



Course Name: Wireless Communications Dept.: Electronic & Electrical Engineering

Exam Duration: 2 hours

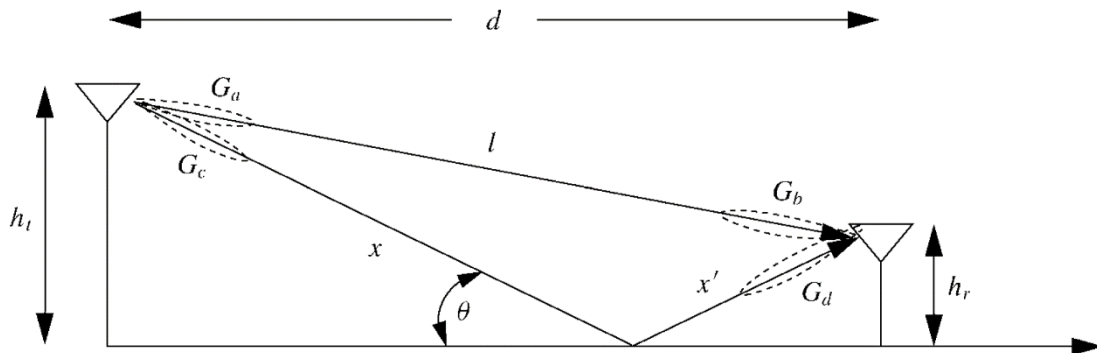
Exam Paper Setter: WANG Rui

Question No.	1	2	3	4	5	6	7	8	9	10
Score	25	25	30	20						

This exam paper contains 4 questions and the score is 100 in total. (Please hand in your exam paper, answer sheet, and your scrap paper to the proctor when the exam ends.)

#### Q1. Channel Model

- (a) Please discuss the relation between narrowband/wideband signals and flat/frequency selective fading channels.
- (b) For the following two-ray model with  $d = 100m$ ,  $h_t = 10m$ , and  $h_r = 2m$ , please estimate the delay spread between two signals (light speed  $c = 3 \times 10^8 m/s$ ).



- (c) Given a cellular system with central frequency at 900MHz. The transmit power of base station is 1W.
- Consider the simplified path-loss model with the path-loss exponent, constant of antenna and channel attenuation characteristics, reference distance as  $\gamma = 4, K = 15 \text{ dB}, d_0 = 2m$ . Suppose a user is 100 m away from the base station with environment noise  $-40 \text{ dBm}$ . What's the receive SNR? ( $\log_{10} 2 \approx 0.301$ )
  - Based on problem (1), assuming log-normal shadowing with standard deviation  $\sigma = 5 \text{ dB}$ , please compute the probability that the user could receive data with a SNR above 10dB. ( $Q(3.408) \approx 0.0003, Q(2.408) \approx 0.008, Q(1.408) \approx 0.0796, Q(0.408) \approx 0.3416$ )

3. If there no LOS component exists from the base station to user, and the environment is filled with scatters, please tell which channel model is appropriate – Rayleigh or Rician? With the channel model and average received signal power 80 dBm, please find the outage probability relative to a threshold  $P_0 = -90$  dBm.

### Q2. Channel Capacity

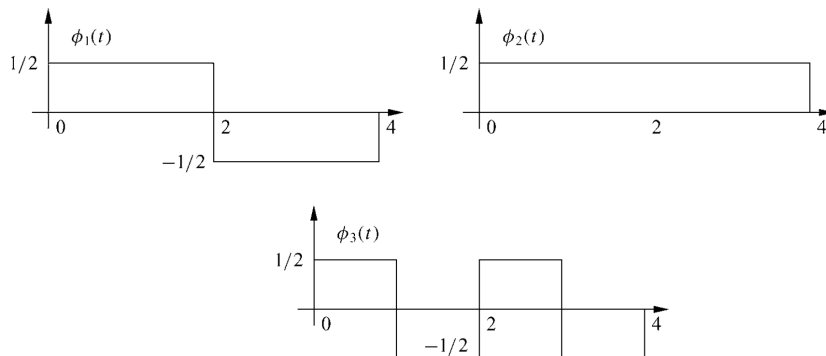
Assume an i.i.d. channel with a flat fading channel amplitude gain  $\sqrt{g[i]}$ , which can take on three possible values:  $\sqrt{g_1} = 0.1$  with probability  $p_1 = 0.2$ ,  $\sqrt{g_2} = 0.3$  with probability  $p_2 = 0.5$ , and  $\sqrt{g_3} = 1$  with probability  $p_3 = 0.3$ . The average transmit power is 1mW, the noise power spectral density is  $\frac{N_0}{2}$  with  $N_0 = 10^{-9}$  W/Hz, and the channel bandwidth is 20 kHz. Assume both the receiver and the transmitter have knowledge of CSI.

