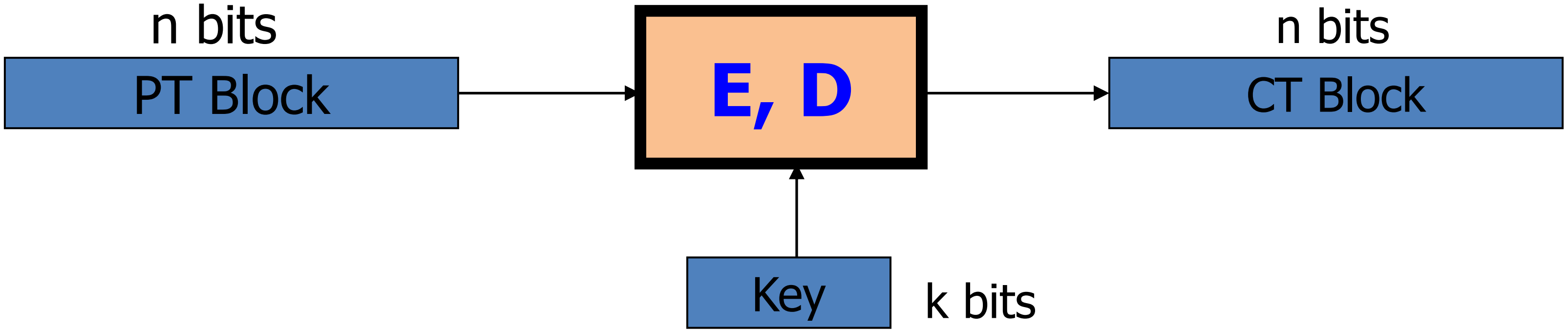
Block ciphers

What is a block cipher?

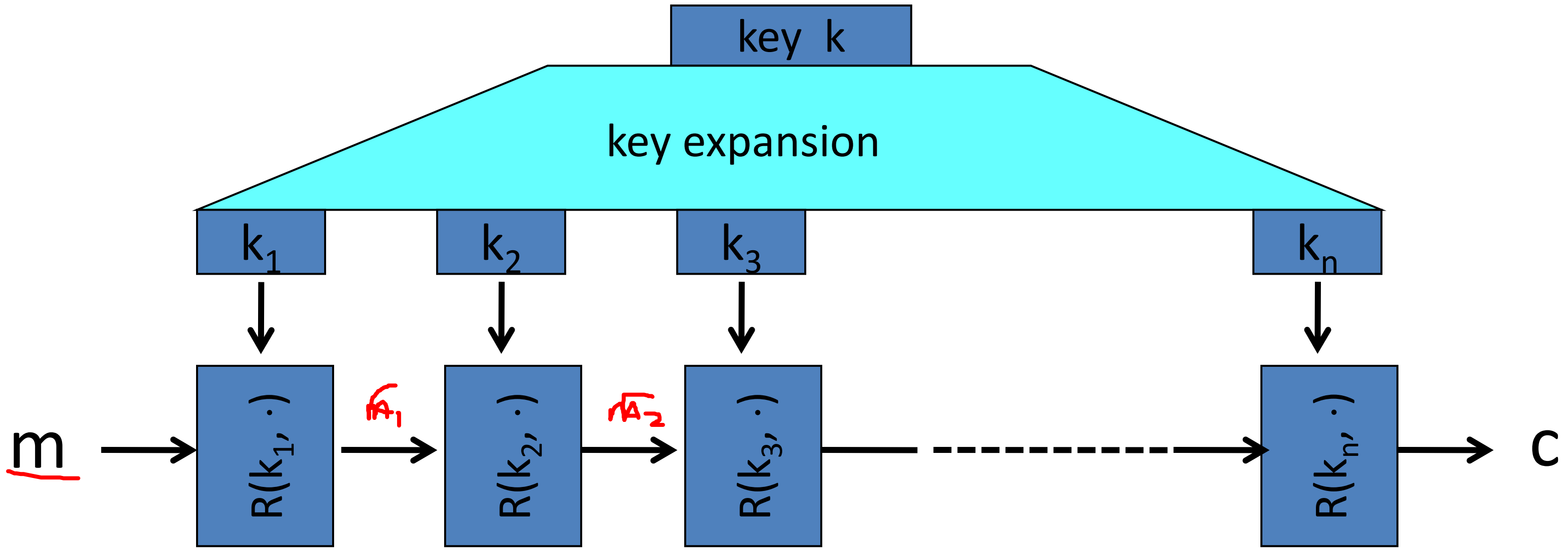
Block ciphers: crypto work horse



Canonical examples:

1. 3DES: n= 64 bits, k = 168 bits
2. AES: n=128 bits, k = 128, 192, 256 bits

Block Ciphers Built by Iteration



R(k,m) is called a round function

for 3DES (n=48), for AES-128 (n=10)

Performance:

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	Cipher	Block/key size	Speed (MB/sec)
stream	RC4		126
	Salsa20/12		643
	Sosemanuk		727
block	3DES	64/168	13
	AES-128	128/128	109

