HW3

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Problem 1

Our objective is to solve the following optimization problem:

$min_{f \in \mathcal{H}} R(f)$, where $\mathcal{H} = \{f : \mathbb{R}^d \to [K] | f \ integrable\}$ and $R(f) = \int_{\mathbb{R}^d} R(f|x)p(x)dx$.

Due to the monotonicity of the integral, the problem is then equivalent to:

$min_{f \in \mathcal{H}} R(f|x) = \Sigma_{y \in [K]} L^{0-1}(y, f(x))P(y|x)$.

Further, we can rewrite $R(f|x)$:

$R(f|x)$

$= \Sigma_{y \in [K]} I_{\{y \neq f(x)\}} P(y|x)$

$= \Sigma_{y \in [K]} (1 - I_{\{y = f(x)\}})P(y|x)$

$= \Sigma_{y \in [K]} P(y|x) - \Sigma_{y \in [K]} I_{\{y = f(x)\}} P(y|x)$

$= 1 - \Sigma_{y \in [K]} I_{\{y = f(x)\}} P(y|x)$

Note that to minimize $R(f|x)$ is equivalent to maximize the following:

$max_{f \in \mathcal{H}} \ \Sigma_{y \in [K]} I_{\{y = f(x)\}} P(y|x)$

Apparently, when $f(x) = argmax_{y \in [K]} P(y|x)$, it reaches the maximum.

Thus, the classifier $f(x) = argmax_{y \in [K]} P(y|x)$ minimizes the risk, which is exactly the Bayes-optimal classifier.

Problem 2

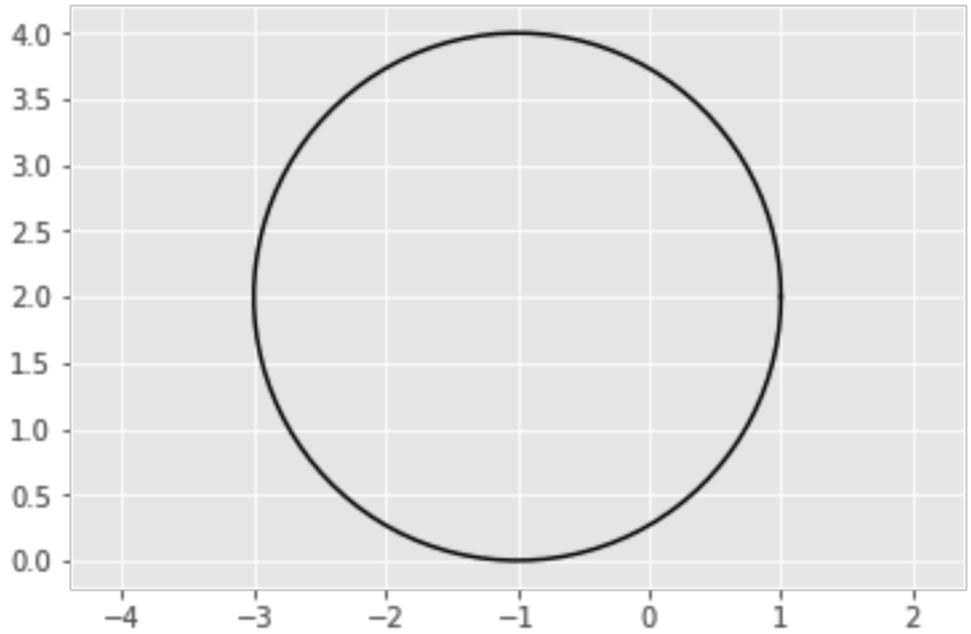
In [1]:

```
import matplotlib.pyplot as plt
plt.style.use("ggplot")
%matplotlib inline
import numpy as np
```

In [2]:

```
# 1
theta = np.linspace(0, 2*np.pi, 100)
r = 2
x1 = r*np.cos(theta) - 1
x2 = 2 - r*np.sin(theta)
plt.axis('equal')
plt.plot(x1, x2, "k")

plt.show()
```