- Find the ergodic capacity of this channel without power adaptation.
- Find the ergodic capacity of this channel with power adaptation (Using water-filling method).
- Find the zero-outage capacity of this channel.
- Which one is the largest? Why?

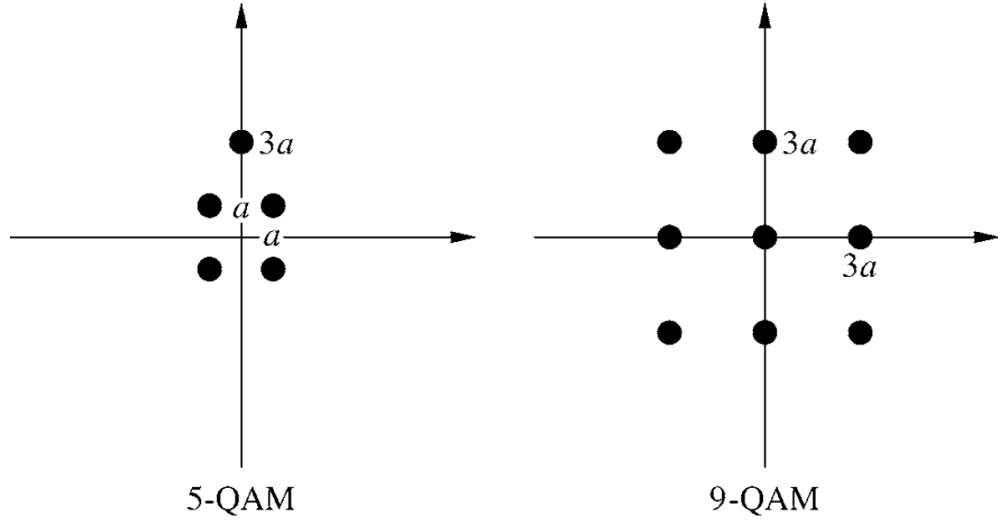
### Q3. Digital Modulation

- What's the dimensions of signal spaces for digital modulation schemes: BPSK, QPSK, 2048QAM, 4-FSK?
- Please explain why the Rayleigh fading channel leads to larger error probability than the AWGN channel for the same transmit signal and average receive signal power.
- Consider the three signal waveforms  $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$  as follows. Please
  - Show that these waveforms are orthonormal
  - Express the waveform  $x(t)$  as a linear combination of  $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$  and find the coefficients, where  $x(t)$  is given as

$$x(t) = \begin{cases} 2 & 0 \leq t < 2 \\ 4 & 2 \leq t \leq 4 \end{cases}$$



- Please use union bound to derive the average error probabilities of the following two constellations for AWGN channel. The variance of noise per signal-space dimension is  $N_0/2$ . Assume all the constellation points are transmitted with equal probability.



#### Q4. MIMO

Consider a  $3 \times 3$  narrowband MIMO channel matrix  $\mathbf{H}$  with the following SVD form:

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3] \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]^H,$$

where the singular values are  $\sigma_1 = \sqrt{3}$ ,  $\sigma_2 = \sqrt{2}$ ,  $\sigma_3 = 1$ . Let  $P$  and  $\sigma^2$  be total transmit power of all antennas and average noise power, respectively.  $P/\sigma^2 = 10dB$ . Assuming channel state information is known at both transmitter and receiver, please find the channel capacities of this channel with bandwidth  $B=10\text{KHz}$  for the following three power allocation schemes.

- Equal power allocation on all the three decomposed SISO channel;
- Water-filling power allocation;
- Allocate all the power to the channel with largest gain  $\sigma_1$ .

What's the water-filling power allocation for  $\frac{P}{\sigma^2} = 100dB$  and  $\frac{P}{\sigma^2} = -100dB$ ? Please provide approximate values of power allocation on the three decomposed channels.