Abstractly: PRPs and PRFs

- Pseudo Random Function (**PRF**) defined over (K,X,Y): PRG

$$F: \; K \times X \;\to\; Y$$

such that exists "efficient" algorithm to evaluate F(k,x)

- Pseudo Random Permutation (**PRP**) defined over (K,X):

$$E: \; K \times X \;\to\; X$$

such that:

1. Exists "efficient" deterministic algorithm to evaluate E(k,x)
2. The function $E(\,k,\,\cdot\,)$ is one-to-one
3. Exists "efficient" inversion algorithm D(k,y)

Running example

- Example PRPs: 3DES, AES, …

  AES: $K \times \overset{m}{X} \to X$ where $K = X = \{0,1\}^{128}$

  3DES: $K \times X \to X$ where $X = \{0,1\}^{64}$, $K = \{0,1\}^{168}$

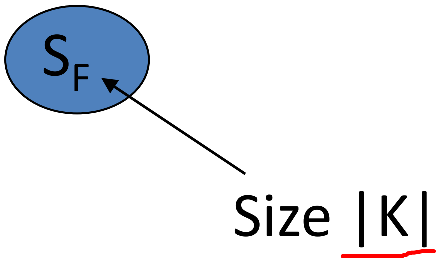
- Functionally, any PRP is also a PRF.
  - A PRP is a PRF where X=Y and is efficiently invertible.

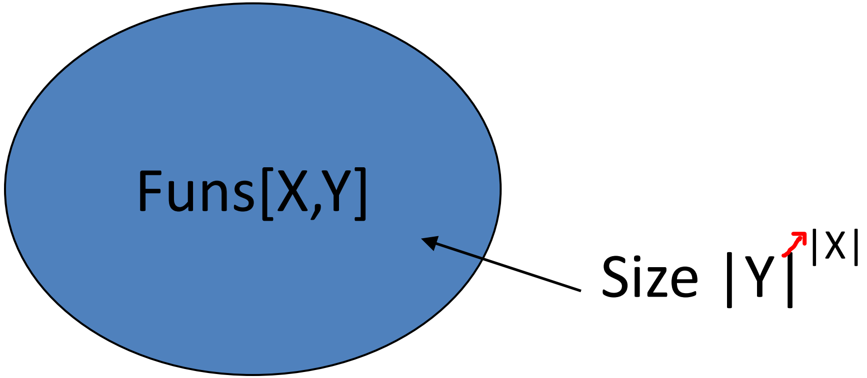
Secure PRFs

- Let $F: K \times X \rightarrow Y$ be a PRF

$$\begin{cases} \text{Funs[X,Y]:} & \text{the set of } \textbf{all} \text{ functions from X to Y} \\ S_F = \{ F(k,\cdot) \text{ s.t. } k \in K \} & \subseteq \text{Funs[X,Y]} \end{cases}$$

- Intuition: a PRF is **secure** if
  a random function in Funs[X,Y] is indistinguishable from
  a random function in S_F





Secure PRFs

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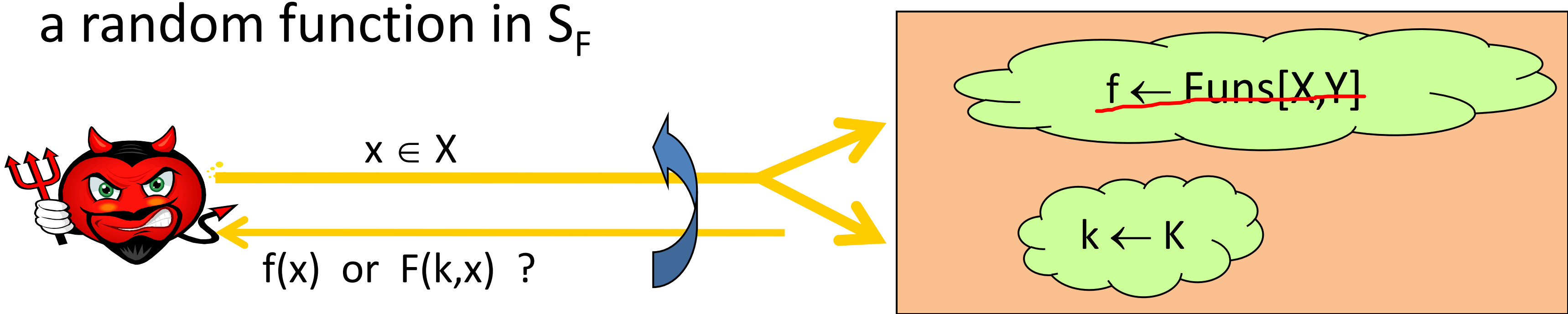
???