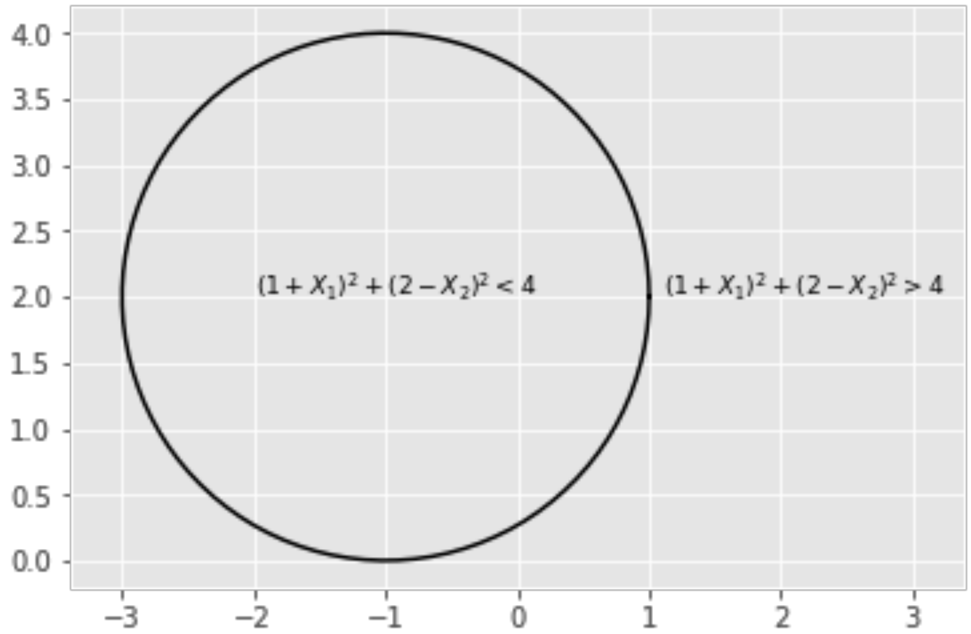


In [3]:

```
# 2
theta = np.linspace(0, 2*np.pi, 100)
r = 2
x1 = r*np.cos(theta) - 1
x2 = 2 - r*np.sin(theta)
plt.axis('equal')
plt.xlim(-5, 5)
plt.plot(x1, x2, "k")

plt.text(-2, 2, r'$(1+X_1)^2+(2-X_2)^2 < 4$', fontsize=9)
plt.text(1.1, 2, r'$(1+X_1)^2+(2-X_2)^2 > 4$', fontsize=9)

plt.show()
```



In [4]:

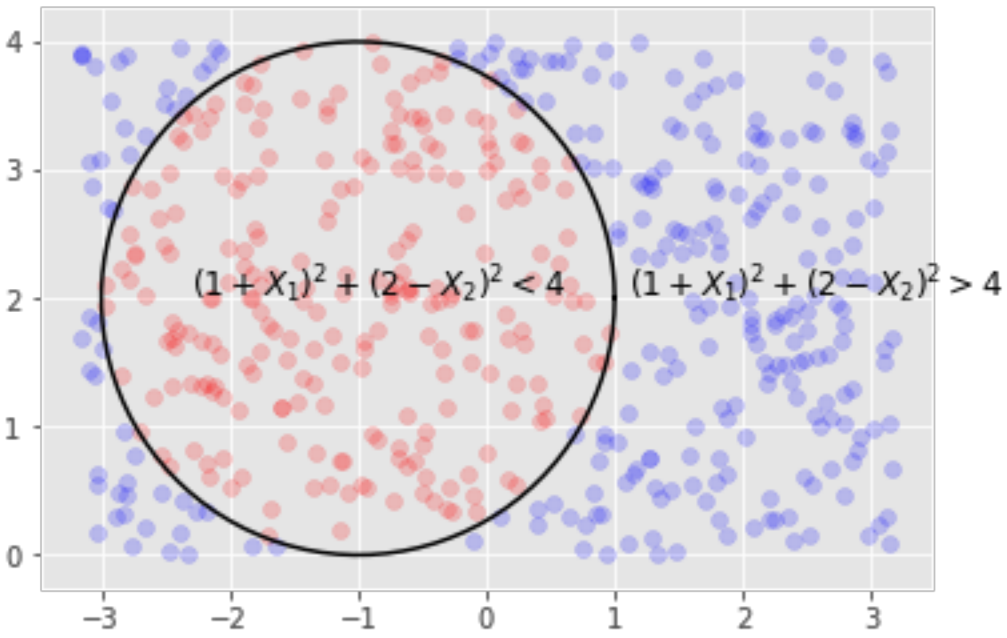
```
# 3
theta = np.linspace(0, 2*np.pi, 100)
r = 2
x1 = r*np.cos(theta) - 1
x2 = 2 - r*np.sin(theta)
plt.axis('equal')
plt.xlim(-5, 5)
plt.plot(x1, x2, "k")

rand1 = np.random.uniform(-3.2, 3.2, 500)
rand2 = np.random.uniform(0, 4, 500)
dist = np.sqrt((rand1+1)**2 + (rand2-2)**2)
inner = [[rand1[i], rand2[i]] for i in range(len(rand1)) if dist[i] < 2]
outer = [[rand1[i], rand2[i]] for i in range(len(rand1)) if dist[i] > 2]

plt.scatter([row[0] for row in inner], [row[1] for row in inner], c="r", alpha =
0.2)
plt.scatter([row[0] for row in outer], [row[1] for row in outer], c="b", alpha =
0.2)

plt.text(-2.3, 2, r'$(1+X_1)^2+(2-X_2)^2 < 4$', fontsize=12)
plt.text(1.1, 2, r'$(1+X_1)^2+(2-X_2)^2 > 4$', fontsize=12)

plt.show()
```



Red: (-1, 1) Blue: (0, 0), (2, 2), (3, 8)

4.

Because it can be rewritten in a linear form: $X_1^2 + X_2^2 + 2 * X_1 - 4 * X_2 + 5 > 4$

Problem 3

In [14]:

```
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.decomposition import PCA
from sklearn.decomposition import IncrementalPCA
from sklearn.linear_model import LogisticRegression
```

In [15]:

```
# training data
X3 = np.loadtxt("/Users/apple/Desktop/semester_2/4.Stat_ML/hw/hw3/train_3.txt",
delimiter=',')
X5 = np.loadtxt("/Users/apple/Desktop/semester_2/4.Stat_ML/hw/hw3/train_5.txt",
delimiter=',')
X8 = np.loadtxt("/Users/apple/Desktop/semester_2/4.Stat_ML/hw/hw3/train_8.txt",
delimiter=',')
X = np.concatenate((X3, X5, X8))

y3 = np.repeat(3, len(X3))
y5 = np.repeat(5, len(X5))
y8 = np.repeat(8, len(X8))
y = np.concatenate((y3, y5, y8))

# testing data
test = np.loadtxt("/Users/apple/Desktop/semester_2/4.Stat_ML/hw/hw3/zip_test.txt
", delimiter=' ')

y3_test = np.repeat(3, len(test[test[:, 0] == 3]))
y5_test = np.repeat(5, len(test[test[:, 0] == 5]))
y8_test = np.repeat(8, len(test[test[:, 0] == 8]))
y_test = np.concatenate((y3_test, y5_test, y8_test))

X3_test = np.delete(test, 0, 1)[test[:, 0] == 3]
X5_test = np.delete(test, 0, 1)[test[:, 0] == 5]
X8_test = np.delete(test, 0, 1)[test[:, 0] == 8]
X_test = np.concatenate((X3_test, X5_test, X8_test))
```

In [16]:

```
# 1. LDA
lda_1 = LinearDiscriminantAnalysis()
lda_1.fit(X, y)

# train
y1 = lda_1.predict(X)
train_err_1 = 1-np.sum(y1 == y)/len(y)

# test
y1_test = lda_1.predict(X_test)
test_err_1 = 1-np.sum(y1_test == y_test)/len(y_test)
```

In [17]:

```
# 2. PCA & LDA
pca = PCA(n_components=49)
pca.fit(X)
X_pca = pca.transform(X)
X_pca_test = pca.transform(X_test)

# fit LDA and predict
lda_2 = LinearDiscriminantAnalysis()
lda_2.fit(X_pca, y)

# train
y2 = lda_2.predict(X_pca)
train_err_2 = 1-np.sum(y2 == y)/len(y)

# test
y2_test = lda_2.predict(X_pca_test)
test_err_2 = 1-np.sum(y2_test == y_test)/len(y_test)
```

In [18]:

```
# 3. filter & LDA
# help function
def filter(arr, size = 16, step = 2):
    # size: the length or width of image
    n = size // step
    res = np.zeros(n**2)
    for i in range(n):
        for j in range(n):
            t1 = i*step*size + j*step
            t2 = i*step*size + j*step + 1
            t3 = i*step*size + j*step + size
            t4 = i*step*size + j*step + size + 1
            res[i*n + j] = np.mean([arr[t1], arr[t2], arr[t3], arr[t4]])
    return res

# filter step
X_filter = np.zeros((len(X), 64))
for i in range(len(X)):
    X_filter[i] = filter(X[i])

X_filter_test = np.zeros((len(X_test), 64))
for i in range(len(X_test)):
    X_filter_test[i] = filter(X_test[i])

# fit LDA
lda_3 = LinearDiscriminantAnalysis()
lda_3.fit(X_filter, y)

# train
y3 = lda_3.predict(X_filter)
train_err_3 = 1-np.sum(y3 == y)/len(y)

# test
y3_test = lda_3.predict(X_filter_test)
test_err_3 = 1-np.sum(y3_test == y_test)/len(y_test)
```

In [19]:

```
# 4 multiple logistic regression
multi_log_reg = LogisticRegression(random_state=0, solver='lbfgs', multi_class='
multinomial', max_iter=500).fit(X, y)

# train
y4 = multi_log_reg.predict(X)
train_err_4 = 1-np.sum(y4 == y)/len(y)

# test
y4_test = multi_log_reg.predict(X_test)
test_err_4 = 1-np.sum(y4_test == y_test)/len(y_test)
```

In [20]:

```
model = ["lda", "pca_lda", "filter_lda", "log_reg"]
train_err = np.array([train_err_1, train_err_2, train_err_3, train_err_4])
test_err = np.array([test_err_1, test_err_2, test_err_3, test_err_4])
```

In [21]:

```
plt.plot(model, train_err)
plt.plot(model, test_err)
plt.legend(("train error", "test error"))

plt.show()
```

As we can see, test error is always greater than train error. For training, multiple linear regression has the lowest error while for testing LDA with filtering has the lowest error.