Supplementary Materials for "A Change-Point Based Control Chart for Detecting Sparse Changes in High-Dimensional (Heteroscedastic) Data"

1 The SS_W Method Based on the Studentized Statistic

Based on the change-poiny model, the studentized statistic(SS) proposed by [1] with n observations and split point k are as follows,

$$T_{n,k}^{SS} = \max_{1 \le r \le p} \frac{\sqrt{k(n-k)}|\bar{X}_{k,r} - \bar{X}_{n-k,r}|}{\sqrt{k\hat{\sigma}_{k,r}^2 + (n-k)\hat{\sigma}_{n-k,r}^2}}, \tag{1}$$

where $\bar{X}_{k,r}$ and $\bar{X}_{n-k,r}$ are the means of the rth variable (X_r) in the pre-shift sample and post-shift sample respectively. $\hat{\sigma}_{k,r}^2 = \sum_{i=1}^{k}(X_{i,r} - \bar{X}_{k,r})^2/k$, and $\hat{\sigma}_{n-k,r}^2 = \sum_{i=k+1}^{n}(X_{i,r} - \bar{X}_{n-k,r})^2/(n-k)$ are the pre-shift sample variance and the post-shift sample variance of variable r (X_r).

The monitoring statistic in SS version at time point n, referred as U_n^{SS}, is the maximum value of $T_{n,k}^{SS}$ over all split points k,

$$U_n^{SS} = \max_{3 \le k \le n-3} T_{n,k}^{SS}. \tag{2}$$

The change-point τ can be estimated directly by the corresponding k when U_n^{SS} exceeds the control limits.

$$\hat{\tau}_n^{SS} = \arg_k \max_{3 \le k \le n-3}(T_{n,k}^{SS}). \tag{3}$$

The window based SS statistic is

$$T_{n,W,k^*}^{SS} = \max_{1 \le r \le p} \frac{\sqrt{k^*(W-k^*)}|\bar{X}_{k^*,r} - \bar{X}_{W-k^*,r}|}{\sqrt{k^*\hat{\sigma}_{k^*,r}^2 + (W-k^*)\hat{\sigma}_{W-k^*,r}^2}}, \tag{4}$$

where k^* ($3 \le k^* \le W-3$) is the split point inside the current window, and $\hat{\sigma}_{k^*,r}^2$ is the variance of observation $1 - k^*$ in window W_n. The corresponding charting statistic based on this window is

$$U_{n,W}^{SS} = \max_{3 \le k^* \le W-3} T_{n,W,k^*}^{SS}. \tag{5}$$

If the control chart signals at time point n, the corresponding change-point estimate is

$$\hat{\tau}_{n,W}^{SS} = n - W + \arg_{k^*} \max_{3 \le k^* \le W-3} T_{n,W,k^*}^{SS}. \tag{6}$$

2 Additional Simulation Results

Table 1 to Table 3 show the DR, CED of the proposed NS_W and SS_W methods in signal detection, and the CPE in signal diagnosis under Model I with τ = 10, 25, 50 respectively. It is the baseline model.

Table 4 to Table 6 show the DR, CED of the proposed NS_W and SS_W methods in signal detection, and the CPE in signal diagnosis under Model II, which simulates the heteroscedasticity, with $\tau = 10$, 25, 50 respectively.

Table 7 to Table 9 show the DR, CED of the proposed NS_W and SS_W methods in signal detection, and the CPE in signal diagnosis under Model III, which considers the dependency among variables, with $\tau = 10$, 25, 50 respectively.

Table 10 to Table 12 show the DR, CED of the proposed NS_W and SS_W methods in signal detection, and the CPE in signal diagnosis under Model IV, an nonnormal distribution, with $\tau = 10$, 25, 50 respectively.

Table 1: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model I with $\tau = 10$.

$Model I$			DR				CED				CPE			
$\tau = 10$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.02	0.005	0.018	0.014	49.57	43.0	52.5	45.0	49.8	44.8	52.2	46.1
		1	0.078	0.018	0.114	0.028	23.8	36.9	17.7	25.4	23.7	37.7	16.3	23.3
		1.5	0.326	0.07	0.587	0.126	12.5	16.9	11.1	13.9	11.8	16.6	10.6	13.3
		2	0.772	0.222	0.984	0.412	10.5	12.3	10.1	11.3	10.3	11.7	10.0	10.7
	30	0.5	0.018	0.012	0.016	0.013	47.2	59.2	49.1	48.1	40.7	53.5	42.3	41.4
		1	0.064	0.024	0.168	0.058	27.7	30.2	22.5	26.5	18.3	22.2	12.8	18.0
		1.5	0.46	0.19	0.815	0.391	20.9	21.8	20.2	20.9	11.0	12.0	10.2	10.8
		2	0.934	0.568	0.998	0.886	20.1	20.1	20.0	20.1	10.2	10.1	10.1	10.1
	40	0.5	0.009	0.021	0.011	0.022	60.0	50.7	46.8	53.6	46.7	40.5	35.5	38.2
		1	0.074	0.068	0.167	0.125	32.4	37.6	32.4	34.9	13.4	19.4	13.9	16.6
		1.5	0.523	0.316	0.841	0.61	30.3	31.0	30.0	30.3	10.6	11.3	10.2	10.5
		2	0.964	0.811	1	0.989	30.0	30.2	30.0	30.0	10.1	10.1	10.1	10.2
50	20	0.5	0.019	0.004	0.023	0.006	39.2	55.0	41.1	33.3	39.6	50.3	41.6	31.2
		1	0.076	0.005	0.157	0.014	22.4	37.0	13.2	25.7	22.5	38.4	12.7	26.1
		1.5	0.467	0.037	0.791	0.104	11.2	14.5	10.3	12.2	10.9	13.2	10.1	11.4
		2	0.961	0.205	0.999	0.459	10.2	11.1	10.0	11.1	10.0	10.2	10.0	10.5
	30	0.5	0.024	0.015	0.021	0.011	50.0	53.0	40.7	48.2	46.0	44.6	33.2	42.5
		1	0.107	0.041	0.248	0.075	25.0	44.1	22.8	24.5	15.4	36.1	12.9	15.0
		1.5	0.718	0.287	0.94	0.504	20.3	21.0	20.0	20.6	10.4	10.9	10.1	10.5
		2	0.995	0.796	1	0.967	20.0	20.3	20.0	20.0	10.1	10.3	10.0	10.1
	40	0.5	0.014	0.024	0.016	0.022	53.2	60.4	43.4	53.2	40.6	47.0	24.1	40.7
		1	0.111	0.069	0.238	0.136	31.6	33.9	31.6	32.5	12.2	16.4	12.5	14.2
		1.5	0.751	0.457	0.968	0.776	30.1	30.6	30.0	30.3	10.2	10.7	10.2	10.4
		2	1	0.946	1	1	30.0	30.0	30.0	30.0	10.1	10.1	10.0	10.0
100	20	0.5	0.011	0.017	0.011	0.017	44.5	56.5	45.5	54.7	44.8	57.5	45.1	53.2
		1	0.031	0.025	0.112	0.046	16.5	33.6	11.7	28.5	15.0	32.4	10.7	27.8
		1.5	0.42	0.093	0.738	0.235	10.6	16.3	10.4	13.0	10.0	15.0	10.0	11.8
		2	0.973	0.433	1	0.716	10.1	11.7	10.0	11.3	10.0	10.9	10.0	10.8
	30	0.5	0.013	0.01	0.019	0.015	43.5	45.0	48.7	44.7	37.4	40.1	40.3	36.8
		1	0.128	0.031	0.275	0.081	22.0	30.6	21.5	23.3	12.1	22.7	11.8	14.5
		1.5	0.795	0.266	0.981	0.548	20.2	21.3	20.0	20.6	10.2	11.2	10.0	10.5
		2	1	0.848	1	0.994	20.0	20.1	20.0	20.0	10.0	10.0	10.1	10.0
	40	0.5	0.007	0.005	0.011	0.013	42.1	63.0	45.0	60.8	30.6	53.2	28.9	52.2
		1	0.165	0.066	0.343	0.137	31.6	35.4	30.6	32.6	12.8	16.7	11.1	13.2
		1.5	0.903	0.502	0.996	0.836	30.1	30.5	30.0	30.1	10.2	10.5	10.2	10.2
		2	1	0.98	1	1	30.0	30.0	30.0	30.0	10.1	10.0	10.1	10.1