Secure PRPs (secure block cipher)

- Let $E: K \times X \rightarrow Y$ be a PRP

  Perms[X]: the set of all **<u>one-to-one</u>** functions from X to Y

  $S_F = \{ E(k,\cdot) \text{ s.t. } k \in K \} \subseteq$ Perms[X,Y]

- <u>Intuition</u>: a PRP is **secure** if

  a random function in Perms[X] is indistinguishable from

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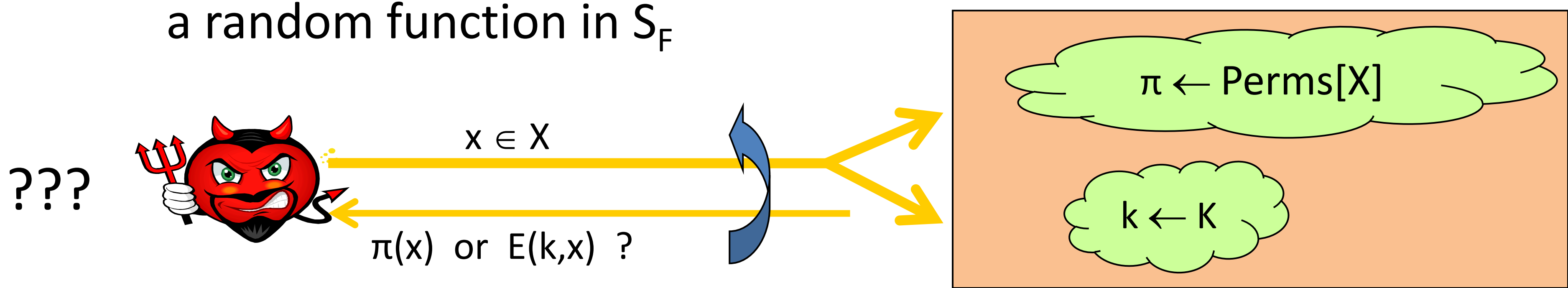


Let $F: K \times X \rightarrow \{0,1\}^{128}$ be a secure PRF.

Is the following G a secure PRF?

$$G(k, x) = \begin{cases} 0^{128} & \text{if } x=0 \\ F(k,x) & \text{otherwise} \end{cases}$$

- ○ No, it is easy to distinguish G from a random function
- ○ Yes, an attack on G would also break F
- ○ It depends on F

An easy application: PRF ⇒ PRG

Let $F: K \times \{0,1\}^n \to \{0,1\}^n$ be a secure PRF.

Then the following $G: K \to \{0,1\}^{nt}$ is a secure PRG:

K x

$$\mathbf{G(k) = F(k,0) \parallel F(k,1) \parallel \cdots \parallel F(k,t\text{-}1)}$$

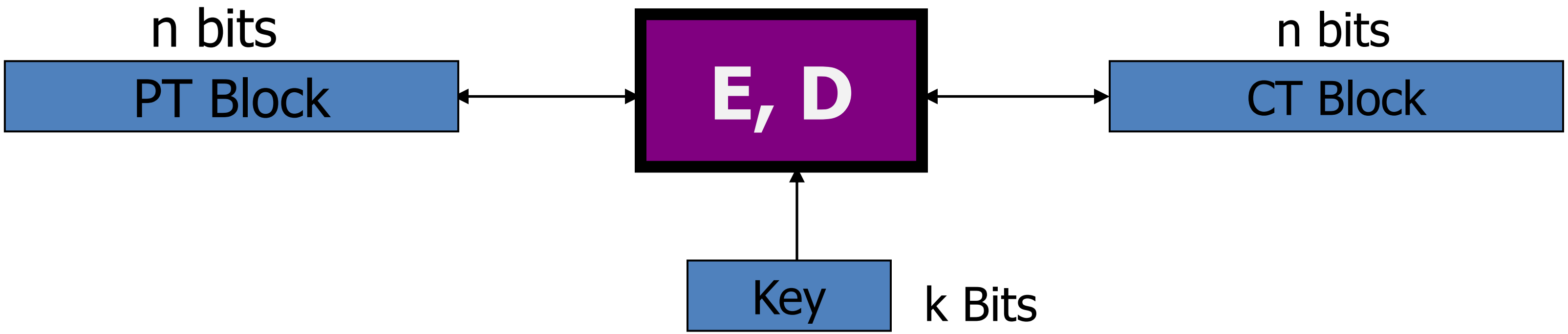
Key property: parallelizable

Security from PRF property: $F(k, \cdot)$ indist. from random function $f(\cdot)$

Block ciphers

The data encryption standard (DES)

