# CW1

## problem 1

### a

$P = (\nu\, c)(\bar{b}\langle c\rangle | b(x).\, P_1)$

$f_n(P) = f_n(\bar{b}\langle c\rangle | b(x).\, P_1) \backslash \{c\}$

$= (\{b, c\} \cup \{b\} \cup f_n(P_1)) \backslash \{c\}$

$= \{b\} \cup f_n(P_1) \backslash \{c\}$

$f_v(P) = f_v(\bar{b}\langle c\rangle | b(x).\, P_1)$

$= \emptyset \cup f_v(b(x).\, P_1)$

$= \{x\} \cup f_v(P_1)$

### b

$Q = (\nu\, b)(b(x).\, Q_1 | \bar{b}\langle c\rangle | x(y).\, Q_2)$

$f_n(Q) = fn(b(x).\, Q_1 | \bar{b}\langle c\rangle | x(y).\, Q_2) \backslash \{b\}$

$= (\{b\} \cup f_n(Q_1) \cup \{b, c\} \cup f_n(Q_2)) \backslash \{b\}$

$= \{c\} \cup ((f_n(Q_1) \cup f_n(Q_2)) \backslash \{b\})$

$f_v(Q) = f_v(b(x).\, Q_1 | \bar{b}\langle c\rangle | x(y).\, Q_2)$

$= (f_v(Q_1) \backslash \{x\}) \cup \emptyset \cup (\{x\} \cup (f_v(Q_2) \backslash \{y\}))$

$= (f_v(Q_1) \backslash \{x\}) \cup (\{x\} \cup (f_v(Q_2) \backslash \{y\}))$

$= f_v(Q_1) \cup (f_v(Q_2) \backslash \{y\})$

### c

$R = (\nu\, a)(!a(x).\, \bar{c}\langle y\rangle | a(x).\, R_1 | b(y).0)$

$f_n(R) = f_n(!a(x).\, \bar{c}\langle y\rangle | a(x).\, R_1 | b(y).0) \backslash \{a\}$

$= (\{a, c\} \cup \{a\} \cup f_n(R_1) \cup \{b\}) \backslash \{a\}$

$= \{b, c\} \cup (f_n(R_1) \backslash \{a\})$

$f_v(R) = f_v(!a(x).\, \bar{c}\langle y\rangle | a(x).\, R_1 | b(y).0)$

$= f_v(!a(x).\, \bar{c}\langle y\rangle) \cup (f_v(R_1) \backslash \{x\}) \cup \{y\}$

$= \{y\} \cup (f_v(R_1) \backslash \{x\}) \cup \{y\}$

$= (f_v(R_1) \backslash \{x\}) \cup \{y\}$

# problem 2

## a

$((\nu\ a, b)(!\bar{b}\langle x\rangle|\bar{b}\langle a\rangle|!(\nu\ c)b(y).\ P))\{b/x, a/y\}$

$= ((\nu\ a, b')(!\bar{b'}\langle x\rangle|\bar{b'}\langle a\rangle|!(\nu\ c)b'(y).\ P))\{b/x, a/y\}$

$= ((\nu\ a, b')(!\bar{b'}\langle b\rangle|\bar{b'}\langle a\rangle|!(\nu\ c)b'(y).\ P\{b/x\}))\{a/y\}$

$= ((\nu\ a, b')(!\bar{b'}\langle b\rangle|\bar{b'}\langle a\rangle|!(\nu\ c)b'(y).\ P\{b/x\}))$

## b

The substitution is not correct.

$((\nu\ a)(\bar{x}\langle x\rangle|a(x).\ (\nu\ a)\bar{y}\langle a\rangle))\{a/x, b/y\}$

$= ((\nu\ a')(\bar{x}\langle x\rangle|a'(x).\ (\nu\ a)\bar{y}\langle a\rangle))\{a/x, b/y\}$

$= ((\nu\ a')(\bar{a}\langle a\rangle|a'(a).\ (\nu\ a)\bar{y}\langle a\rangle))\{b/y\}$

$= ((\nu\ a')(\bar{a}\langle a\rangle|a'(a).\ (\nu\ a)\bar{b}\langle a\rangle))$

The righthand side given in the question is $((\nu\ a)(\bar{a}\langle a\rangle|a(a).\ (\nu\ a)\bar{b}\langle a\rangle))$. It not correct since $a$ clashes.

# problem 3

## a

if $a \notin f_n(P)$, then:

lefthandside $(\nu\ a)Q|P|!(P|(\nu\ a)Q)$

$\equiv (\nu\ a)(P|Q)|!((\nu\ a)(P|Q))$, which is equal to the right hand side.

But $a \notin f_n(P)$ is not given, so they are not structurally congruent.

## b

The left hand side is $(\nu\ a)\bar{c}\langle a\rangle|(!(\bar{a}\langle x\rangle|b(y).0))$

$\equiv (\nu\ a')\bar{c}\langle a'\rangle|(!(\bar{a'}\langle x\rangle|b(y).0))$

$\equiv (\nu\ a')\bar{a}\langle a'\rangle|(!(\bar{a'}\langle x\rangle|b(y).0))$

$\equiv (\nu\ c)\bar{a}\langle c\rangle|(!(\bar{c}\langle x\rangle|b(y).0))$

The right hand side is $\bar{a}\langle c\rangle|(\nu\ c)(!(\bar{c}\langle x\rangle|b(y).0))$

The only difference between them is the position of the restriction. Therefore, they are not structurally congruent.

## c

They are not structurally congruent. On the left hand side, $a$ in $a(x).\ \bar{a}\langle a\rangle$ is bounded, but on the right hand side, $b$ in $b(x).\ \bar{b}\langle b\rangle$ is a free name.

# problem 4

## a

$(\nu\, b)(a(x).\, \bar{x}\langle b\rangle)|!(\bar{a}\langle b\rangle|b(x).0)$

$\equiv (\nu\, b)(a(x).\, \bar{x}\langle b\rangle)|(\bar{a}\langle b\rangle|b(x).0)|!(\bar{a}\langle b\rangle|b(x).0)$

$\equiv (\nu\, b)(a(x).\, \bar{x}\langle b\rangle)|\bar{a}\langle b\rangle|b(x).0|!(\bar{a}\langle b\rangle|b(x).0)$

$\equiv (\nu\, c)(a(x).\, \bar{x}\langle c\rangle)|\bar{a}\langle b\rangle|b(x).0|!(\bar{a}\langle b\rangle|b(x).0)$

$\rightarrow (\nu\, c)\bar{b}\langle c\rangle|b(x).0|!(\bar{a}\langle b\rangle|b(x).0)$

$\rightarrow 0|!(\bar{a}\langle b\rangle|b(x).0)$

$\equiv !(\bar{a}\langle b\rangle|b(x).0)$

## b

$!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)|\bar{a}\langle e\rangle$

$\equiv a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)|\bar{a}\langle e\rangle|!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)$

$\rightarrow \bar{b}\langle e\rangle|\bar{c}\langle e\rangle|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)|!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)$

$\equiv a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)|\bar{a}\langle e\rangle|!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)$

$\equiv \bar{b}\langle e\rangle|\bar{c}\langle e\rangle|b(y).\, (\bar{a}\langle y\rangle|c(y).0)|!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)$

$\rightarrow \bar{c}\langle e\rangle|\bar{a}\langle e\rangle|c(e).0|!a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)$

$\equiv !a(x).\, (\bar{b}\langle x\rangle|\bar{c}\langle x\rangle)|!b(y).\, (\bar{a}\langle y\rangle|c(y).0)|\bar{a}\langle e\rangle$

## c

$\bar{a}\langle a, b, c\rangle|!a(x, y, z).\, \bar{y}\langle x, z, y\rangle|!b(x, z, y).\, \bar{x}\langle x, z, y\rangle$

$\rightarrow \bar{b}\langle a, c, b\rangle|!b(x, z, y).\, \bar{x}\langle x, z, y\rangle|!a(x, y, z).\, \bar{y}\langle x, z, y\rangle$

$\rightarrow \bar{a}\langle a, c, b\rangle|!a(x, y, z).\, \bar{y}\langle x, z, y\rangle|!b(x, z, y).\, \bar{x}\langle x, z, y\rangle$

## d

$\bar{a}\langle c\rangle|\bar{a}\langle d\rangle|\bar{a}\langle e\rangle|\bar{a}\langle f\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\equiv \bar{a}\langle c\rangle|\bar{a}\langle d\rangle|\bar{a}\langle e\rangle|\bar{a}\langle f\rangle|a(x).\, (\nu\, b)\bar{x}\langle b\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\rightarrow (\nu\, b)\bar{c}\langle b\rangle|\bar{a}\langle d\rangle|\bar{a}\langle e\rangle|\bar{a}\langle f\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\rightarrow (\nu\, b)\bar{c}\langle b\rangle|(\nu\, b)\bar{d}\langle b\rangle|\bar{a}\langle e\rangle|\bar{a}\langle f\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\rightarrow (\nu\, b)\bar{c}\langle b\rangle|(\nu\, b)\bar{d}\langle b\rangle|(\nu\, b)\bar{e}\langle b\rangle|\bar{a}\langle f\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\rightarrow (\nu\, b)\bar{c}\langle b\rangle|(\nu\, b)\bar{d}\langle b\rangle|(\nu\, b)\bar{e}\langle b\rangle|(\nu\, b)\bar{f}\langle b\rangle|!a(x).\, (\nu\, b)\bar{x}\langle b\rangle$

$\equiv (\nu\, b)\bar{c}\langle b\rangle|(\nu\, b)\bar{d}\langle b\rangle|(\nu\, b)\bar{e}\langle b\rangle|(\nu\, b)\bar{f}\langle b\rangle|\mathrm{NN}(a)$