1 & \text{if } P_{change}[i] > R_{random}[i] \end{cases} \tag{4.26}$$

It means when the value of $P_{change}[i]$ is smaller than the value of $R_{random}[i]$, Δn_i is assigned to 1. When the value of $P_{change}[i]$ is larger, Δn_i is assigned to 0.

5 Simulation Result and Data Analysis

5.1 The effects of lanes

5.1.1 The effects of the number of the lanes

We discuss the effects on our model from the number of lanes of roads.

We use V_i, α_i index to evaluate the degree of congestion. The number of lanes of roads increases 1,2,3 respectively. And a simulation within 60 minutes is implemented.

The figure below shows the variance of V_i and α_i within 60 minutes when different number of lanes of roads is added.

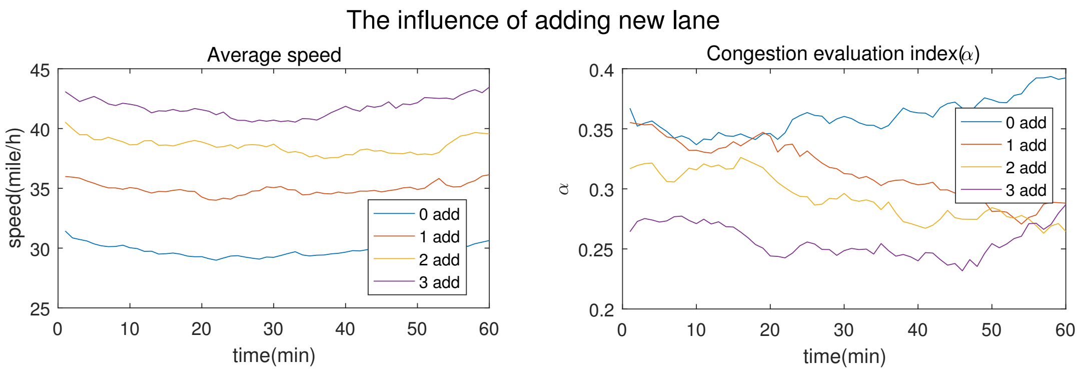


Figure 3: the influence of adding new lane

From the above figure, when the number of lanes of roads increases, V_i will remarkably increase, and the value of α_i will remarkably decrease. And it means that when the number of lanes increases, there will be less congestion, and the road will be more smooth.

5.1.2 When to set lanes to be dedicated to the self-driving cars

For each road, we can get the tipping point where we can set up accommodation lanes to make the road more smooth.

We took the 90th road for example. We compare the V_{90} when two and no accommodation lanes are set up with different proportion of self-driving cars.

The horizontal ordinate represents the different proportion of self-driving cars, and the vertical ordinate represents the value of V_{90}.

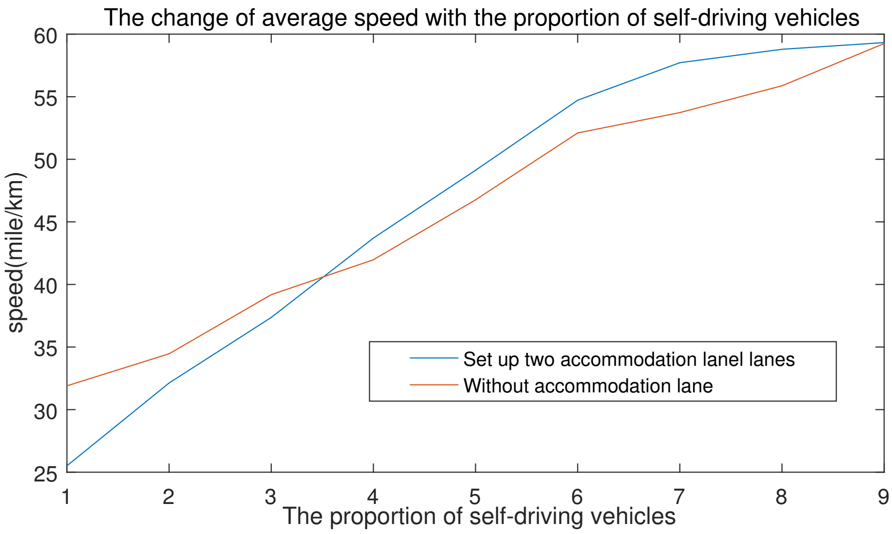


Figure 4: the change of average speed with the proportion of self-driving vehicles

In the above figure, there is an intersection of the two curves. The intersection can be regarded as a tipping point when we will need to set up accommodation lanes for self-driving cars. We can know that the tipping point is approximately 35%.

When the proportion of self-driving is lower than 35%, because the number of the self-driving cars is small, the utilization rate of the road is not high, Therefore, V_{90} is lower than the road without accommodation lanes.

When the proportion of self-driving is higher than 35%, because the self-driving cars will drive on the road smoothly at a fast velocity. The utilization rate of the road is high. Therefore, V_{90} is larger, and the road is more smooth.

5.2 The effects of percentage of self-driving in Peak and Non-Peak travel hours

The traffic flow is influenced by percentage of self-driving in Peak and Non-Peak travel hours. We discuss the situation in Peak and Non-Peak travel hours.

5.2.1 the effects of Peak and Non-Peak travel hours

- **the actual traffic flow in different time in Peak travel hours.** The quantity of the vehicle density in some time(15min,30min,45min,60min from the beginning) is described by the color in the following figure.

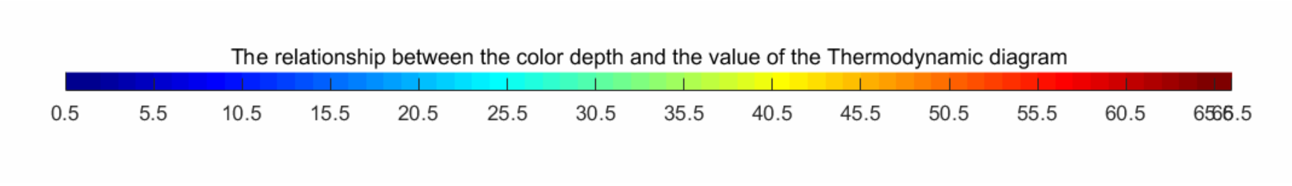


Figure 5: the relationship between color depth and value of thermodynamic diagram

We offer a standard color card to show the relationship between the color depth and the value of the thermodynamic diagram.

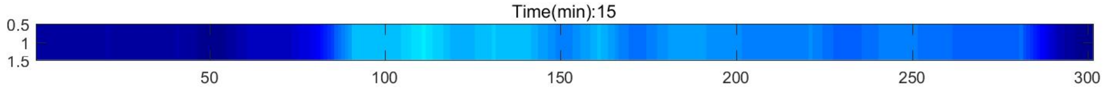


Figure 6: The vehicle density distribution of the road at 15min

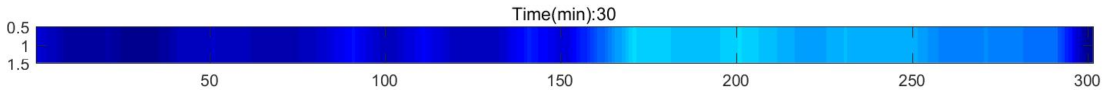


Figure 7: The vehicle density distribution of the road at 30min

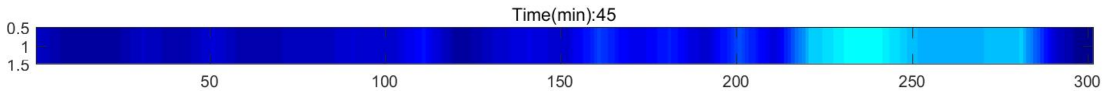


Figure 8: The vehicle density distribution of the road at 45min

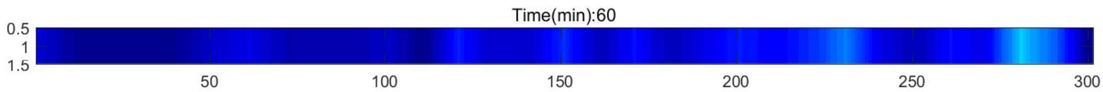


Figure 9: The vehicle density distribution of the road at 60min

In Non-Peak travel hours, the color is light, so the actual road is very smooth, and all the vehicles can travel fast. And the density of the road is small, so there is almost no congestion.

- **the actual traffic flow in defferent time in Non-Peak travel hours.** The quantity of the vehicle density in some time(15min,30min,45min,60min from the beginning) is described by the color in the following figure.

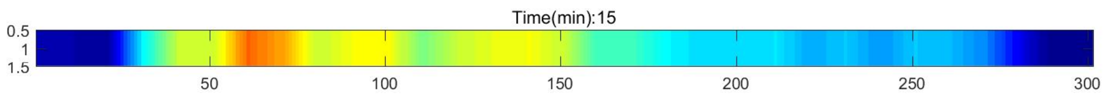


Figure 10: The vehicle density distribution of the road at 15min

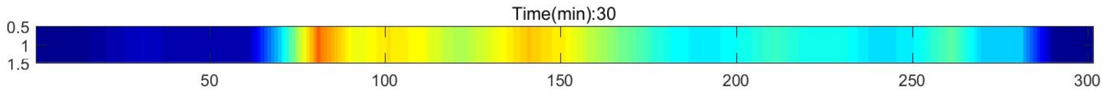


Figure 11: The vehicle density distribution of the road at 30min

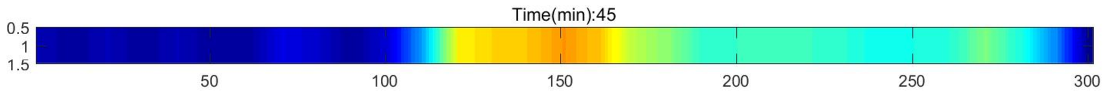


Figure 12: The vehicle density distribution of the road at 45min

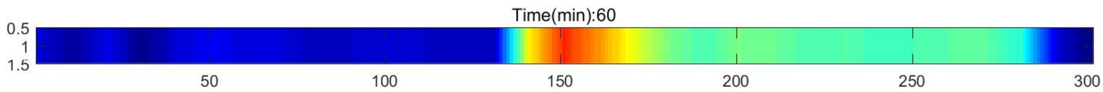


Figure 13: The vehicle density distribution of the road at 60min

In Peak travel hours, the density of the middle part of the road is so large. Therefore, there must be a congestion in the middle part of the road.

5.2.2 The Effects of Different Proportion of Self-Driving

When different proportion of self-driving are set, we use the evaluation criteria V_i, α_i in 4.6 to evaluate the degree of congestion.

When the proportion of self-driving is $0\%, 10\%, 50\%, 90\%$, the image of variations of V_i and α_i within 60 minutes are obtained respectively.

The figure below shows within the peak travel hours, the variations of V_i and α_i within 60 minutes.

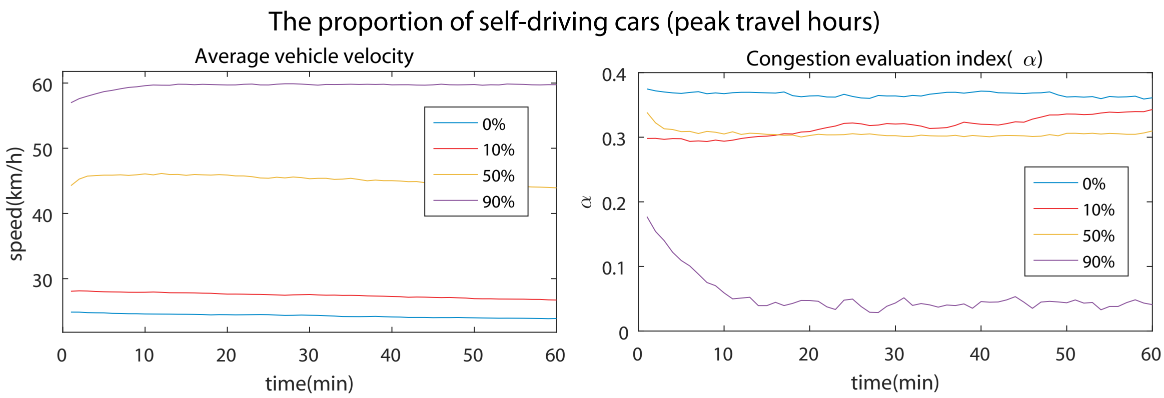


Figure 14: The proportion of self-driving cars(peak travel hours)

The figure below shows within the peak travel hours, the variations of V_i and α_i within 60 minutes.

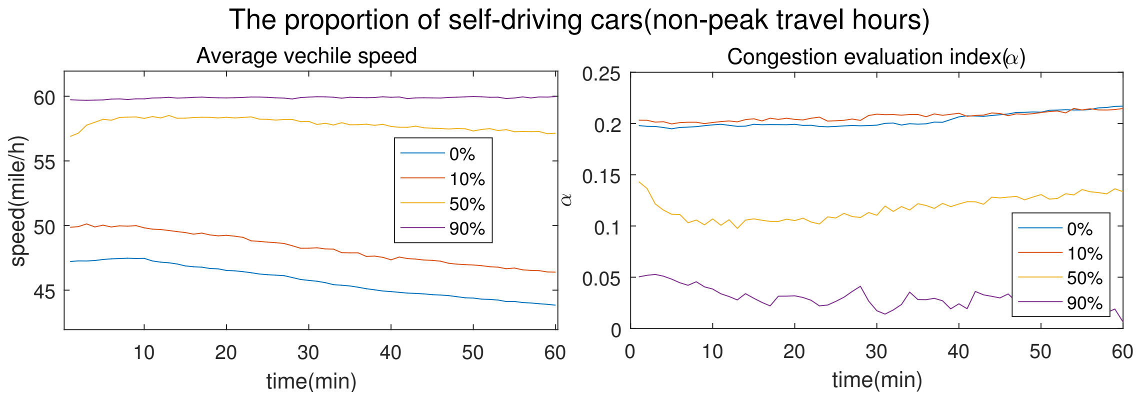


Figure 15: The proportion of self-driving cars(non-peak travel hours)

The above figure shows in peak travel hours or non-peak hours, when the proportion of self-driving increases, the V_i will increase and the α_i will decrease, which indicates the congestion will decrease with the proportion of self-driving increasing.

And when proportion transfers from 50% to 90%, the variation is remarkable. When the proportion reach 90%, the V_i approaches the maximum allowed speed 60 miles per hour. So the operation efficiency of the road approaches maximum value. And there is little congestion occurred.

5.3 The effects of cooperating systems

And then we discuss the effect of the cooperating systems of the cellular automaton. The rule of the cooperating system has been established earlier.

We mainly use two indexes of V_i and α_i to analyze the model. So taking advantage of the rule of our cooperative relationship, we simulate the cellular automaton under the different percentage of the self-driving cars for 4 roads in one hour.

Figure 16: The influence of self-driving car cooperation

The numbers from 1 to 4 on the abscissa refer to four different roads. Every road has six data and set the two adjacent data as a group which refers to the situations that the proportion of self-driving is 10%, 50% to 90%. In each group, the column on the left indicates no cooperating rules adding, and is opposite to the right.

We can know from the figure that V_i has increased and α_i has decreased when adding the cooperation rules. And different roads have the similar situations.

And we also can know from the figure that the more dense traffic flow is, the more obvious the raise of the speed is. Because when the traffic flow is dense, it's easy to cause a traffic jam and make the vehicle slow down.

Equally, the overall impact of the V_i and α_i about the cooperation rules is slightly small as the self-driving vehicle can accurately control the speed of itself.

6 Model Validation

6.1 The Hydromechanics model in Traffic Flow

The related search use The Hydromechanics model to describe the process of traffic flow.

- **Aggregation wave.** A traffic flow wave moves from a low density to a high density state. For example, when the front road is very crowded, the front vehicles will speed down, and the rear car will speed down one after the other. The process is described as an aggregation wave that passes a signal from one vehicle to another vehicle.

- **Seperation wave.** A traffic flow wave moves from a high density to a low density state. For example, when the front road is very smooth, the front vehicles will speed up, and the rear car will speed up one after the other. The process is described as an aggregation wave that passes a signal from one vehicle to another vehicle.

- **Fluctuation of traffic flow.** Because of the existence of Aggregation wave and Seperation wave, some congested or smooth situation in the road will be passed by the wave rather than gather at a certain section of the road.

- **The speed of wave W.** The speed of Aggregation wave or Seperation wave.

6.2 The calculation of W

The state of the traffic flow is described by the K, v.

Suppose the initial state is (K_1, v_1), which means there are K_1 vehicles per meter, and the velocity of the vehicles is v_1. And the state of the next moment is (K_2, v_2).

The W is calculated by the equation:

$$W = \frac{K_1 v_1 - K_2 v_2}{K_1 - K_2} \tag{6.1}$$

6.3 The discussion about the wave

When $K_2 > K_1$, it means the traffic flow wave moves from a low density to a high density state. So the wave is an aggregation wave. And if $Q_2 > Q_1$, the W is a positive value, which means the direction of the aggregation wave is the same as that of the traffic flow. If $Q_1 < Q_2$, the W is a negative value, which means the direction of the aggregation wave is opposite to that of the traffic flow.

And we can discuss the situation when $K_2 < K_1$The table below shows the different situation.

Table 2: the direction and class of the wave in different situation

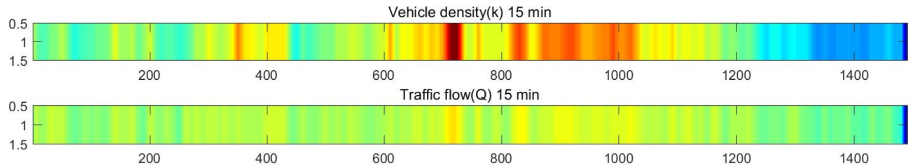
	$K_1 < K_2$	$K_1 > K_2$
$Q_1 < Q_2$	**Direction**: Opposite to that of traffic flow; **Class**: Aggregation wave	**Direction**:The same as that of traffic flow; **Class**: Seperation wave
$Q_1 > Q_2$	**Direction**: The same as that of traffic flow; **Class**: Aggregation wave	**Direction**:Opposite to that of traffic flow; **Class**: Seperation wave

6.4 Use the Hydromechanics model to verify our CA model

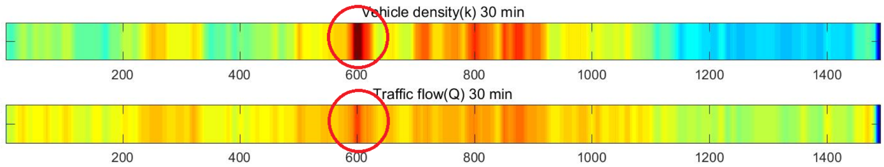
In our simulation, we get the space distribution of vehicle density K and traffic flow Q in some time(15min, 30min, 45min, 60min from the beginning).

The depth of the color in the figure represents the quantity of the vehicle density K and traffic flow Q. And deeper the color is, larger the vehicle and the traffic flow is. The horizontal axis represents the distance from start of a section of the road.

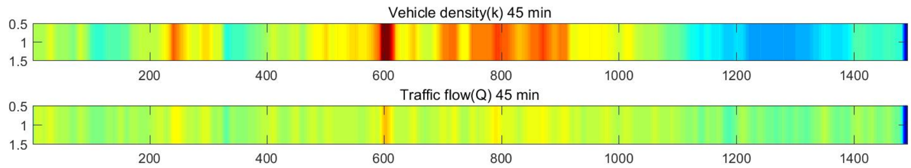
We pay attention to the congestion part of the section, and tag this part with a red circle.



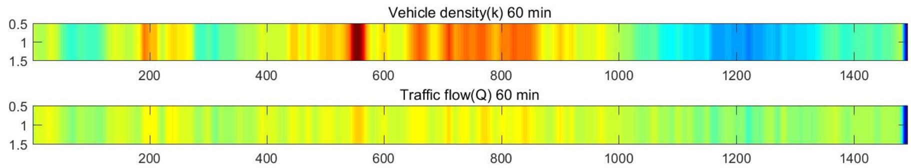
(a) Spatial distribution of k,Q at 15 minutes



(b) Spatial distribution of k,Q at 30 minutes



(c) Spatial distribution of k,Q at 45 minutes



(d) Spatial distribution of k,Q at 60 minutes

Figure 17: Spatial distribution of k,Q at different moments

In the figure, we can find the K_{red} and Q_{red} is the largest in the road. So K_2 must be smaller than K_1, Q_2 must be smaller than Q_1. So according to the Table 1, there must be a Seperation wave of which the direction is to the end of the road and an Aggregation wave of which the direction is to the start of the road.

Therefore, the left part of the tagged part will be more crowded, the right part of the tagged part will be more smooth.

And it means with the time goes, the crowded part(tagged) will shift to the left. So we observe the change of location of the tagged part. From the above figure, we find that actual change of the location of the tagged part is consistent with the classic Hydromechanics model in traffic flow.

Therefore, our model and simulation are reliable.

7 Sensitivity Analysis

7.1 Initial velocity of simulation

Our initial simulation condition is set to the starting speed of all vehicles is 50mile/h. In order to verify the stability of our model to the parameter $v_{initial}$, we set the parameter to 20,30,40,50 mile/hour, and get the following figure by simulation

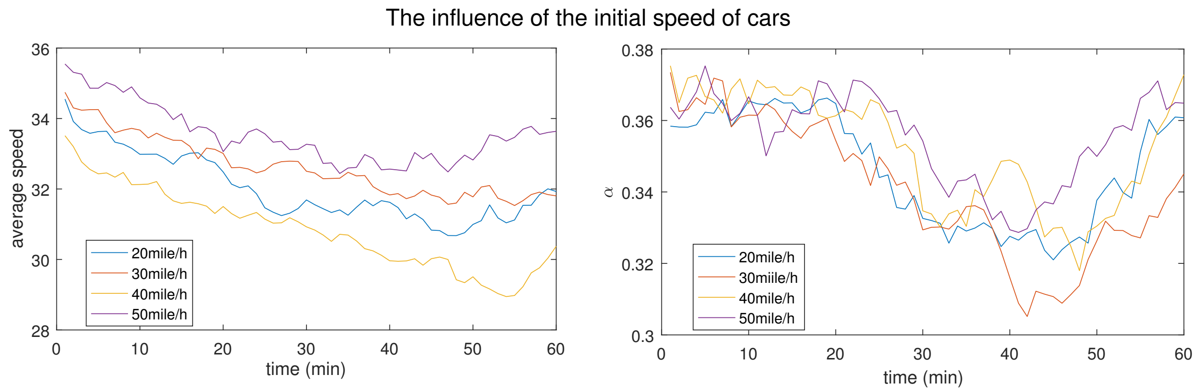


Figure 18: The proportion of self-driving cars(peak travel hours)

Through the picture, we can find that at different initial speeds, after a period of time, the simulation results are basically the same. And we can find the changing ranges of A and B are both small. (The simulation conditions are Highway 90, 10% of the self-driving vehicle ratio.)

So we verify the stability of our model to the parameter $v_{initial}$

7.2 Change of TH

In the previous papaer, we assume that the minimum TH is 4s and the average of TH is 6s. In order to find how the simulation change with HT. We set the parameter to (3,7)s, (4,7)s, (5,7)s and (6,7)s, which means (the shortest TH, the average HT). We get the following figure by simulation.

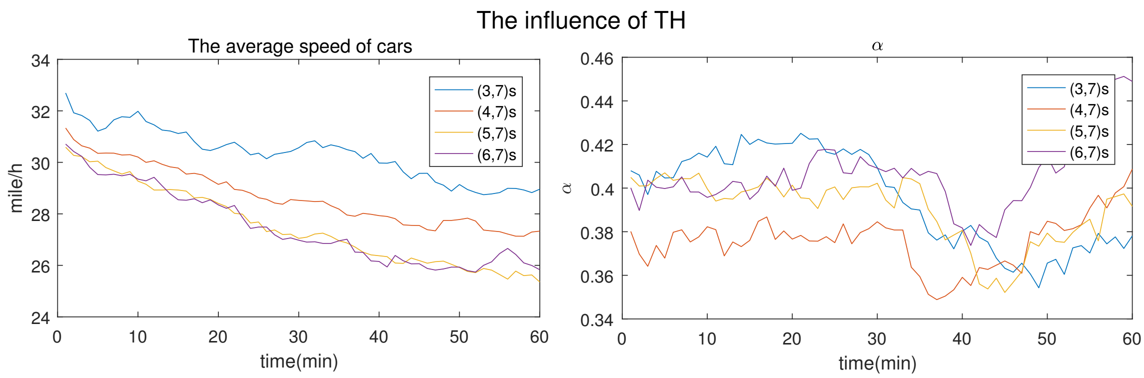


Figure 19: The influence of TH

We can find that the average speed of the vehicle decreases and the inhomogeneity of velocity decreases with the increase of TH. The result of this simulation is the same as our analysis. Because the increase of TH, the distance between two cars increase, and the distribution is more uniform.

8 Strengths and Weaknesses

8.1 Strengthes

- **Our model is reliable.**Our model is related with some classic model such as Hydrodynamics and Cellular Automata model, these model are verified to be reliable.

- **Our models are capable of simulating the situation in real process.**The self-driving can improve the efficiency of transportation in our results of models, which is consisted with the real process.

- **Two comprehensive evaluation criteria of the degree of congestion are given.**Our model considers the weaknesses to use average velocity to measure the degree of congestion, so we build the other evaluation criteria.

8.2 Weaknessess

- **Our model consider little about some exceptional case.**Once there are exceptional cases like road closure, fog weather and other unpredictable factors, our model will be influenced remarkably.

9 A letter to the Governor's office

As the population grows and the number of private cars has increasingly becoming large, the drives will experience long delays when there is congestion in the road. However, it costs a lot to build a new road, and the fundamental problem still can not be solved.

With the progress of science and technology, the technology of self-driving has become more and more mature. We can consider increasing the proportion of self-driving to reduce road congestion. In our paper, our model discuss the influence on the traffic flow from different proportion of self-driving in peak travel hours and non-peak travel hours. Because the self-driving is controlled by sensors and computers, they will spend little time to make a corresponding reaction. Therefore, the Time Headway between two self-drivings can be very short, and the distance between them can be close.

In our paper, our model discusses the effects on the number of lanes. The conclusion is obtained that when the proportion of the self-driving is greater than 35%, it is a good idea to set up two lanes in the road to make the road more smooth. Then we discuss the influence on the traffic flow from different proportion of self-driving in peak travel hours and non-peak travel hours. Because the self-driving is controlled by sensors and computers, they will spend little time to make a corresponding reaction. Therefore, the Time Headway between two self-driving cars can be very short, and the distance between them can be close. In our model, we can find the larger the proportion of the self-driving is, the more smooth the road is, and the less congestion there is.

At the same time, the self-driving vehicles can arrive at the destination automatically. The driver will not be banned from drinking, sleeping when the vehicles travel on the road. We make some suggestion for some policy changes:

1.Give allowance or impose less taxes when citizens purchase self-driving.

2.More road toll is paid when the manual-driving passes the toll-gate.

3.When the proportion of self-driving is greater than 35%, use a few lanes as a special driveway for self-driving vehicles.

Therefore, use a large proportion of self-driving can reduce the congestion effectively. If our suggestion is adopted, the transportation in Washington will be very smooth, and citizens will experience a comfortable and time-saving travel.

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