Block ciphers: crypto work horse



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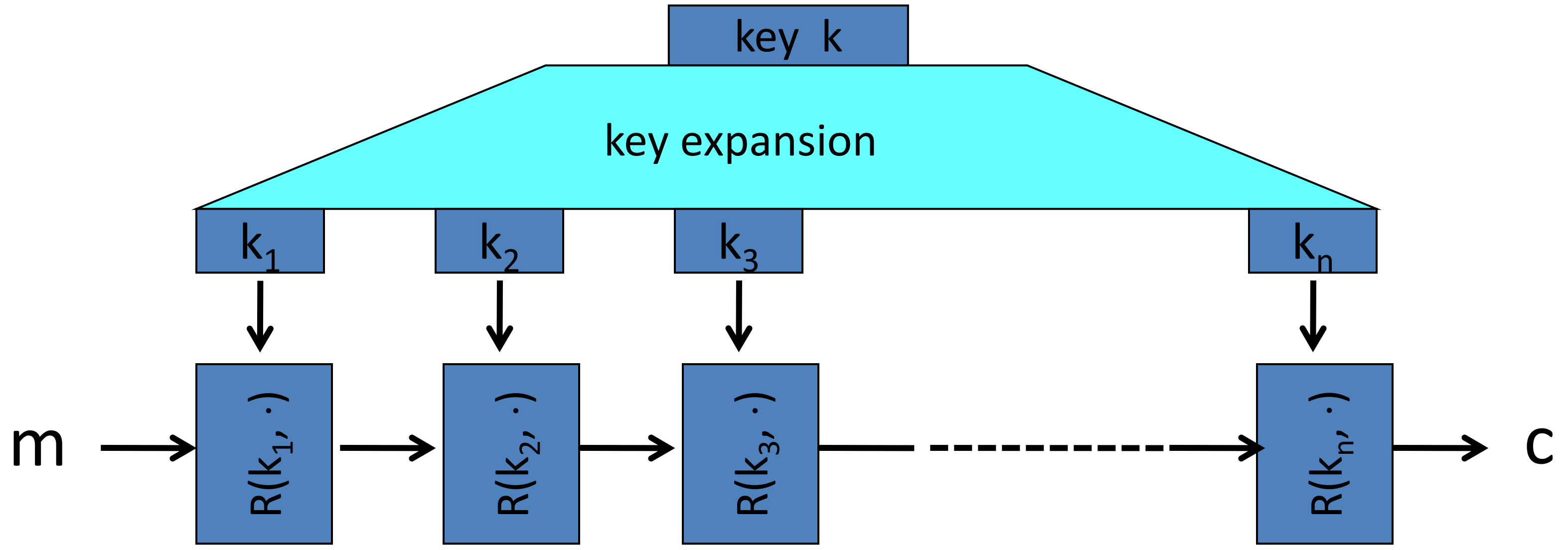
Block Ciphers Built by Iteration



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for 3DES (n=48), for AES-128 (n=10)

The Data Encryption Standard (DES)

- Early 1970s: Horst Feistel designs Lucifer at IBM

  key-len = 128 bits ; block-len = 128 bits

- 1973: NBS asks for block cipher proposals.

  IBM submits variant of Lucifer.

- 1976: NBS adopts DES as a federal standard

  key-len = 56 bits ; block-len = 64 bits

- 1997: DES broken by exhaustive search

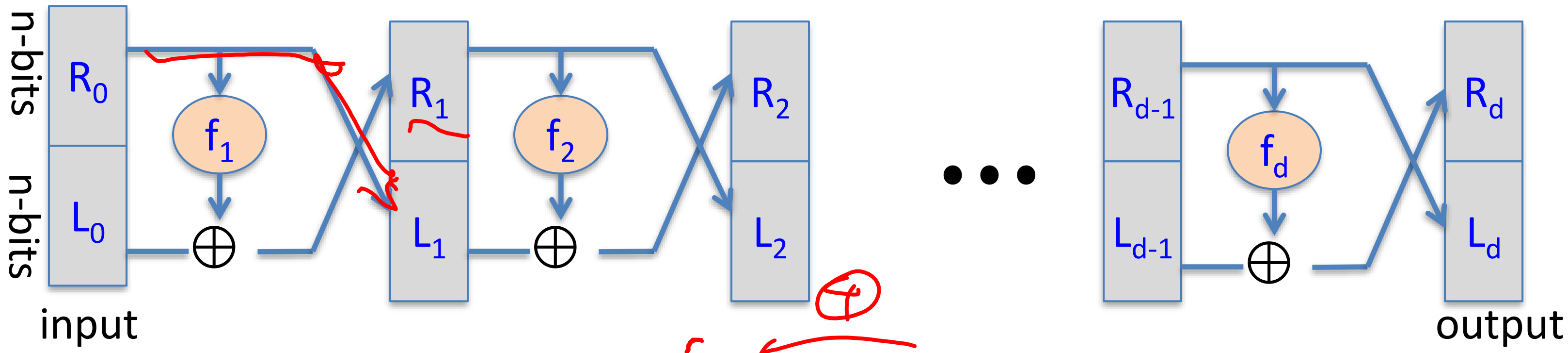
- 2000: NIST adopts Rijndael as AES to replace DES

Widely deployed in banking (ACH) and commerce

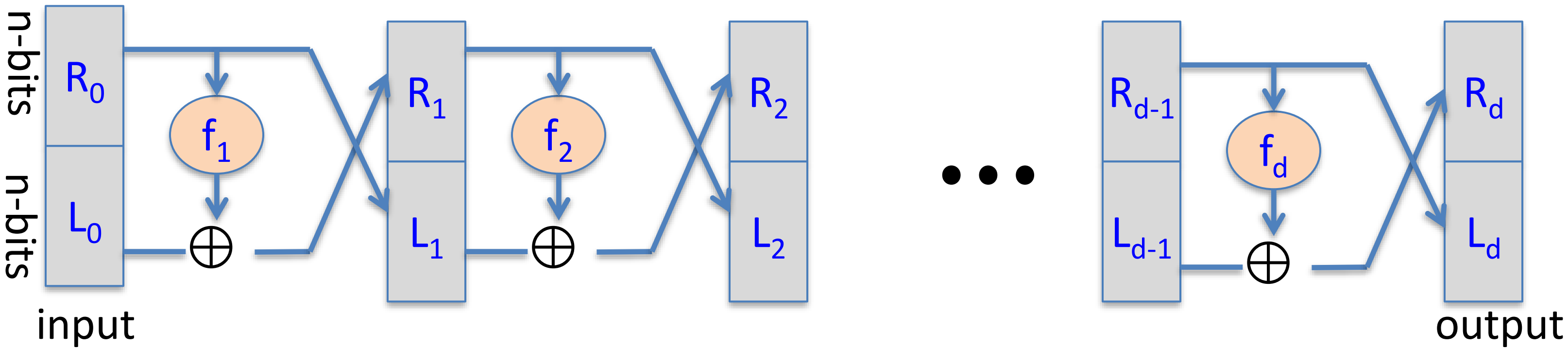
DES: core idea – Feistel Network

Given functions $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Goal: build invertible function $F: \{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$



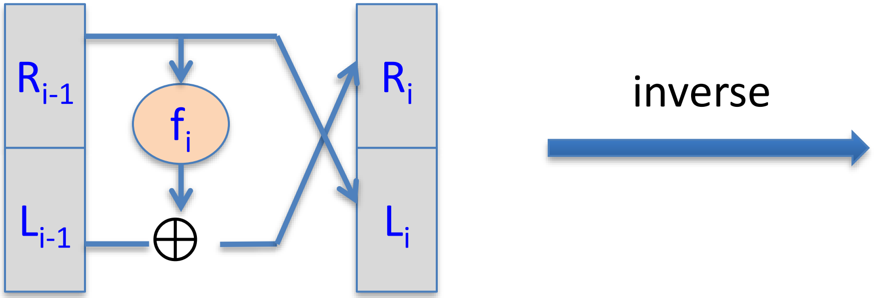
In symbols:
$$\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1} \\ L_i = R_{i-1} \end{cases}$$



Claim: for all $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

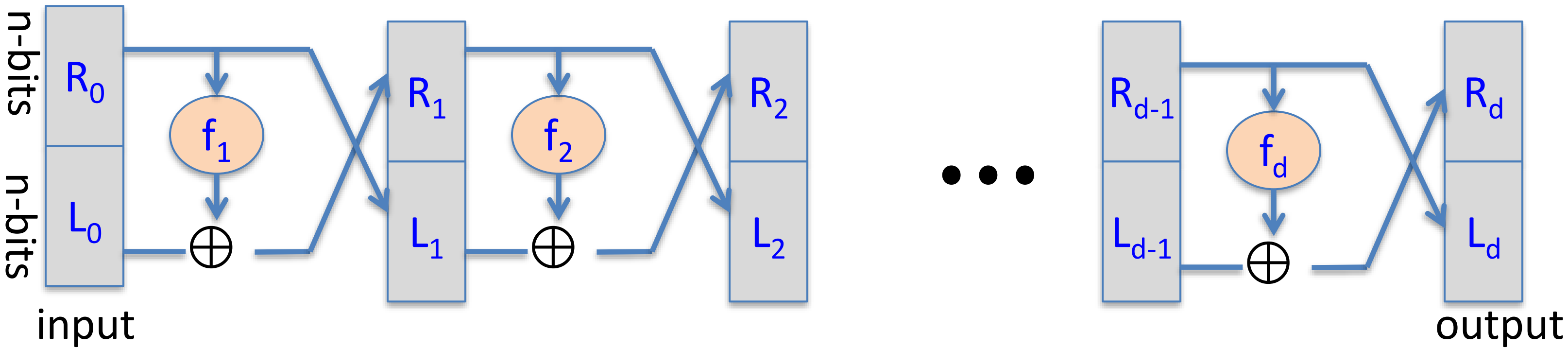
Feistel network F: $\{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ is invertible

Proof: construct inverse



$R_{i-1} = L_i$

$L_{i-1} =$



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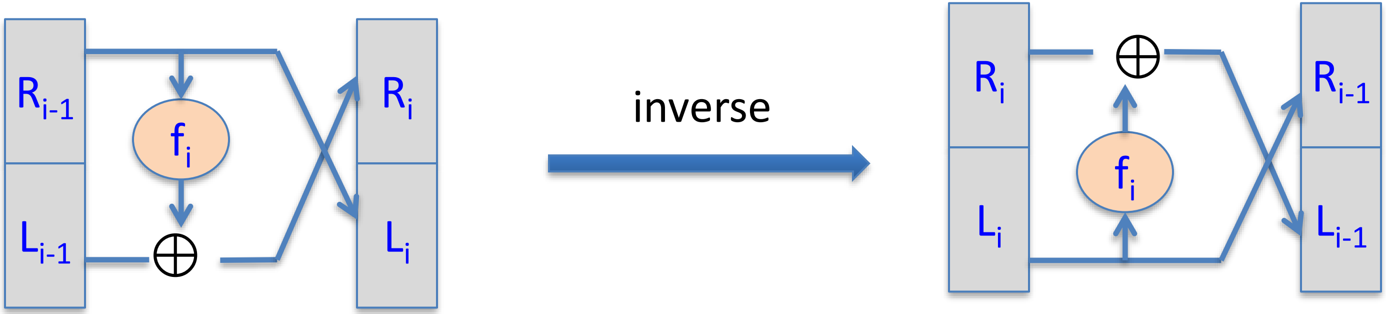
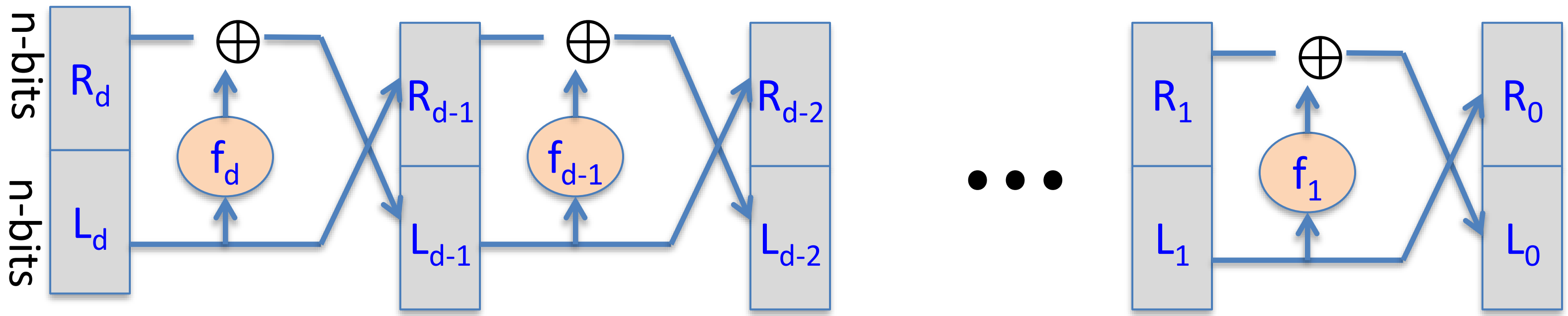


Decryption circuit



- Inversion is basically the same circuit,
  with $f_1, \ldots, f_d$ applied in reverse order

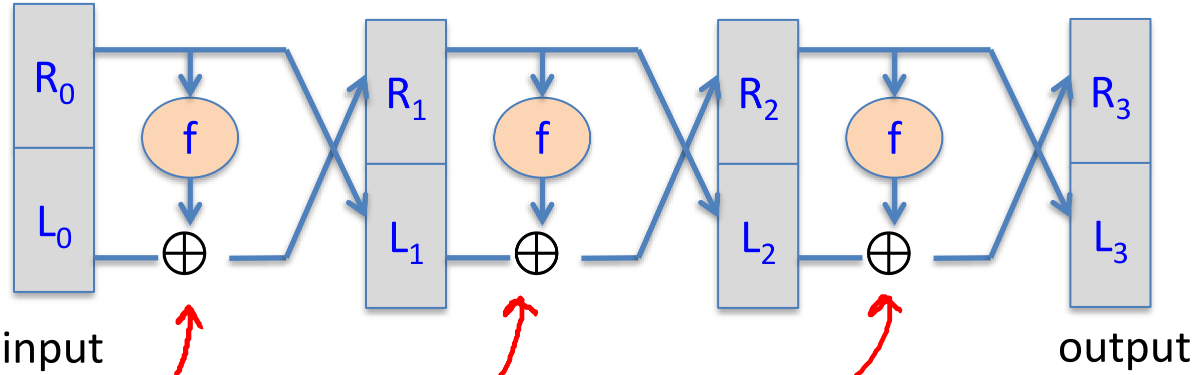
- General method for building invertible functions (block ciphers) from arbitrary functions.

- Used in many block ciphers … but not AES

"Thm:" (Luby-Rackoff '85):

$f: K \times \{0,1\}^n \longrightarrow \{0,1\}^n$ a secure PRF

$\Rightarrow$ 3-round Feistel $F: K^3 \times \{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ a secure PRP

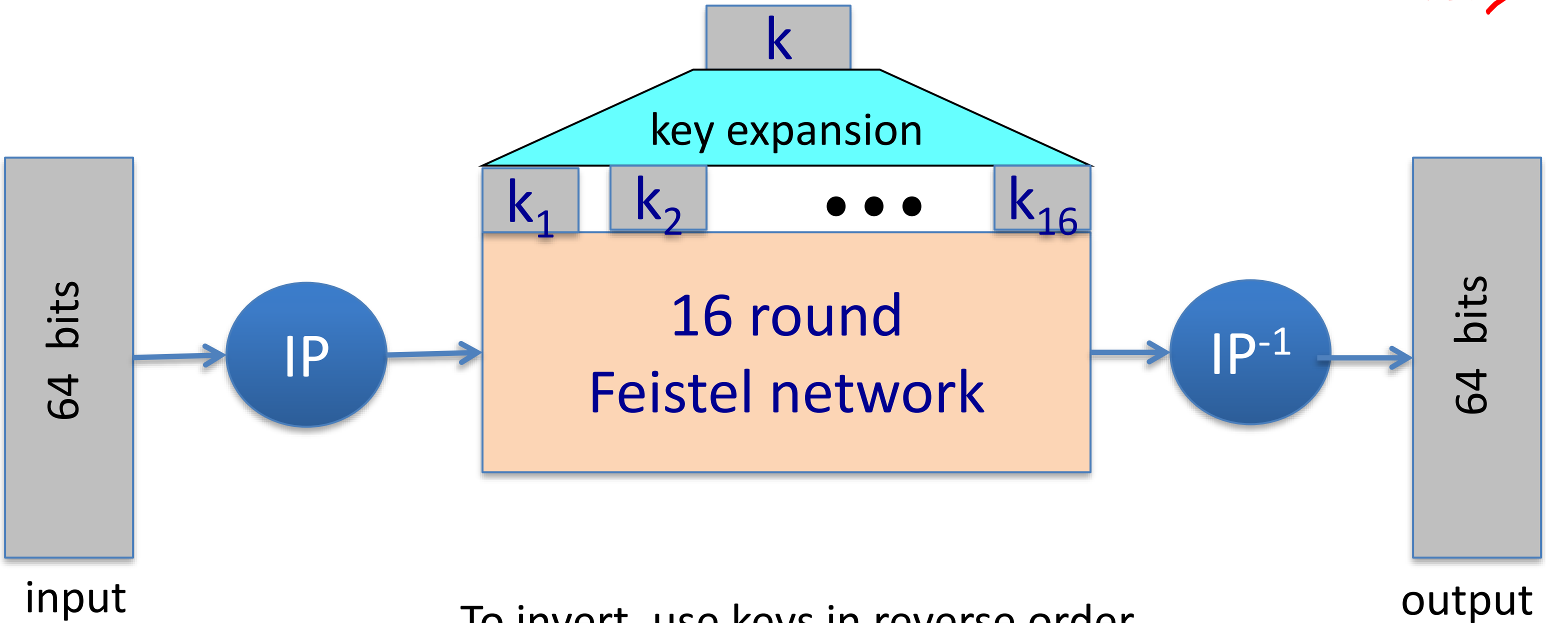


$f(k_0, R_0)$ $f(k_1, R_1)$ $f(k_2, R_2)$ $\Longrightarrow$ 3 independent keys

DES: 16 round Feistel network

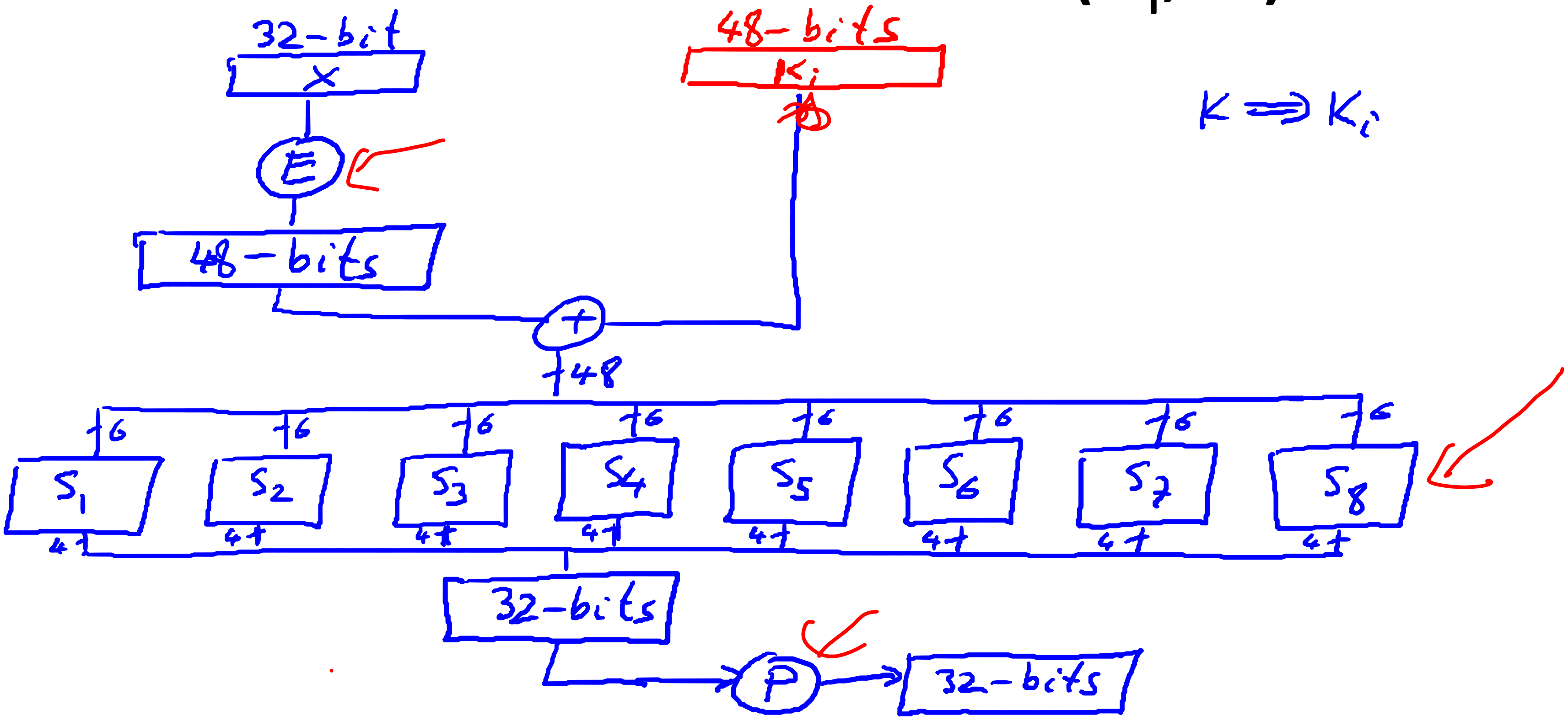
$f_1, \ldots, f_{16}: \{0,1\}^{32} \longrightarrow \{0,1\}^{32}$, $f_i(x) = \mathbf{F}(k_i, x)$

from key k



To invert, use keys in reverse order

The function $F(k_i, x)$



S-box: function $\{0,1\}^6 \longrightarrow \{0,1\}^4$, implemented as look-up table.

The S-boxes

$$S_i: \{0,1\}^6 \longrightarrow \{0,1\}^4$$

0 1101 1

S_5		Middle 4 bits of input															
		0000	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**
Outer bits	**00**	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

Example: a bad S-box choice

Suppose:

$$S_i(x_1, x_2, \ldots, x_6) = \left(x_2 \oplus x_3,\ \ x_1 \oplus x_4 \oplus x_5,\ \ x_1 \oplus x_6,\ \ x_2 \oplus x_3 \oplus x_6 \right)$$

or written equivalently: $\underline{S_i(\mathbf{x}) = A_i \cdot \mathbf{x} \pmod 2}$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} x_2 \oplus x_3 \\ x_1 \oplus x_4 \oplus x_5 \\ x_1 \oplus x_6 \\ x_2 \oplus x_3 \oplus x_6 \end{pmatrix}$$

We say that S_i is a linear function.

Example: a bad S-box choice

Then entire DES cipher would be linear: $\exists$ fixed binary matrix B s.t.

$$\mathrm{DES}(k,m) = \overset{832}{\underset{64}{B}} \cdot \begin{pmatrix} m \\ k_1 \\ k_2 \\ \vdots \\ k_{16} \end{pmatrix} = \begin{pmatrix} c \end{pmatrix} \pmod 2$$

But then: $\mathrm{DES}(k,m_1) \oplus \mathrm{DES}(k,m_2) \oplus \mathrm{DES}(k,m_3) = \mathrm{DES}(k,\ m_1 \oplus m_2 \oplus m_3)$

$$k = \begin{pmatrix} k_1 \\ \vdots \\ k_{16} \end{pmatrix}$$

$$B \begin{pmatrix} m_1 \\ k \end{pmatrix} \oplus B \begin{pmatrix} m_2 \\ k \end{pmatrix} \oplus B \begin{pmatrix} m_3 \\ k \end{pmatrix} = B \begin{pmatrix} m_1 \oplus m_2 \oplus m_3 \\ \cancel{k \oplus k} \oplus k \end{pmatrix}$$

Dan Boneh

Choosing the S-boxes and P-box

Choosing the S-boxes and P-box at random would result in an insecure block cipher (key recovery after $\approx 2^{24}$ outputs) [BS'89]

Several rules used in choice of S and P boxes:

- No output bit should be close to a linear func. of the input bits
- S-boxes are 4-to-1 maps

  ⋮

Block ciphers

Exhaustive Search Attacks

Exhaustive Search for block cipher key

Goal: given a few input output pairs $\big(m_i, c_i = E(k, m_i)\big)$ i=1,..,3
find key k.

Lemma: Suppose DES is an ***ideal cipher***
(2^{56} random invertible functions $\pi_1, \ldots, \pi_{2^{56}} : \{0,1\}^{64} \to \{0,1\}^{64}$)

Then ∀ m, c there is at most <u>**one**</u> key k s.t. c = DES(k, m)

with prob. ