Natural Language Processing

Lecture 10:
Machine Learning: Linear and Log-Linear Models

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COMS W4705
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Machine Learning and NLP

- We have encountered many different situations where we had to make a prediction:
  - Text classification, language modeling, POS tagging, constituency/dependency parsing,
  - These are all classification problems of some form.
- Today: Some machine learning background. Linear/log-linear models. Basic neural networks.

Generative Algorithms

- Assume the observed data is being “generated” by a “hidden” class label.
- Build a different model for each class.
- To predict a new example, check it under each of the models and see which one matches best.
- Model $P(x|y)$ and $P(y)$. Then use bases rule

$$P(y|x) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

Discriminative Algorithms

- Model conditional distribution of the label given the data

$$P(y|x)$$

- Learns decision boundaries that separate instances of the different classes.

- To predict a new example, check on which side of the decision boundary it falls.

Machine Learning Definition

- “Creating systems that improve from experience.”
- *“A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.”*
  (Tom Mitchel, Machine Learning 1997)

Inductive Learning (a.k.a. *Science*)

- **Goal:** given a set of input/output pairs (training data), find the function $f(x)$ that maps inputs to outputs.
  Problem: We did not see all possible inputs!

- Learn an approximate function $h(x)$ from the training data and hope that this function *generalize well* to unseen inputs.

- **Ockham's razor:** Choose the *simplest* hypothesis that is consistent with the training data.

- vs. deductive learning: start from general rule and infer new knowledge from it.

Supervised Learning

- Given: Training data consisting of training examples $(\mathbf{x_1}, y_1), \ldots, (\mathbf{x_n}, y_n)$, where $\mathbf{x_i}$ is an input example (a d-dimensional vector of attribute values) and y_i is the label.

example					label
1	x_{11}	x_{12}	$\ldots$	x_{1d}	y_1
…	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
i	x_{i1}	x_{i2}	$\ldots$	x_{id}	y_i
…	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
n	x_{n1}	x_{n2}	$\ldots$	x_{nd}	y_n

- Goal: learn a hypothesis function h(x) that approximates the true relationship between x and y. This functions should

  1) ideally be consistent with the training data.
  2) generalize to unseen examples.

- In NLP, y_i are often drawn from a finite, discrete set. What are some examples for $(\mathbf{x_i}, y_i)$ pairs in NLP?

Classification and Regression

- Recall: In **supervised learning**, training data consisting of training examples
  $(\mathbf{x_1}, y_1), \ldots, (\mathbf{x_n}, y_n)$, where $\mathbf{x_j}$ is an input example (a d-dimensional vector of attribute values) and y_j is the label.

- Two types of supervised learning problems:

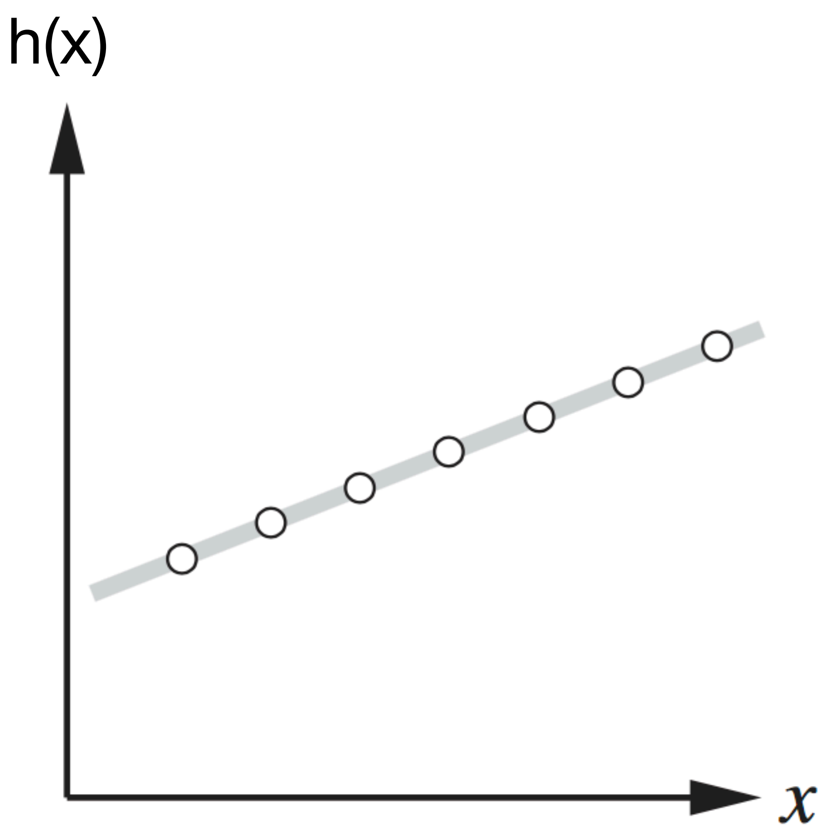
  - In classification: y_j is a finite, discrete set.
    Typically $y_j \in \{-1, +1\}$. i.e. predict a label from a set of labels.
    Learn a **classifier** function:

$$h : \mathbb{R}^d \to \{-1, +1\}$$

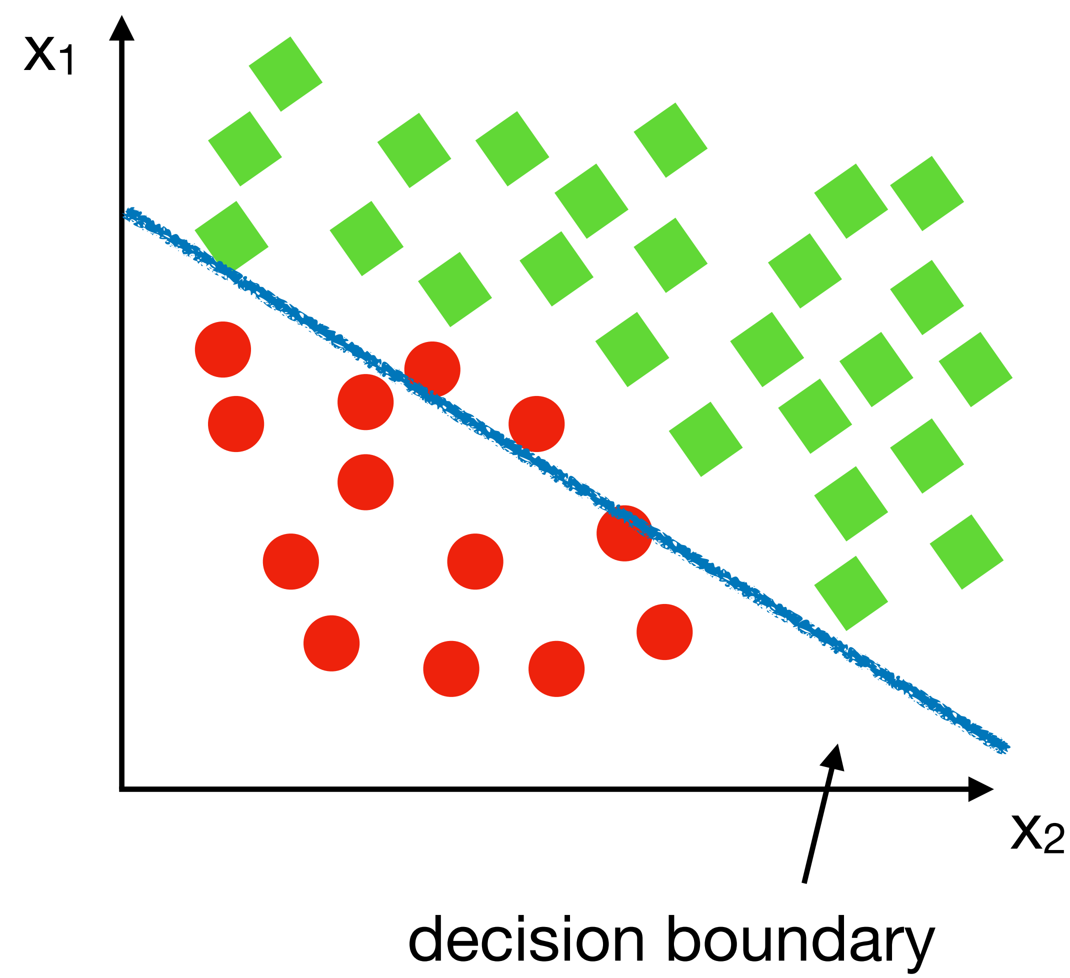
  - In regression: $\mathbf{x_j} \in \mathbb{R}^d$, $y_i \in \mathbb{R}$. i.e. predict a numeric value.
    Learn a **regressor** function:

$$h : \mathbb{R}^d \to \mathbb{R}$$

Linear Classification and Regression

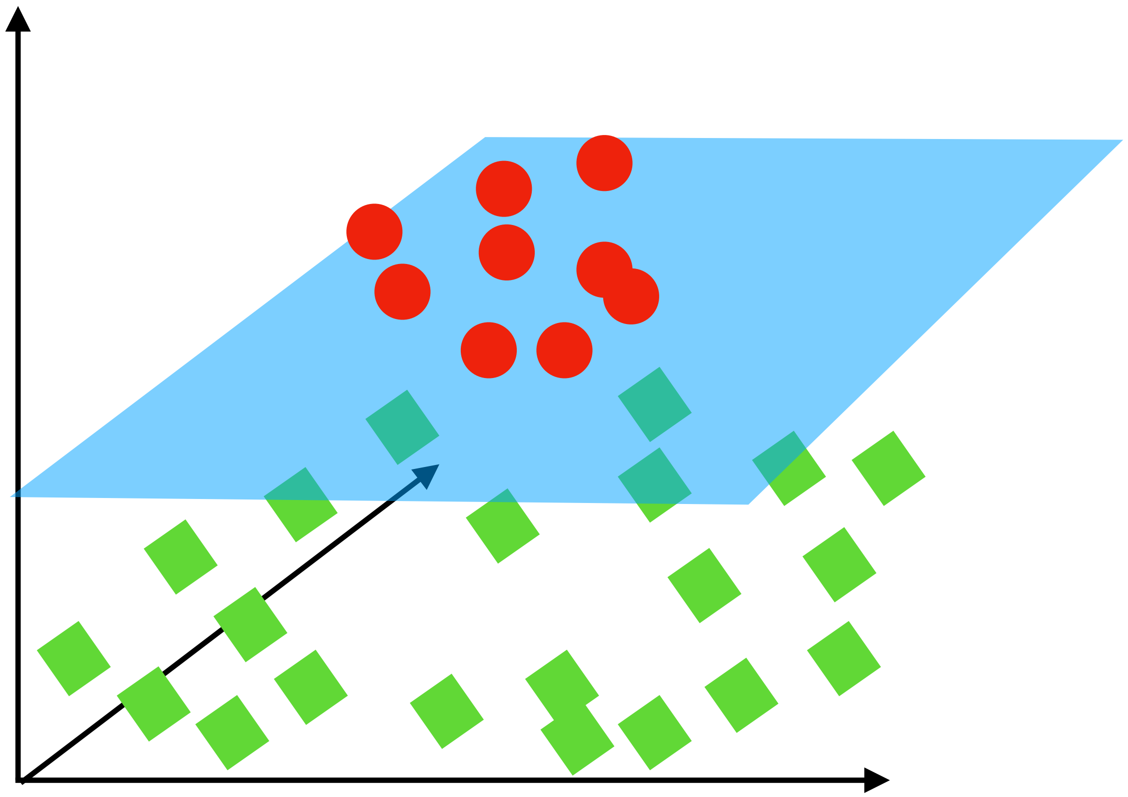


Regression



Classification

Linear Classification



Training ML models

Training Data → ML algorithm → function h(x)=y

- How can we be confident about the learned function?
- Can compute empirical error/risk on the training set:

$$E_{train}(h) = \sum_{i=1}^{n} loss(y_i, h(x_i))$$

- Typical loss functions:
  - Least square loss (L2): $loss(y_i, h(x_i)) = (y_i - h(x_i))^2$
  - Classification error: $loss(y_i, h(x_i)) = \begin{cases} 1 \text{ if } sign(h(x_i)) \neq sign(y_i) \\ 0 \text{ otherwise.} \end{cases}$

Training ML models

Training Data $\longrightarrow$ ML algorithm $\longrightarrow$ function h(x)=y

- Empirical error/risk:

$$E_{train}(h) = \sum_{i=1}^{n} loss(y_i, h(x_i))$$

- Training aims to minimize E_{train} .

- We hope that this also minimizes E_{test}, the test error.

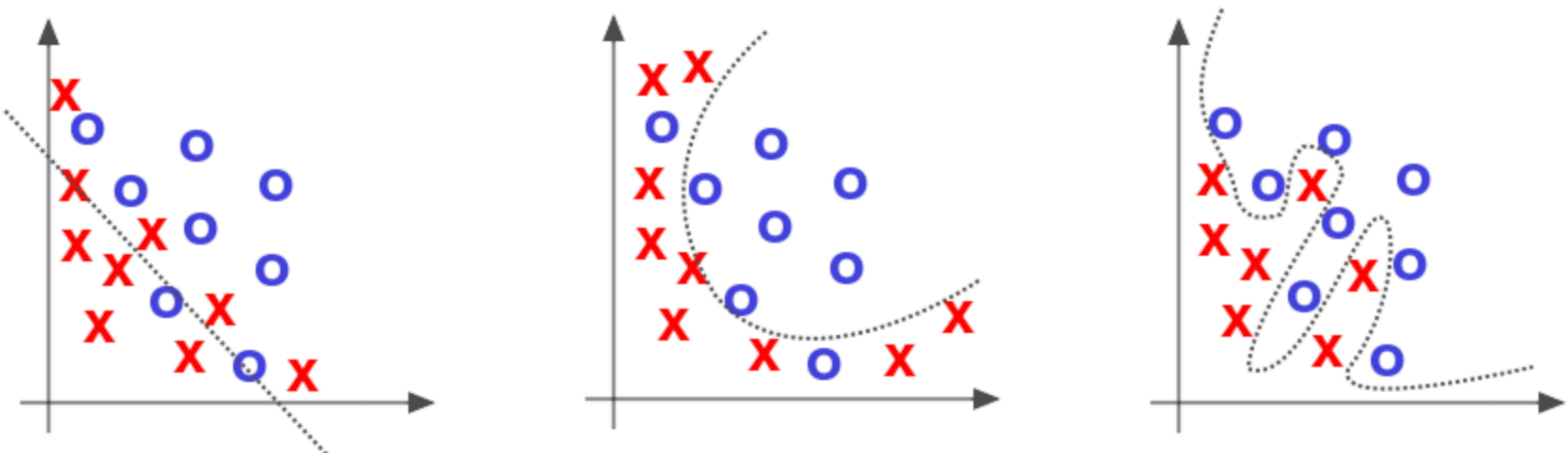
Overfitting

- Problem: Minimizing empirical risk can lead to **overfitting.**
    - This happens when a model works well on the training data, but it does not **generalize** to testing data.
    - Data sets can be noisy. Overfitting can model the noise in the data.

Preventing Overfitting

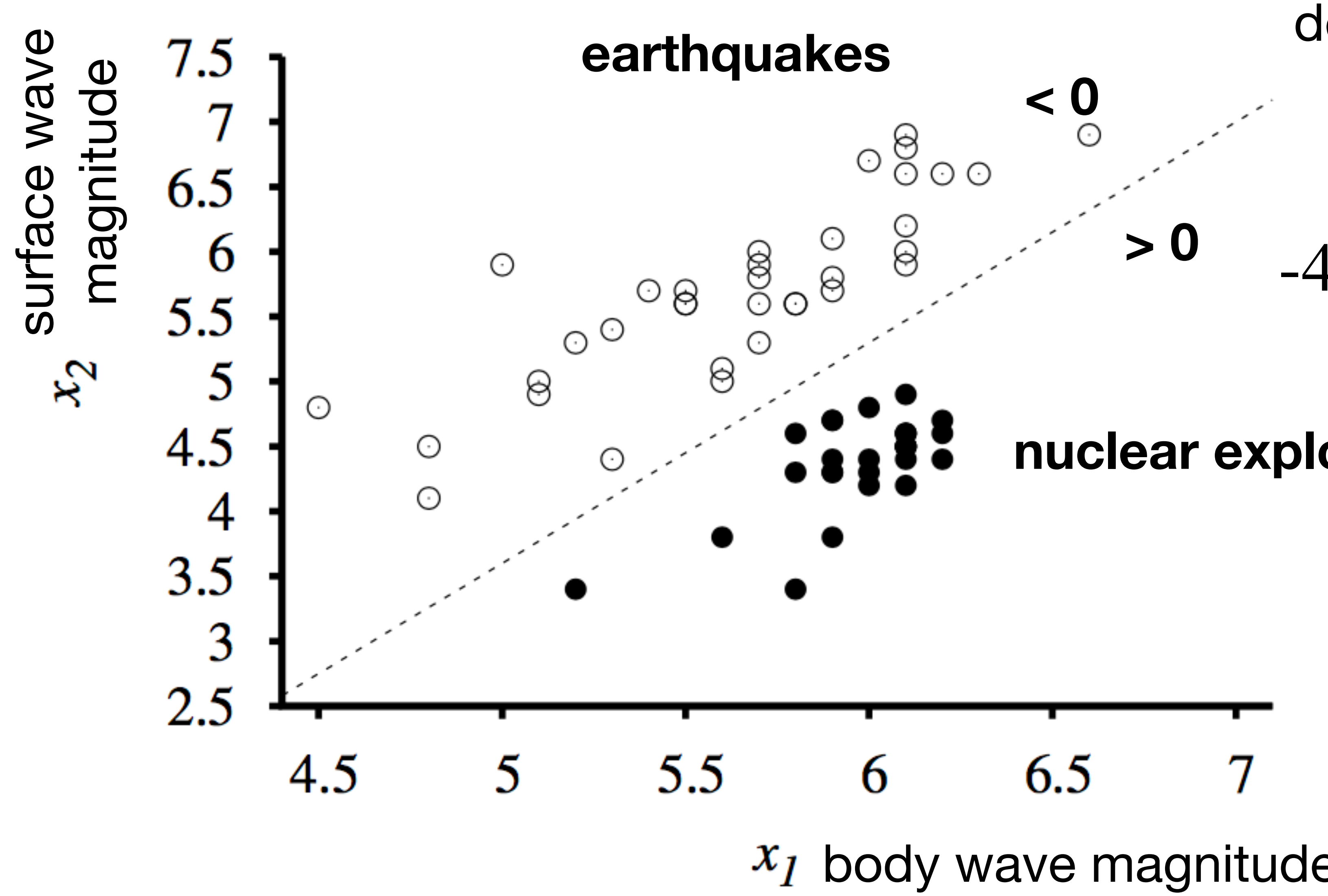
- Solutions: Simpler models.
    - Reduce the number of features (feature selection).
    - Model selection.
    - Regularization.
    - Cross validation.
- However: Adding wrong assumptions (bias) to the training algorithm can lead to **underfitting**!

Goodness of Fit



Under Fit　　Appropriate　　Over Fit

Linear Classification



decision boundary
linear separator

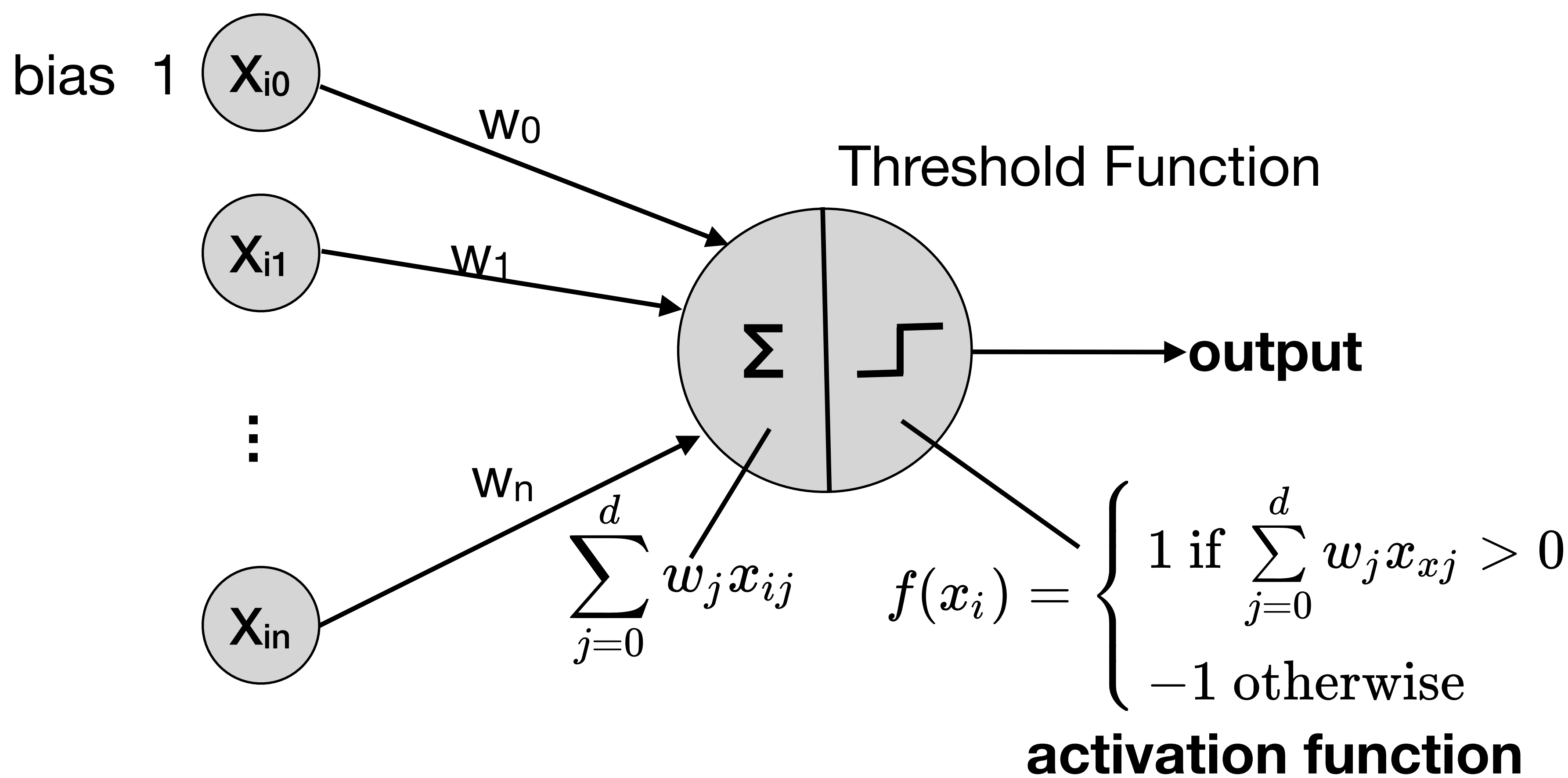
$$x_2 = 1.7\ x_1 - 4.9$$

$$-4.9 + 1.7x_1 - x_2 = 0$$

$$h(\mathbf{w}) = 1w_0 \cdot + x_1 w_1 + \ldots + x_n w_n$$

$$= \mathbf{w} \cdot \mathbf{x}$$

Linear Model



$$f(x_i) = sign(\sum_{j=0}^{d} w_j x_{ij})$$

Linear Models

- We have chosen a function class defines a function class (linear separators).
    - Specified by parameter **w**.
- Need to estimate **w** on the basis of the training set.
- What loss should we use? One option: minimize classification error:

$$loss(y_i, h(x_i)) = \begin{cases} 1 \text{ if } sign(h(x_i)) \neq sign(y_i) \\ 0 \text{ otherwise.} \end{cases}$$

Perceptron Learning

- Problem: Threshold function is not differentiable, so we cannot find a closed-form solution or apply gradient descent.
- Instead use iterative perceptron learning algorithm:
  - Start with arbitrary hyperplane.
  - Adjust it using the training data.
  - Update rule: $w_j \leftarrow w_j + (y - h_{\mathbf{w}}(\mathbf{x})) \times x_j$
- **Perceptron Convergence Theorem** states that any linear function can be learned using this algorithm in a finite number of iterations.

Perceptron Learning Algorithm

Input: Training examples $(x_1, y_1),\ldots,(x_n, y_n)$
Output: A perceptron defined by $(w_0, w_1,\ldots,w_d)$

Initialize $w_j \leftarrow 0$, for j=0…d

while not converged:

"convergence" means that the weights don't change for one entire iteration through the training data.

shuffle training examples.
for each training example $(\mathbf{x}_i, y_i)$:

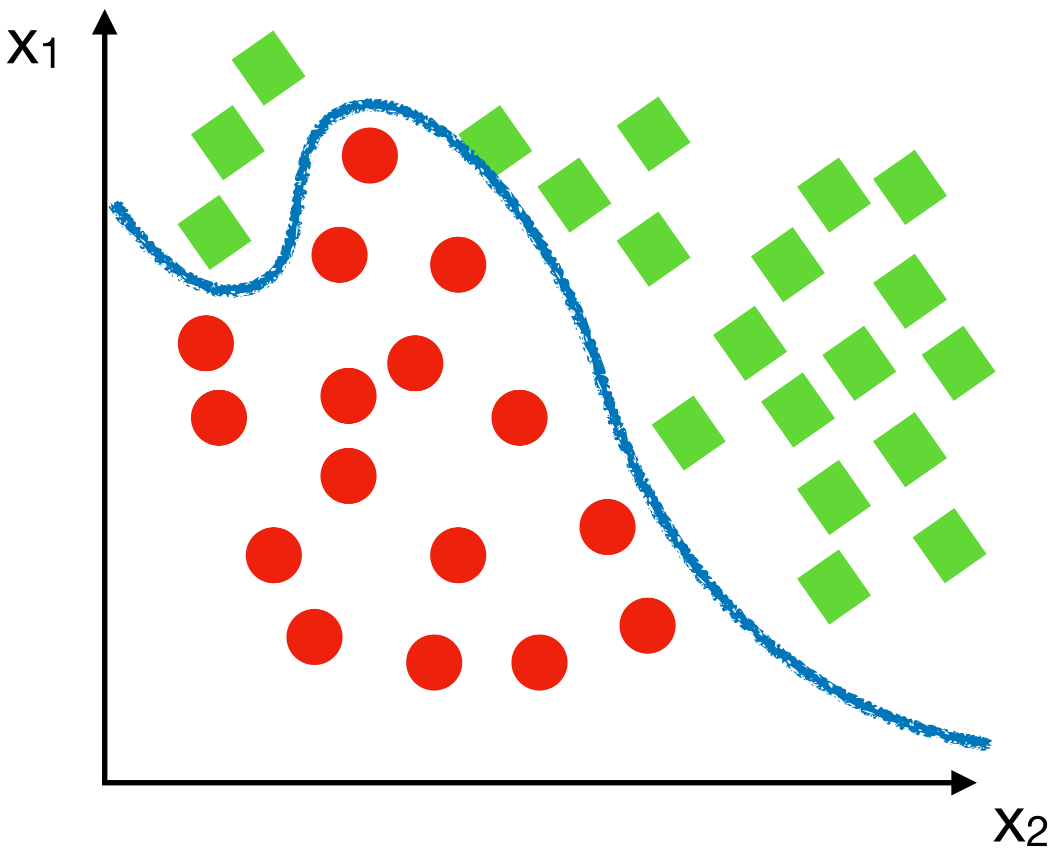
if output - target != 0: #(output and prediction do not match)

for each weight w_j:

$$w_j \leftarrow w_j + (y - h_{\mathbf{w}}(\mathbf{x})) \times x_j$$

Perceptron

- Simple learning algorithm. Guaranteed to converge after a finite number of steps.
  - But **only** if the data is linearly separable.



perceptron cannot learn this

Feature Functions

- In NLP we often need to make multi-class decisions. Linear models provide only binary decisions.
- Use a feature function $\phi(x, y)$ where x is an input object and y is a possible output.
- The values of ϕ are d-dimensional vectors.

$$\phi(x, y) : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}^d$$

Linear Models with Feature Function

- Prediction becomes $\arg\max_{y'} \phi(x, y) \cdot \mathbf{w}$
- Compute dot product between feature value and weights.
  - then choose the y' that gives the highest product.
- Can still use perceptron for training.

Log-Linear Model

(a.k.a. "Maximum Entropy Models")

- Define conditional probability P(y|x)

$$\sum_y P(y|x;\mathbf{w}) = \frac{\exp\left(\mathbf{w}\cdot\phi(x,y)\right)}{\sum_{y'\in\mathcal{Y}}\exp(\mathbf{w}\cdot\phi(x,y))}$$

- $\exp(\mathrm{d}) = \mathrm{e}^{\mathrm{z}}$ is positive for any z.

- $\sum_y P(y|x;\mathbf{w}) = 1$

- But how should we estimate **w**?

Log-Likelihood

- Define the log-likelihood of some model **w** on the training data $(x_1, y_1), \ldots, (x_n, y_n)$ as

$$LL(\mathbf{w}) = \sum_{i=1}^{n} \log P(y_i | x_i; \mathbf{w})$$

- We want to compute the maximum likelihood

$$LL^*(\mathbf{w}) = \arg\max_{\mathbf{w}} \sum_{i=1}^{n} \log P(y_i | x_i; \mathbf{w})$$

- Unfortunately, there is no general analytical solution. Can use gradient-based optimization.

Simple Gradient Ascent

Initialize w ←any setting in the parameter (weight) space

for a set number of iterations T:

for each w_i in $\boldsymbol{w}$:

$$w_i' \leftarrow w_i + \alpha \frac{\partial}{\partial w_i} LL(\mathbf{w})$$

update each w_i to w'_i

- Follow the gradients (partial derivatives) to find a parameter setting that maximizes $LL(\boldsymbol{w})$
- $\alpha > 0$ is the **learning rate** or **step size.**

Partial Derivative of the Log Likelihood

$$\frac{\partial}{\partial_i} LL(\mathbf{w}) = \frac{\partial}{\partial_i} \sum_{i=1}^{n} \log P(y_i | x_i ; \mathbf{w})$$

$$= \sum_i \phi_j(x_i, y_i) - \sum_i \sum_y P(y | x_i ; \mathbf{w}) \phi_j(x_i, y)$$

Regularization

- Problem: Parameter estimation can overfit the training data.
- Can include a regularization term. For example L_2 regularizer:

$$LL(\mathbf{w}) = \sum_{i=1}^{n} \log P(y_i | x_i ; \mathbf{w}) - \frac{\lambda}{2} |\mathbf{w}|^2$$

Regularization

$$LL(\mathbf{w}) = \sum_{i=1}^{n} \log P(y_i|x_i;\mathbf{w}) - \frac{\lambda}{2}|\mathbf{w}|^2$$

- λ > 0 controls the strength of the regularization.
- Since we are maximizing $\mathbf{w}^* = \arg\max_{\mathbf{w}} LL(\mathbf{w})$, there is now a trade-off between fit and model 'complexity'.

$$\frac{\partial}{\partial_j} LL(\mathbf{w}) = \sum_i \phi_j(x_i, y_i) - \sum_i \sum_y P(y|x_i;\mathbf{w})\phi_j(x_i, y) - \lambda w_j$$

POS Tagging with Log-Linear Models

- Previously we used a generative model (HMM) for POS tagging.

- Now we want to use a discriminative model for

$$P(t_1, t_2, \ldots, t_n | w_1, w_2, \ldots, w_n)$$

$$= \prod_{i=1}^{m} P(t_i | t_1, \ldots, t_{i-1}, x_1, \ldots, x_m)$$

- Next tag is conditioned on previous tag sequence and all observed words.

Maximum Entropy Markov Models (MEMM)

- Make an independence assumption (similar to HMM):

$$P(t_1, t_2, \ldots, t_n | w_1, w_2, \ldots, w_n)$$

$$= \prod_{i=1}^{m} P(t_i | t_1, \ldots, t_{i-1}, w_1, \ldots, w_m)$$

$$= \prod_{i=1}^{m} P(t_i | t_{i-1}, w_1, \ldots, w_m)$$

- Probability only depends on the previous tag.

MEMMs

$$\prod_{i=1}^{m} P(t_i | t_{i-1}, w_1, \ldots, w_m)$$

- Model each term using a log-linear model

$$P(t_i | t_{i-1}, w_1, \ldots, w_m) = \frac{\exp(\mathbf{w} \cdot \phi(w_1, \ldots, w_m, i, t_{i-1}, t_i))}{\sum_{t'} \exp(\mathbf{w} \cdot \phi(w_1, \ldots, w_m, i, t_{i-1}, t'))}$$

- ϕ is a feature function defined over:
    - the observed words $w_1,\ldots,w_m$
    - the position of the current word
    - the previous tag t_{i-1}
    - the suggested tag for the current word t_i

- t' is a variable ranging over all possible tags.

MEMMs

$$P(t_i|t_{i-1}, w_1, \ldots, w_m) = \frac{\exp(\mathbf{w} \cdot \phi(w_1, \ldots, w_m, i, t_{i-1}, t_i))}{\sum_{t'} \exp(\mathbf{w} \cdot \phi(w_1, \ldots, w_m, i, t_{i-1}, t'))}$$

- Training: same as any log-linear model.
- Decoding: Need to find $\arg\max_{t_1, \ldots t_m} P(t_i, \ldots, t_m | t_{i-1}, w_1, \ldots, w_m)$
  - Can use Viterbi algorithm!

Feature Function

(Ratnaparkhi, 1996)

- $\phi(w_1, \ldots, w_m, i, t_{i-1}, t_i)$ is a feature vector of length d.
- (w_i, t_i), (w_{i-1}, t_i), (w_{i-2}, t_i), (w_{i+1}, t_i), (w_{i+2}, t_i)
- (t_{i-1}, t_i)
- (w_i contains numbers, t_i),
  (w_i contains uppercase characters, t_i)
  (w_i contains a hyphen, t_i)
- ($prefix_1$ of w_i,t_i), ($prefix_2$ of w_i,t_i), ($prefix_3$ of w_i,t_i), ($prefix_4$ of w_i,t_i)
  ($suffix_1$ of w_i,t_i), ($suffix_2$ of w_i,t_i), ($suffix_3$ of w_i,t_i), ($suffix_4$ of w_i,t_i)

Feature Example

The stories about well-heeled communities and developers ...
DT NNS IN ??

- (*well-helled*,JJ), (*about*,JJ), (*stories*,JJ), (*communities*, JJ), (*and*,JJ)
- (IN,JJ)
- (w_i contains a hyphen, JJ)
- (*w*,JJ), (*we*,JJ), (*wel*,JJ), (*well*, JJ)
  (*d*,JJ), (*ed*,JJ), (*led*,JJ), (*eled*, JJ)