Table 2: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model I with $\tau = 25$

$Model I$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.028	0.017	0.021	0.016	24.3	35.0	24.0	45.0	38.5	49.2	37.9	59.9
		1	0.063	0.013	0.119	0.035	13.8	21.5	11.1	15.9	29.1	36.1	26.2	31.9
		1.5	0.322	0.072	0.652	0.164	9.2	13.9	8.6	11.2	25.3	29.1	25.2	26.8
		2	0.82	0.253	0.993	0.481	7.3	9.7	6.0	8.9	25.0	25.6	24.8	25.3
	30	0.5	0.023	0.013	0.035	0.025	31.5	33.8	23.7	30.6	41.5	44.3	34.6	42.0
		1	0.192	0.068	0.324	0.132	15.9	17.8	12.9	16.4	28.0	29.3	25.6	27.7
		1.5	0.728	0.324	0.953	0.65	11.2	13.7	9.4	12.3	25.1	25.4	25.0	25.0
		2	0.99	0.81	1	0.983	7.5	10.7	5.6	8.5	24.9	25.0	24.9	25.0
	40	0.5	0.012	0.026	0.04	0.046	31.3	33.1	24.3	28.7	34.6	34.8	30.3	33.6
		1	0.236	0.178	0.495	0.35	19.0	20.8	18.5	19.7	25.7	26.0	25.1	25.6
		1.5	0.892	0.736	0.996	0.965	16.3	17.7	15.2	16.0	25.0	25.1	24.9	25.0
		2	1	0.983	1	1	15.0	15.5	15.0	15.0	24.9	25.1	25.0	25.0
50	20	0.5	0.026	0.008	0.026	0.008	32.5	22.5	22.7	27.5	47.4	38.9	39.2	43.4
		1	0.098	0.011	0.178	0.016	13.8	33.2	12.0	15.6	28.7	46.5	27.3	31.1
		1.5	0.537	0.057	0.825	0.099	9.0	11.0	8.0	10.5	25.4	25.6	25.1	25.6
		2	0.96	0.238	1	0.458	6.4	9.5	5.3	9.2	25.0	25.5	24.8	25.2
	30	0.5	0.024	0.017	0.029	0.026	24.2	27.9	24.3	36.0	32.1	36.6	33.4	45.3
		1	0.231	0.066	0.493	0.151	15.3	16.4	13.1	14.4	26.2	26.4	25.6	26.1
		1.5	0.921	0.482	0.997	0.764	10.2	13.4	7.9	11.6	25.0	25.7	24.9	25.1
		2	1	0.948	1	0.998	6.1	9.7	5.1	7.5	24.9	25.1	24.9	25.0
	40	0.5	0.021	0.025	0.048	0.042	28.8	32.0	27.3	30.5	32.1	35.4	32.9	33.4
		1	0.364	0.245	0.684	0.458	19.5	20.9	18.2	20.0	25.3	25.8	25.2	25.4
		1.5	0.987	0.881	1	0.996	15.4	16.6	15.0	15.4	24.9	25.0	24.9	24.9
		2	1	1	1	1	15.0	15.0	15.0	15.0	24.9	24.9	25.0	25.0
100	20	0.5	0.006	0.019	0.011	0.021	25.8	28.2	18.6	29.5	39.7	41.5	34.6	43.8
		1	0.042	0.036	0.105	0.044	11.3	24.2	9.5	19.1	26.5	38.8	25.3	33.1
		1.5	0.439	0.121	0.782	0.272	8.8	11.0	8.2	10.2	25.2	26.5	25.0	25.6
		2	0.982	0.466	1	0.798	6.5	9.8	5.3	8.4	24.9	25.6	24.9	25.2
	30	0.5	0.018	0.012	0.034	0.022	20.8	26.3	20.1	29.3	29.9	35.5	32.6	38.8
		1	0.243	0.066	0.533	0.16	14.2	18.2	13.3	15.6	25.4	29.3	25.3	26.0
		1.5	0.962	0.549	0.999	0.858	9.8	12.8	7.4	11.5	24.9	25.1	24.8	25.0
		2	1	0.987	1	1	5.7	9.3	5.0	7.2	24.9	25.0	24.8	24.9
	40	0.5	0.025	0.013	0.052	0.029	27.4	32.7	25.1	26.6	32.3	36.2	29.8	29.7
		1	0.508	0.234	0.822	0.484	18.7	19.8	17.3	19.5	25.4	25.6	25.1	25.2
		1.5	1	0.939	1	0.999	15.1	16.4	15.0	15.1	24.9	25.0	24.9	24.9
		2	1	1	1	1	15.0	15.0	15.0	15.0	24.9	25.0	24.9	24.9

Table 3: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model I with $\tau = 50$

$Model I$			DR				CED				CPE			
$\tau = 50$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.016	0.014	0.022	0.011	17.8	26.8	7.0	2.7	58.8	66.3	47.9	42.5
		1	0.073	0.026	0.13	0.036	9.2	9.0	9.9	10.3	49.8	48.5	50.9	50.4
		1.5	0.347	0.066	0.636	0.151	8.1	8.6	8.1	9.3	49.5	49.0	49.8	49.5
		2	0.831	0.254	0.979	0.466	7.3	9.5	5.8	8.8	49.9	50.5	49.6	50.0
	30	0.5	0.023	0.025	0.034	0.017	14.8	19.4	12.2	12.6	51.2	58.2	47.3	50.1
		1	0.161	0.072	0.35	0.154	13.4	12.9	13.4	14.3	50.2	48.2	50.2	49.6
		1.5	0.715	0.311	0.952	0.63	11.2	13.0	9.2	12.5	49.9	49.9	49.8	49.8
		2	0.991	0.801	1	0.979	7.5	10.5	5.6	8.3	49.8	49.9	49.8	49.8
	40	0.5	0.014	0.027	0.025	0.036	28.2	23.3	21.8	20.7	55.6	52.5	52.3	49.2
		1	0.273	0.206	0.514	0.392	17.5	18.2	16.4	17.2	50.2	49.9	49.8	49.6
		1.5	0.902	0.739	0.996	0.961	12.4	14.5	9.7	11.8	49.8	50.0	49.8	49.9
		2	1	0.988	1	1	7.6	9.5	5.9	7.1	49.9	49.8	49.9	49.9
50	20	0.5	0.014	0.006	0.023	0.005	14.3	5.0	6.1	6.0	54.4	48.0	47.3	47.6
		1	0.09	0.019	0.166	0.029	9.3	2.9	10.0	11.9	49.8	42.2	50.4	51.9
		1.5	0.506	0.055	0.807	0.128	8.5	9.6	7.5	9.7	49.7	49.8	49.6	50.2
		2	0.962	0.222	0.999	0.501	6.4	9.3	5.2	8.9	49.8	49.8	49.8	49.9
	30	0.5	0.028	0.013	0.044	0.025	17.7	11.9	13.0	13.0	54.1	52.8	51.2	46.8
		1	0.249	0.087	0.489	0.152	13.5	14.6	12.9	14.5	50.1	51.4	49.8	49.7
		1.5	0.918	0.457	0.998	0.752	10.1	12.7	8.0	11.6	49.9	49.8	49.9	49.8
		2	1	0.956	1	0.999	6.3	9.9	5.1	7.6	49.9	49.8	49.8	49.9
	40	0.5	0.016	0.019	0.051	0.035	21.9	19.2	17.5	18.1	50.8	49.2	47.9	49.4
		1	0.385	0.242	0.705	0.487	16.9	17.6	15.3	17.1	50.1	50.2	50.0	49.9
		1.5	0.992	0.903	1	0.995	10.5	13.4	8.2	10.6	49.8	49.9	49.8	49.9
		2	1	1	1	1	6.2	7.9	5.2	6.4	49.8	49.9	49.9	49.9
100	20	0.5	0.008	0.021	0.008	0.017	20.6	4.3	3.1	-1.8	59.5	44.7	43.8	39.4
		1	0.052	0.033	0.114	0.058	11.6	7.6	9.3	7.2	52.0	47.6	49.5	47.1
		1.5	0.431	0.13	0.781	0.271	8.8	9.6	8.3	9.2	49.9	50.3	49.9	50.1
		2	0.977	0.453	1	0.784	6.5	8.8	5.2	8.1	49.9	49.6	49.9	49.7
	30	0.5	0.019	0.015	0.033	0.022	11.8	12.0	14.4	16.6	47.6	45.4	50.1	49.2
		1	0.263	0.08	0.518	0.175	13.5	14.8	12.7	13.5	50.1	50.1	49.9	49.8
		1.5	0.966	0.509	0.999	0.879	9.6	12.6	7.5	11.5	49.9	49.8	49.7	49.8
		2	1	0.978	1	1	5.6	9.2	5.0	7.1	49.8	49.8	49.8	49.9
	40	0.5	0.024	0.017	0.051	0.027	20.4	18.5	20.2	15.7	54.3	46.5	54.1	45.0
		1	0.531	0.249	0.848	0.493	16.3	17.7	14.9	17.3	49.9	50.0	50.1	49.8
		1.5	1	0.952	1	0.999	9.1	12.9	6.9	9.8	49.8	49.9	49.7	49.8
		2	1	1	1	1	5.4	7.7	5.0	5.9	49.7	49.8	49.8	49.8

Table 4: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model II with $\tau = 10$

$Model I$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0	0.014	0.005	0.022	NaN	55.4	27.0	41.1	NaN	54.3	21.6	37.9
		1	0.028	0.022	0.062	0.05	11.4	38.0	11.1	20.2	9.8	36.3	9.5	17.6
		1.5	0.287	0.102	0.507	0.185	10.7	17.7	10.5	13.5	9.8	16.0	9.8	11.7
		2	0.788	0.295	0.973	0.57	10.2	12.1	10.1	11.5	9.8	10.9	9.9	10.4
	30	0.5	0.004	0.016	0.004	0.026	41.3	42.2	22.5	45.8	32.5	36.2	7.5	37.9
		1	0.048	0.053	0.133	0.119	20.7	26.1	21.0	25.2	9.7	16.0	10.1	14.7
		1.5	0.43	0.318	0.778	0.564	20.1	21.0	20.1	20.9	9.8	10.2	9.8	10.1
		2	0.935	0.763	0.998	0.96	20.0	20.3	20.0	20.1	9.9	10.0	9.9	10.0
	40	0.5	0.003	0.019	0.003	0.035	43.3	57.4	31.7	46.3	18.7	49.3	8.7	30.1
		1	0.048	0.112	0.116	0.22	30.6	36.3	30.3	31.5	10.1	17.7	9.7	11.0
		1.5	0.442	0.491	0.793	0.83	30.1	30.8	30.1	30.2	9.8	10.6	9.8	10.0
		2	0.959	0.941	1	0.999	30.0	30.1	30.0	30.0	10.0	9.9	9.9	9.9
50	20	0.5	0.005	0.005	0.004	0.004	47.0	49.0	13.8	46.3	42.8	49.8	10.0	41.0
		1	0.037	0.011	0.095	0.028	11.4	22.7	10.7	21.3	10.3	20.2	9.9	19.0
		1.5	0.389	0.077	0.695	0.161	10.6	14.8	10.4	12.1	9.8	12.6	9.8	10.0
		2	0.946	0.326	1	0.618	10.1	11.4	10.0	11.4	9.8	10.1	9.9	10.2
	30	0.5	0.006	0.015	0.006	0.019	26.7	37.3	21.7	33.2	17.5	31.3	7.5	20.5
		1	0.085	0.07	0.179	0.143	20.5	28.2	20.6	23.1	9.8	18.4	9.7	11.8
		1.5	0.647	0.401	0.933	0.726	20.3	21.2	20.1	20.5	9.9	10.6	9.8	9.9
		2	0.997	0.91	1	0.998	20.0	20.1	20.0	20.0	9.9	9.9	9.8	9.8
	40	0.5	0.001	0.026	0.002	0.027	30.0	54.6	30.0	43.5	11.0	44.7	10.0	23.6
		1	0.082	0.125	0.177	0.278	30.4	33.5	30.4	32.7	9.6	13.6	9.7	12.2
		1.5	0.705	0.7	0.95	0.956	30.1	30.5	30.0	30.2	9.8	10.2	9.8	9.9
		2	0.999	0.997	1	1	30.0	30.0	30.0	30.0	9.9	9.8	9.9	9.9
100	20	0.5	0	0.019	0.003	0.021	NaN	53.4	26.7	43.1	NaN	54.6	25.7	42.8
		1	0.026	0.037	0.043	0.056	10.8	32.7	10.6	18.9	9.5	31.8	9.9	17.4
		1.5	0.33	0.149	0.615	0.331	10.8	16.8	10.4	13.2	9.9	15.0	9.8	11.5
		2	0.95	0.593	1	0.882	10.1	11.7	10.0	10.8	9.8	10.3	9.8	10.0
	30	0.5	0.002	0.016	0.006	0.024	20.0	36.6	20.8	35.6	5.0	26.4	9.3	25.3
		1	0.079	0.066	0.178	0.135	20.8	26.4	20.3	23.6	9.6	15.5	9.7	12.5
		1.5	0.717	0.44	0.976	0.807	20.1	20.6	20.0	20.4	9.9	9.8	9.8	9.9
		2	1	0.962	1	1	20.0	20.1	20.0	20.0	9.8	9.8	9.8	9.8
	40	0.5	0.003	0.017	0.009	0.034	31.7	57.9	31.1	48.2	10.0	43.9	11.7	34.0
		1	0.112	0.146	0.236	0.324	30.3	32.0	30.4	31.6	9.7	11.9	10.4	11.1
		1.5	0.871	0.812	0.987	0.974	30.1	30.3	30.0	30.0	9.8	9.9	9.8	9.8
		2	1	1	1	1	30.0	30.0	30.0	30.0	9.9	9.9	9.9	9.9

Table 5: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model II with $\tau = 25$.

$ModelI$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.002	0.019	0.006	0.013	7.5	32.6	7.5	40.4	18.5	45.9	22.2	54.9
		1	0.044	0.029	0.07	0.037	10.0	25.9	9.1	15.7	24.8	40.6	24.1	29.0
		1.5	0.251	0.082	0.523	0.186	8.6	13.1	9.1	11.1	24.6	28.1	24.9	25.7
		2	0.795	0.309	0.979	0.603	7.6	10.2	6.3	9.6	24.7	25.3	24.7	25.0
	30	0.5	0.007	0.017	0.013	0.033	17.1	35.6	15.4	28.9	23.7	42.0	22.2	36.4
		1	0.102	0.092	0.241	0.207	14.7	21.4	13.4	17.1	24.5	30.3	24.4	25.9
		1.5	0.683	0.455	0.927	0.749	11.7	14.0	10.0	13.1	24.6	25.2	24.6	25.1
		2	0.994	0.902	1	0.995	7.7	10.6	5.9	8.6	24.7	24.8	24.7	24.8
	40	0.5	0.001	0.031	0.012	0.047	30.0	36.1	23.8	30.0	19.0	36.8	22.6	31.7
		1	0.176	0.247	0.386	0.499	19.1	21.4	19.3	20.9	24.7	25.5	24.3	25.1
		1.5	0.863	0.85	0.991	0.986	16.4	17.6	15.2	15.5	24.6	24.9	24.6	24.6
		2	0.997	0.996	1	1	15.0	15.2	15.0	15.0	24.8	24.8	24.8	24.8
50	20	0.5	0.005	0.008	0.009	0.011	8.0	32.5	23.3	37.3	20.0	45.0	38.7	53.0
		1	0.052	0.02	0.095	0.034	11.9	22.8	10.8	17.8	26.2	35.4	25.5	32.0
		1.5	0.406	0.075	0.744	0.181	9.2	12.1	8.6	11.7	24.7	25.5	24.6	26.2
		2	0.941	0.302	0.999	0.631	6.8	10.4	5.5	9.8	24.7	25.2	24.7	24.9
	30	0.5	0.006	0.017	0.009	0.03	12.5	27.6	12.2	26.8	16.8	32.4	19.9	35.0
		1	0.167	0.112	0.332	0.245	14.4	17.8	14.1	17.4	24.2	26.1	24.6	26.2
		1.5	0.874	0.581	0.994	0.9	11.1	13.9	9.1	12.6	24.7	24.8	24.6	24.7
		2	1	0.98	1	1	6.5	9.9	5.3	7.8	24.8	24.9	24.7	24.8
	40	0.5	0.009	0.032	0.016	0.06	21.1	31.6	23.1	29.2	19.8	36.7	22.4	29.5
		1	0.314	0.358	0.551	0.613	19.2	21.6	18.9	20.2	24.4	24.7	24.1	24.6
		1.5	0.989	0.974	1	0.998	15.5	16.6	15.0	15.1	24.5	24.5	24.7	24.6
		2	1	1	1	1	15.0	15.0	15.0	15.0	24.7	24.8	24.8	24.8
100	20	0.5	0.001	0.017	0.003	0.02	0.0	31.5	15.0	32.3	10.0	44.7	29.3	47.2
		1	0.025	0.039	0.044	0.082	11.2	15.9	9.2	14.1	25.6	29.9	24.4	27.1
		1.5	0.358	0.196	0.654	0.374	9.4	13.0	9.3	11.3	24.7	26.8	24.7	25.4
		2	0.953	0.601	1	0.893	7.2	10.3	5.6	8.6	24.8	25.4	24.8	24.9
	30	0.5	0.006	0.01	0.007	0.024	13.3	36.0	16.4	27.5	24.7	42.5	22.3	34.1
		1	0.149	0.108	0.36	0.25	14.6	18.0	14.1	16.8	24.2	26.2	24.3	25.0
		1.5	0.946	0.679	1	0.95	10.6	14.1	8.6	12.1	24.6	24.8	24.6	24.7
		2	1	0.998	1	1	6.0	9.3	5.1	7.4	24.8	24.8	24.8	24.8
	40	0.5	0.006	0.026	0.012	0.046	32.5	29.8	20.4	31.4	34.0	33.3	22.3	30.4
		1	0.408	0.385	0.712	0.69	19.7	21.4	18.6	20.6	24.3	24.7	24.2	24.3
		1.5	0.999	0.988	1	1	15.1	15.8	15.0	15.0	24.6	24.7	24.6	24.7
		2	1	1	1	1	15.0	15.0	15.0	15.0	24.8	24.8	24.8	24.8

Table 6: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model II with $\tau = 50$.

$ModelI$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.005	0.013	0.009	0.012	-11.0	6.9	-11.1	15.4	27.6	45.4	27.1	57.0
		1	0.035	0.028	0.074	0.049	8.3	12.3	8.6	12.6	47.1	50.9	48.1	50.4
		1.5	0.275	0.097	0.545	0.202	8.3	10.9	8.7	10.6	48.8	50.3	49.4	49.8
		2	0.816	0.308	0.982	0.61	7.5	10.3	6.0	9.2	49.6	49.8	49.5	49.6
	30	0.5	0.01	0.021	0.021	0.033	1.5	14.3	8.1	11.4	32.8	47.4	40.3	43.8
		1	0.109	0.081	0.238	0.196	13.6	15.7	13.4	16.0	48.6	48.8	49.1	49.3
		1.5	0.657	0.452	0.941	0.762	11.4	13.6	9.9	12.6	49.5	49.6	49.5	49.5
		2	0.988	0.896	1	0.997	7.8	10.7	5.9	8.5	49.7	49.7	49.6	49.6
	40	0.5	0.006	0.031	0.012	0.062	13.3	24.5	15.8	21.9	41.5	50.6	45.2	47.4
		1	0.202	0.262	0.423	0.505	17.4	19.3	17.0	19.0	49.7	49.2	49.3	49.3
		1.5	0.878	0.835	0.995	0.993	13.1	14.9	10.3	11.8	49.6	49.5	49.5	49.5
		2	0.999	0.996	1	1	7.9	9.5	5.9	7.1	49.6	49.7	49.5	49.5
50	20	0.5	0.011	0.006	0.012	0.006	-8.6	13.3	-12.9	16.7	32.7	54.2	27.3	54.3
		1	0.067	0.016	0.104	0.038	5.1	15.6	8.2	8.2	44.5	53.6	47.1	47.1
		1.5	0.401	0.085	0.734	0.193	8.4	9.6	8.5	10.6	49.1	48.9	49.5	49.4
		2	0.95	0.33	1	0.615	6.6	10.4	5.3	9.8	49.6	50.1	49.5	49.8
	30	0.5	0.011	0.016	0.015	0.027	1.8	14.4	11.3	17.2	35.5	44.4	45.7	50.4
		1	0.17	0.125	0.331	0.225	12.9	16.2	13.4	15.8	48.5	49.3	49.0	49.7
		1.5	0.857	0.62	0.993	0.902	10.8	13.8	9.0	12.3	49.5	49.5	49.5	49.6
		2	1	0.979	1	1	6.4	9.6	5.2	7.6	49.6	49.8	49.6	49.6
	40	0.5	0.011	0.042	0.022	0.054	13.6	20.6	10.9	23.7	40.1	46.5	39.0	47.8
		1	0.289	0.337	0.567	0.639	17.5	19.4	17.0	18.7	49.4	49.1	49.2	49.1
		1.5	0.983	0.971	1	1	11.5	13.6	9.1	10.5	49.6	49.6	49.6	49.5
		2	1	1	1	1	6.7	8.3	5.4	6.5	49.7	49.6	49.7	49.7
100	20	0.5	0.001	0.015	0.006	0.02	-5.0	6.7	-7.5	12.0	33.0	45.0	28.7	51.0
		1	0.027	0.039	0.055	0.087	11.1	14.6	9.5	11.9	49.5	52.2	48.7	50.8
		1.5	0.342	0.162	0.674	0.386	9.2	11.9	8.9	10.0	49.4	50.7	49.7	49.4
		2	0.964	0.58	1	0.907	7.0	9.3	5.5	8.6	49.7	49.7	49.6	49.6
	30	0.5	0.004	0.011	0.019	0.022	17.5	15.5	8.9	13.6	60.3	46.6	43.3	49.8
		1	0.173	0.115	0.378	0.25	13.4	15.7	13.5	16.1	48.8	48.9	49.2	49.4
		1.5	0.943	0.682	0.998	0.938	10.5	13.8	8.5	11.9	49.5	49.6	49.6	49.5
		2	1	0.995	1	1	6.0	9.3	5.0	7.3	49.8	49.8	49.6	49.7
	40	0.5	0.015	0.035	0.018	0.039	18.0	23.3	15.8	24.2	39.9	47.9	46.7	48.9
		1	0.427	0.368	0.741	0.701	17.3	19.5	16.8	19.4	48.9	49.0	49.2	49.4
		1.5	1	0.976	1	1	10.3	13.4	8.3	10.3	49.4	49.6	49.4	49.7
		2	1	1	1	1	5.9	8.0	5.1	6.2	49.7	49.7	49.6	49.7

Table 7: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model III with $\tau = 10$.

$Model I$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.006	0.003	0.008	0.004	45.0	30.0	28.8	23.8	43.3	30.3	27.8	22.5
		1	0.048	0.009	0.09	0.023	12.4	19.4	11.0	17.6	11.8	19.9	10.2	16.3
		1.5	0.265	0.037	0.456	0.087	10.8	11.2	10.5	10.9	10.2	9.9	10.0	10.0
		2	0.712	0.169	0.907	0.336	10.3	11.1	10.3	10.9	10.0	10.2	9.9	10.2
	30	0.5	0.007	0.005	0.007	0.005	22.9	44.0	28.6	31.0	11.3	40.0	20.9	30.4
		1	0.065	0.018	0.142	0.054	22.4	30.8	21.1	21.0	12.2	21.0	11.2	10.7
		1.5	0.452	0.157	0.666	0.274	20.4	20.6	20.3	20.3	10.5	10.4	10.1	10.1
		2	0.898	0.526	0.983	0.721	20.0	20.4	20.0	20.2	10.0	10.3	10.0	10.1
	40	0.5	0.002	0.003	0.009	0.008	30.0	50.0	36.7	52.5	19.0	38.0	16.1	36.6
		1	0.056	0.044	0.14	0.113	30.7	30.7	30.3	31.8	10.6	10.2	10.3	12.4
		1.5	0.469	0.285	0.737	0.466	30.1	30.2	30.1	30.2	10.2	10.3	10.1	10.2
		2	0.937	0.738	0.992	0.921	30.0	30.0	30.0	30.0	10.0	10.1	10.0	10.0
50	20	0.5	0.005	0.003	0.008	0.001	24.0	28.3	24.4	85.0	24.8	31.7	23.3	82.0
		1	0.054	0.001	0.086	0.01	11.4	10.0	12.3	17.5	9.8	7.0	11.4	15.2
		1.5	0.369	0.036	0.53	0.07	10.8	11.1	10.9	11.0	10.1	10.0	10.1	9.9
		2	0.839	0.157	0.941	0.274	10.3	10.8	10.2	10.5	10.0	10.1	9.9	10.0
	30	0.5	0.006	0.004	0.01	0.006	20.8	28.8	27.5	43.3	10.7	22.5	16.1	37.3
		1	0.114	0.031	0.2	0.05	21.4	22.6	20.6	23.3	11.1	12.6	10.3	13.2
		1.5	0.567	0.179	0.818	0.307	20.2	21.4	20.2	20.5	10.0	10.8	10.0	10.2
		2	0.967	0.581	0.994	0.757	20.0	20.3	20.0	20.1	10.0	10.1	10.0	10.1
	40	0.5	0.002	0.003	0.01	0.007	30.0	41.7	31.5	30.7	13.0	26.3	11.6	10.6
		1	0.092	0.053	0.18	0.094	31.1	31.6	30.3	30.5	11.4	11.2	10.4	10.7
		1.5	0.648	0.365	0.854	0.528	30.2	30.1	30.0	30.2	10.1	9.9	10.0	10.1
		2	0.986	0.837	0.999	0.931	30.0	30.1	30.0	30.0	10.1	10.1	10.0	10.0
100	20	0.5	0.001	0.004	0.003	0.005	15.0	32.5	11.7	38.0	17.0	29.8	14.0	37.2
		1	0.026	0.012	0.075	0.034	12.5	12.5	10.8	13.7	11.4	10.2	9.7	12.2
		1.5	0.286	0.091	0.471	0.137	10.7	11.8	11.1	12.1	10.0	10.6	10.0	11.0
		2	0.827	0.316	0.931	0.436	10.3	10.8	10.3	10.4	9.9	10.2	9.9	10.0
	30	0.5	0.007	0.011	0.009	0.006	43.6	32.7	20.0	28.3	38.6	24.2	9.3	21.7
		1	0.078	0.025	0.166	0.044	21.1	20.8	20.5	24.0	10.5	11.0	10.0	14.0
		1.5	0.608	0.167	0.827	0.31	20.2	21.1	20.1	20.2	9.9	10.8	9.9	10.1
		2	0.983	0.603	1	0.797	20.0	20.1	20.0	20.1	10.0	10.0	9.9	10.0
	40	0.5	0.002	0.003	0.006	0.003	30.0	45.0	33.3	61.7	16.0	26.3	16.5	57.0
		1	0.11	0.035	0.245	0.095	30.5	30.3	30.3	30.6	10.3	10.2	10.2	10.9
		1.5	0.768	0.366	0.924	0.544	30.1	30.1	30.0	30.1	10.0	10.0	9.9	10.0
		2	1	0.864	1	0.949	30.0	30.0	30.0	30.0	10.0	10.0	10.0	10.0

Table 8: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model III with $\tau = 25$.

$Model I$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.011	0.007	0.01	0.005	11.8	22.1	11.5	41.0	28.8	35.0	25.7	56.6
		1	0.055	0.017	0.1	0.025	12.2	17.4	9.9	9.6	27.9	34.3	25.1	24.9
		1.5	0.304	0.049	0.452	0.095	9.1	9.2	8.7	9.4	25.0	25.9	24.7	25.3
		2	0.76	0.207	0.912	0.344	7.7	9.4	7.4	8.9	24.9	25.2	24.8	24.9
	30	0.5	0.014	0.008	0.03	0.016	15.7	33.1	18.5	16.3	22.6	44.9	28.8	27.0
		1	0.134	0.049	0.253	0.113	14.4	15.2	13.1	12.9	25.0	26.4	24.5	24.3
		1.5	0.638	0.289	0.838	0.492	11.7	12.8	11.1	12.2	25.1	25.1	24.8	24.9
		2	0.966	0.741	0.995	0.88	8.5	10.8	7.6	9.6	24.9	24.9	24.7	24.9
	40	0.5	0.006	0.014	0.014	0.024	24.2	24.6	20.7	25.0	32.0	30.9	27.4	29.2
		1	0.216	0.162	0.397	0.258	18.9	20.0	19.3	19.8	24.6	25.5	24.6	24.7
		1.5	0.824	0.652	0.943	0.795	16.7	17.6	16.2	16.6	24.9	25.1	24.7	24.8
		2	0.995	0.966	1	0.994	15.1	15.6	15.1	15.3	24.9	25.0	24.9	25.0
50	20	0.5	0.01	0.002	0.025	0.006	6.0	37.5	10.8	10.0	19.8	52.5	25.9	25.7
		1	0.072	0.009	0.103	0.013	9.3	17.8	10.0	10.4	23.9	34.8	24.6	27.2
		1.5	0.39	0.04	0.572	0.078	8.8	9.4	9.1	9.9	24.9	25.2	24.8	25.5
		2	0.839	0.196	0.944	0.295	7.6	9.3	7.3	8.9	24.8	25.0	24.8	24.9
	30	0.5	0.016	0.007	0.021	0.01	14.7	14.3	16.0	16.0	24.4	25.0	24.2	23.6
		1	0.21	0.051	0.308	0.102	14.0	18.0	14.1	12.6	25.1	27.2	24.7	24.9
		1.5	0.748	0.375	0.89	0.505	11.6	13.1	11.1	11.8	24.9	25.1	24.6	24.9
		2	0.997	0.809	0.999	0.93	8.0	10.5	7.2	9.4	24.7	25.0	24.5	25.0
	40	0.5	0.014	0.006	0.023	0.022	26.8	22.5	20.2	23.6	27.7	23.5	23.4	25.1
		1	0.298	0.16	0.464	0.31	19.3	19.0	18.7	19.1	24.8	24.7	24.7	25.1
		1.5	0.903	0.729	0.973	0.85	16.4	17.2	15.9	16.7	24.8	24.8	24.5	24.8
		2	1	0.984	1	0.998	15.0	15.3	15.0	15.1	24.9	25.0	24.8	24.9
100	20	0.5	0.004	0.006	0.004	0.016	1.3	8.3	12.5	12.5	17.5	22.8	26.5	27.7
		1	0.029	0.019	0.078	0.053	10.0	12.4	9.9	9.9	24.3	27.4	24.4	25.1
		1.5	0.317	0.092	0.47	0.182	9.4	10.3	9.3	9.7	24.9	25.5	24.8	25.6
		2	0.812	0.333	0.933	0.495	8.1	9.3	7.7	8.3	24.9	25.4	24.8	25.0
	30	0.5	0.009	0.004	0.015	0.004	20.6	8.8	15.7	13.8	33.3	22.0	24.7	25.8
		1	0.188	0.061	0.312	0.125	13.8	14.8	13.5	13.6	25.1	25.0	24.8	24.9
		1.5	0.8	0.356	0.91	0.553	11.9	12.9	11.0	12.2	24.9	25.0	24.6	24.8
		2	0.998	0.851	1	0.916	8.0	10.6	7.2	9.2	24.8	24.9	24.7	24.9
	40	0.5	0.021	0.01	0.02	0.011	22.9	27.0	21.3	20.9	28.0	30.0	24.7	24.5
		1	0.348	0.187	0.539	0.291	19.2	19.2	19.5	19.6	25.2	24.9	24.7	24.9
		1.5	0.968	0.748	0.992	0.886	16.3	17.3	15.6	16.5	24.9	25.1	24.7	24.9
		2	1	0.986	1	0.997	15.0	15.3	15.0	15.1	24.8	24.9	24.8	24.9

Table 9: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model III with $\tau = 50$.

$ModelI$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.008	0.018	0.027	0.009	-2.5	-5.8	7.2	0.0	36.3	33.5	47.3	40.4
		1	0.038	0.014	0.087	0.022	5.3	3.2	8.9	7.3	46.6	42.2	49.4	48.1
		1.5	0.292	0.063	0.488	0.12	8.4	8.6	8.1	8.8	49.5	49.1	49.4	50.1
		2	0.769	0.227	0.902	0.379	7.8	7.8	7.1	8.2	49.8	48.6	49.6	49.7
	30	0.5	0.014	0.011	0.02	0.017	9.3	7.3	10.5	5.3	44.4	41.4	43.9	41.8
		1	0.138	0.058	0.247	0.081	12.2	13.4	13.3	13.1	48.2	47.5	49.2	49.5
		1.5	0.644	0.294	0.813	0.473	11.7	13.2	10.7	11.6	49.7	49.8	49.6	49.7
		2	0.968	0.717	0.998	0.911	8.4	10.7	7.8	9.6	49.8	49.9	49.5	49.7
	40	0.5	0.021	0.023	0.016	0.028	17.4	14.6	14.7	20.7	45.6	45.2	43.4	50.8
		1	0.227	0.156	0.38	0.31	17.6	17.1	17.1	16.4	49.7	49.4	49.7	49.3
		1.5	0.848	0.687	0.945	0.844	13.7	15.1	12.5	13.7	49.9	49.9	49.7	49.8
		2	0.997	0.965	1	0.996	8.8	10.5	7.9	8.8	49.8	49.9	49.6	49.9
50	20	0.5	0.023	0.002	0.012	0.006	-2.0	-10.0	-0.4	1.7	37.7	33.0	38.3	39.2
		1	0.071	0.013	0.117	0.019	9.2	7.3	9.3	5.3	48.9	49.4	48.7	45.9
		1.5	0.377	0.034	0.573	0.092	8.9	9.0	8.5	8.9	49.4	48.8	49.2	49.2
		2	0.876	0.193	0.947	0.33	7.6	9.0	7.2	8.3	49.8	49.7	49.7	49.7
	30	0.5	0.015	0.008	0.032	0.025	1.7	5.6	13.6	10.6	38.4	38.9	45.8	45.1
		1	0.181	0.063	0.276	0.124	12.9	14.1	13.0	13.9	48.9	49.6	49.2	49.5
		1.5	0.748	0.351	0.894	0.497	11.4	12.4	10.6	12.4	49.6	49.6	49.5	50.0
		2	0.989	0.812	0.998	0.912	8.0	10.3	7.3	9.3	49.6	49.6	49.5	49.9
	40	0.5	0.014	0.015	0.025	0.027	15.4	12.3	20.0	18.5	46.6	40.9	50.4	48.0
		1	0.293	0.191	0.453	0.304	17.5	18.0	17.5	16.8	49.6	49.8	49.4	48.9
		1.5	0.921	0.734	0.977	0.878	13.2	14.4	11.8	13.0	49.6	50.0	49.5	49.6
		2	0.999	0.989	1	0.998	8.3	9.8	7.6	8.4	49.7	49.8	49.6	49.6
100	20	0.5	0.004	0.015	0.005	0.011	1.3	-4.0	2.0	5.5	41.0	34.6	43.0	45.5
		1	0.036	0.009	0.078	0.043	8.3	10.6	9.3	5.9	47.6	50.1	49.1	47.3
		1.5	0.309	0.108	0.459	0.176	9.1	7.7	9.2	8.6	49.7	48.7	49.7	49.4
		2	0.815	0.315	0.928	0.498	8.1	8.4	7.7	8.2	49.8	49.3	49.8	49.9
	30	0.5	0.009	0.009	0.019	0.013	7.2	3.3	13.4	3.8	40.7	39.3	49.3	37.5
		1	0.178	0.055	0.309	0.125	12.8	12.8	13.6	12.6	49.4	48.6	49.3	49.3
		1.5	0.791	0.35	0.901	0.558	11.6	12.7	10.8	11.8	49.7	50.0	49.5	49.7
		2	1	0.83	1	0.945	7.7	10.3	7.3	9.3	49.5	49.8	49.6	49.9
	40	0.5	0.018	0.015	0.046	0.033	18.3	22.3	16.8	18.0	47.4	52.1	45.5	48.3
		1	0.384	0.192	0.546	0.318	17.5	17.7	17.0	17.7	49.3	49.7	49.3	49.4
		1.5	0.958	0.76	0.99	0.902	12.8	14.4	11.2	12.9	49.7	49.9	49.3	49.7
		2	1	0.994	1	0.999	7.7	9.6	7.3	8.6	49.6	49.9	49.5	49.8

Table 10: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model IV with $\tau = 10$.

$ModelI$ $\tau=25$			DR				CED				CPE			
			$v=10\%$		$v=25\%$		$v=10\%$		$v=25\%$		$v=10\%$		$v=25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.007	0.005	0.024	0.007	34.3	46.0	34.6	22.1	34.6	47.2	33.9	21.6
		1	0.044	0.011	0.092	0.018	16.4	20.5	14.5	21.4	16.0	21.3	14.2	20.4
		1.5	0.275	0.043	0.463	0.081	11.6	11.6	11.1	11.9	10.9	10.7	10.4	10.9
		2	0.716	0.171	0.887	0.294	10.4	10.8	10.2	11.0	10.2	10.2	10.0	10.5
	30	0.5	0.013	0.003	0.013	0.006	47.7	55.0	25.0	45.0	41.6	52.3	13.5	38.2
		1	0.067	0.021	0.164	0.045	24.0	26.4	22.7	22.1	14.7	17.7	12.7	11.6
		1.5	0.467	0.128	0.681	0.244	20.5	21.3	20.2	20.6	10.5	11.0	10.0	10.3
		2	0.904	0.492	0.983	0.697	20.1	20.2	20.0	20.1	10.1	10.1	10.0	10.0
	40	0.5	0.009	0.006	0.007	0.004	45.6	46.7	36.4	46.3	33.1	32.2	23.4	22.3
		1	0.07	0.031	0.169	0.076	32.0	30.2	30.5	30.2	12.6	10.9	11.1	11.0
		1.5	0.481	0.246	0.726	0.428	30.3	30.6	30.1	30.1	10.6	10.9	10.3	10.2
		2	0.943	0.703	0.993	0.848	30.0	30.0	30.0	30.0	10.2	10.1	10.0	10.1
50	20	0.5	0.01	0.002	0.011	0	26.0	10.0	21.4	NaN	25.3	9.5	21.5	NaN
		1	0.069	0.007	0.124	0.011	12.3	25.7	12.7	15.0	11.8	23.0	12.0	13.5
		1.5	0.383	0.029	0.545	0.075	11.5	14.0	10.8	10.8	10.9	13.3	9.9	9.9
		2	0.83	0.152	0.949	0.243	10.4	11.0	10.2	10.6	9.9	10.4	9.9	10.1
	30	0.5	0.015	0.005	0.016	0.007	47.7	21.0	24.7	30.7	43.6	9.2	17.1	21.1
		1	0.098	0.039	0.205	0.041	21.0	21.7	21.1	21.7	10.8	11.9	10.8	11.9
		1.5	0.586	0.165	0.789	0.256	20.2	20.7	20.1	20.4	10.1	10.4	10.0	10.1
		2	0.978	0.546	0.994	0.695	20.0	20.2	20.0	20.1	10.0	10.1	9.9	10.0
	40	0.5	0.011	0.007	0.005	0.002	42.7	35.7	31.0	30.0	27.9	19.0	9.2	14.5
		1	0.102	0.043	0.217	0.093	31.6	32.9	30.2	30.3	12.7	13.2	10.1	10.3
		1.5	0.636	0.288	0.854	0.478	30.1	30.3	30.1	30.2	10.1	10.3	10.0	10.0
		2	0.982	0.769	1	0.895	30.0	30.0	30.0	30.0	10.1	10.1	10.0	10.0
100	20	0.5	0.004	0.006	0.005	0.008	12.5	45.8	21.0	18.1	8.5	47.0	19.8	16.6
		1	0.046	0.028	0.071	0.025	11.1	18.0	12.1	14.4	10.2	16.9	10.6	13.0
		1.5	0.327	0.083	0.462	0.139	10.8	11.3	11.1	12.6	10.1	10.5	10.1	11.5
		2	0.818	0.247	0.913	0.424	10.5	11.2	10.3	10.6	10.0	10.5	9.9	10.0
	30	0.5	0.011	0.003	0.011	0.004	23.6	21.7	29.5	21.3	16.7	14.0	21.5	12.5
		1	0.108	0.021	0.208	0.054	20.8	24.8	21.6	20.4	10.8	16.1	11.3	9.9
		1.5	0.669	0.168	0.82	0.279	20.2	21.1	20.2	20.4	10.0	10.9	10.2	10.2
		2	0.986	0.572	0.997	0.742	20.0	20.1	20.0	20.1	10.0	10.0	10.0	10.1
	40	0.5	0.008	0.003	0.013	0.002	45.0	46.7	35.0	32.5	28.9	33.0	15.7	14.0
		1	0.153	0.04	0.258	0.072	30.3	30.5	30.3	32.0	10.0	10.7	10.5	12.4
		1.5	0.782	0.338	0.931	0.499	30.1	30.2	30.0	30.1	10.1	10.1	10.1	10.1
		2	0.997	0.838	1	0.918	30.0	30.0	30.0	30.0	10.0	10.1	10.0	10.0

Table 11: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model IV with $\tau = 25$.

$ModelI$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.011	0.004	0.017	0.005	22.3	28.8	14.7	31.0	37.6	42.0	31.2	47.2
		1	0.063	0.011	0.113	0.019	11.7	20.0	10.1	13.4	27.7	35.6	25.9	27.7
		1.5	0.32	0.043	0.469	0.082	9.0	11.4	8.8	9.6	25.0	26.2	24.9	25.5
		2	0.744	0.203	0.913	0.356	7.7	8.8	7.5	8.4	25.0	24.8	24.9	25.1
	30	0.5	0.015	0.005	0.03	0.013	29.7	11.0	22.2	21.2	39.5	23.6	34.2	33.8
		1	0.159	0.04	0.284	0.079	14.0	15.3	13.9	13.2	25.3	26.6	25.4	24.8
		1.5	0.651	0.264	0.81	0.444	11.7	13.0	11.1	12.2	25.3	24.9	24.7	25.0
		2	0.97	0.725	0.994	0.863	8.4	10.7	7.8	9.9	24.8	24.9	24.8	25.0
	40	0.5	0.013	0.014	0.033	0.022	31.9	25.4	22.3	20.2	32.8	28.4	27.3	26.5
		1	0.211	0.13	0.424	0.288	19.5	19.8	19.2	18.9	25.2	25.0	25.0	25.1
		1.5	0.842	0.623	0.943	0.803	16.6	17.4	16.1	16.9	25.1	25.2	24.9	25.0
		2	0.995	0.941	1	0.988	15.2	15.6	15.0	15.2	24.9	24.9	24.8	24.9
50	20	0.5	0.01	0.003	0.018	0.01	17.0	3.3	20.8	8.0	30.4	23.7	36.9	24.3
		1	0.077	0.009	0.127	0.016	12.5	21.1	10.3	8.8	28.0	34.7	25.7	25.1
		1.5	0.397	0.044	0.575	0.069	8.7	11.1	9.1	10.2	25.1	26.4	25.0	25.6
		2	0.875	0.184	0.953	0.294	7.9	8.7	7.2	8.3	25.0	24.8	24.8	24.9
	30	0.5	0.017	0.008	0.033	0.009	15.3	17.5	15.9	12.8	24.3	30.5	28.0	26.0
		1	0.196	0.044	0.333	0.11	13.9	14.4	13.9	14.0	25.1	25.2	24.9	25.3
		1.5	0.761	0.352	0.89	0.473	11.7	12.6	10.9	11.5	25.1	25.1	24.7	25.0
		2	0.992	0.781	0.996	0.907	8.4	10.6	7.3	9.4	24.8	24.9	24.6	24.9
	40	0.5	0.013	0.01	0.035	0.02	21.2	20.0	26.4	20.3	28.2	25.6	28.8	26.9
		1	0.317	0.173	0.476	0.279	19.4	20.2	19.4	19.4	25.3	25.9	24.9	25.0
		1.5	0.911	0.688	0.968	0.813	16.2	16.9	15.9	16.7	24.9	25.1	24.7	24.9
		2	1	0.976	1	0.994	15.1	15.4	15.0	15.2	24.9	25.0	24.8	24.9
100	20	0.5	0.003	0.009	0.009	0.012	21.7	8.3	5.0	22.5	39.7	25.2	19.1	35.8
		1	0.036	0.015	0.095	0.034	8.9	11.7	10.4	9.0	25.0	26.8	25.0	24.6
		1.5	0.317	0.079	0.492	0.158	9.0	9.5	9.1	9.6	25.0	24.8	24.9	25.6
		2	0.826	0.315	0.931	0.444	8.0	8.8	7.6	8.2	24.9	24.9	24.8	25.0
	30	0.5	0.015	0.005	0.022	0.013	19.0	13.0	17.0	12.3	28.4	23.0	25.7	22.5
		1	0.206	0.06	0.317	0.094	14.3	14.9	14.1	14.4	25.8	26.5	24.4	25.2
		1.5	0.775	0.331	0.899	0.502	11.5	12.8	11.1	12.2	24.9	24.9	24.8	24.8
		2	0.992	0.781	1	0.901	8.0	10.4	7.3	9.3	24.8	25.0	24.7	25.0
	40	0.5	0.019	0.009	0.043	0.021	24.5	26.1	22.3	23.1	29.9	29.9	25.2	26.8
		1	0.404	0.16	0.547	0.265	18.9	19.4	19.2	19.5	24.8	25.2	24.8	24.7
		1.5	0.955	0.698	0.989	0.832	16.3	17.4	15.7	16.9	24.7	24.9	24.7	24.9
		2	1	0.981	1	0.993	15.0	15.4	15.0	15.1	24.8	24.9	24.8	24.9

Table 12: The DR, CED, and CPE of the proposed NS_W and SS_W methods under Model IV with $\tau = 50$.

$Model I$			DR				CED				CPE			
$\tau = 25$			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W	NS_W	SS_W
20	20	0.5	0.01	0.009	0.021	0.01	0.0	3.9	4.3	6.0	39.5	43.0	44.4	47.8
		1	0.062	0.013	0.1	0.028	6.7	5.8	8.9	3.4	46.7	44.9	49.8	44.1
		1.5	0.316	0.06	0.499	0.12	8.5	7.8	8.4	8.1	49.6	48.7	49.6	48.4
		2	0.762	0.217	0.899	0.336	7.7	8.2	7.4	8.8	49.9	49.4	49.9	49.8
	30	0.5	0.018	0.011	0.027	0.017	5.6	5.5	10.9	10.9	40.9	39.7	46.0	45.8
		1	0.156	0.051	0.282	0.095	12.5	8.8	13.7	13.9	49.2	46.1	50.1	50.2
		1.5	0.649	0.283	0.83	0.462	11.5	12.6	10.8	11.8	49.9	50.0	49.6	49.7
		2	0.971	0.711	0.993	0.867	8.6	10.6	7.4	9.5	49.8	50.0	49.7	49.9
	40	0.5	0.026	0.023	0.034	0.022	11.3	13.3	15.4	16.1	41.8	43.7	50.1	50.0
		1	0.231	0.157	0.386	0.271	16.7	16.6	17.3	16.4	49.9	48.7	49.7	49.4
		1.5	0.844	0.639	0.943	0.806	13.5	14.3	12.5	13.5	49.8	50.0	49.7	49.8
		2	0.994	0.969	1	0.993	9.0	10.8	7.9	9.1	49.8	49.9	49.6	49.7
50	20	0.5	0.01	0.004	0.022	0.006	-11.5	16.3	5.9	2.5	28.9	56.5	46.6	40.0
		1	0.095	0.008	0.131	0.012	7.7	8.8	9.0	5.8	47.5	48.8	49.6	47.1
		1.5	0.427	0.057	0.584	0.091	8.9	8.4	8.6	9.8	49.8	49.7	49.4	50.2
		2	0.855	0.189	0.96	0.308	7.2	8.3	6.9	8.4	49.5	49.7	49.6	49.7
	30	0.5	0.018	0.008	0.023	0.012	5.0	13.1	13.7	9.6	41.2	46.0	48.9	45.6
		1	0.213	0.059	0.352	0.104	13.2	12.3	13.1	11.8	49.8	48.7	49.2	47.6
		1.5	0.751	0.337	0.89	0.506	11.3	12.2	10.5	11.7	49.7	49.7	49.5	49.8
		2	0.989	0.793	1	0.889	8.0	10.1	7.3	9.4	49.8	49.7	49.6	49.9
	40	0.5	0.017	0.013	0.037	0.018	17.4	15.4	18.0	12.8	49.1	43.1	44.6	45.3
		1	0.318	0.19	0.458	0.288	17.1	17.2	17.1	16.6	49.6	49.8	49.8	50.1
		1.5	0.908	0.689	0.973	0.857	13.3	14.5	11.9	13.3	49.9	50.0	49.3	49.6
		2	1	0.979	1	0.993	8.2	9.7	7.4	8.5	49.7	49.8	49.4	49.8
100	20	0.5	0.005	0.015	0.006	0.014	-6.0	-7.3	8.3	-0.4	34.4	31.1	50.3	39.6
		1	0.046	0.017	0.071	0.035	9.1	2.1	10.1	3.6	49.7	41.6	50.0	43.0
		1.5	0.336	0.096	0.499	0.168	9.1	7.7	9.0	7.6	49.8	47.5	49.8	49.1
		2	0.842	0.341	0.925	0.442	7.9	8.2	7.7	8.3	49.9	49.4	49.7	49.9
	30	0.5	0.013	0.011	0.027	0.008	14.6	13.2	12.8	5.6	50.7	49.1	46.8	40.6
		1	0.199	0.061	0.35	0.086	14.1	13.6	13.8	13.6	50.3	49.5	49.6	48.9
		1.5	0.799	0.337	0.921	0.525	11.3	12.5	10.7	11.4	49.7	49.8	49.6	49.9
		2	0.995	0.821	0.999	0.898	8.0	10.5	7.1	9.5	49.7	49.8	49.6	49.9
	40	0.5	0.023	0.014	0.04	0.017	17.2	14.6	19.3	12.1	50.2	44.6	50.0	44.1
		1	0.396	0.189	0.571	0.303	17.0	17.4	17.0	16.8	49.9	49.7	49.7	49.4
		1.5	0.963	0.748	0.992	0.845	12.5	14.5	11.5	13.1	49.8	50.0	49.5	49.8
		2	1	0.984	1	0.995	7.8	9.8	7.0	8.4	49.7	49.9	49.4	49.7

3 Additional Diagnostic Results

Table 13 is the change-point estimates and detection rate of variables of the NS_W and SS_W method unde Model I to IV with various setting of parameters.

Table 13: Post-signal diagnosis under Model I to Model IV with various setting of v. When $p = 100$, $\tau = 25$, $W = 40$.

			CPE		DR_v	
$Model$	v	δ	NS_W	SS_W	NS_W	SS_W
I	10%	1	25.4	25.6	0.105	0.099
		1.5	24.9	25.0	0.261	0.134
		2	24.9	25.0	0.805	0.414
	25%	1	25.1	25.2	0.045	0.042
		1.5	24.9	24.9	0.263	0.089
		2	24.9	24.9	0.814	0.423
II	10%	1	24.3	24.7	0.103	0.102
		1.5	24.6	24.7	0.263	0.153
		2	24.8	24.8	0.843	0.547
	25%	1	24.2	24.3	0.043	0.041
		1.5	24.6	24.7	0.241	0.114
		2	24.8	24.8	0.836	0.535
III	10%	1	25.2	24.9	0.179	0.138
		1.5	24.9	25.1	0.405	0.228
		2	24.8	24.9	0.828	0.518
	25%	1	24.7	24.9	0.090	0.060
		1.5	24.7	24.9	0.338	0.168
		2	24.8	24.9	0.807	0.486
IV	10%	1	24.8	25.2	0.200	0.156
		1.5	24.7	24.9	0.396	0.227
		2	24.8	24.9	0.818	0.501
	25%	1	24.8	24.7	0.121	0.072
		1.5	24.7	24.9	0.350	0.165
		2	24.8	24.9	0.821	0.479

4 Additional Comparison Results

Table 14 is the detection rate and detection delay of the DFEWMA chart under Model I with various setting of parameters. For the reference sample, m_0 is set to 100.

Table 14: The DR and CED of the DFEWMA chart under $ModelI$ with various setting of p, W, δ, τ, and v.

$ModelI$			$\tau = 10$				$\tau = 25$				$\tau = 50$			
			$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$		$v = 10\%$		$v = 25\%$	
p	W	δ	DR	CED	DR	CED	DR	CED	DR	CED	DR	CED	DR	CED
20	20	1	0.89	24.4	1	9.5	0.893	22.1	1	9.1	0.87	18.3	1	8.3
		1.5	1	12.1	1	6.1	1	11.4	1	6.1	1	11.0	1	5.7
		2	1	9.1	1	5.0	1	8.6	1	4.9	1	8.1	1	4.4
	30	1	0.944	23.8	1	9.5	0.944	23.3	1	9.2	0.942	19.7	1	8.5
		1.5	1	12.3	1	6.1	1	11.8	1	6.0	1	11.2	1	5.2
		2	1	9.1	1	4.9	1	9.0	1	4.8	1	8.2	1	4.2
	40	1	0.949	24.3	1	9.6	0.95	22.8	1	9.6	0.94	19.6	1	8.9
		1.5	1	12.0	1	6.2	1	11.7	1	6.1	1	10.5	1	5.3
		2	1	8.9	1	5.0	1	8.7	1	4.9	1	8.0	1	4.5
50	20	1	1	13.5	1	6.4	1	12.6	1	6.3	1	11.9	1	6.0
		1.5	1	7.8	1	4.4	1	7.7	1	4.4	1	7.2	1	3.7
		2	1	6.3	1	3.7	1	6.0	1	3.6	1	5.4	1	3.2
	30	1	1	13.1	1	6.3	1	13.0	1	6.4	1	12.3	1	5.6
		1.5	1	7.8	1	4.4	1	7.7	1	4.2	1	7.2	1	3.9
		2	1	6.2	1	3.7	1	6.1	1	3.7	1	5.3	1	2.8
	40	1	1	13.6	1	6.3	1	12.9	1	6.2	1	12.0	1	5.7
		1.5	1	7.8	1	4.4	1	7.6	1	4.3	1	7.1	1	3.6
		2	1	6.2	1	3.7	1	6.0	1	3.6	1	5.6	1	3.2
100	20	1	1	9.3	1	5.1	1	9.0	1	5.1	1	8.5	1	4.2
		1.5	1	6.1	1	3.7	1	5.9	1	3.6	1	5.3	1	3.1
		2	1	4.9	1	3.1	1	4.8	1	3.0	1	4.3	1	2.5
	30	1	1	9.3	1	5.1	1	9.4	1	5.1	1	8.5	1	4.6
		1.5	1	6.0	1	3.7	1	6.0	1	3.6	1	5.3	1	3.1
		2	1	5.0	1	3.2	1	4.8	1	3.1	1	4.2	1	2.6
	40	1	1	9.3	1	5.1	1	9.2	1	5.1	1	9.0	1	4.5
		1.5	1	6.1	1	3.7	1	6.0	1	3.6	1	5.5	1	2.9
		2	1	4.9	1	3.1	1	4.9	1	3.0	1	4.4	1	2.8

5 Additional Case Study Results

Table 15 is the detection rate, detection delay and change-point estimates of the proposed NS_W and SS_W charts in the case study of scemiconductor data.

Table 15: The DR and CED of the proposed NS_W and SS_W methods in signal detection, and the CPE in post-signal diagnosis with various setting of W and τ.

		NS_W			SS_W		
W	τ	DR	CED	CPE	DR	CED	CPE
20	10	1	10.00	14.15	0.029	48.79	50.00
	25	1	4.99	26.24	0.039	26.67	44.95
	50	1	4.92	51.19	0.036	17.22	60.97
30	10	1	20.00	20.05	0.04	49.75	45.38
	25	1	5.02	26.35	0.083	18.31	36.01
	50	1	4.95	51.22	0.071	12.75	54.49
40	10	1	30.00	25.96	0.096	62.19	53.81
	25	1	15.00	31.94	0.114	30.22	45.69
	50	1	4.93	51.12	0.33	10.64	55.95

References

[1] Chang, J., Zheng, C., Zhou, W. X., and Zhou, W. (2017). Simulation-based hypothesis testing of high dimensional means under covariance heterogeneity. Biometrics, 73(4), 1300-1310.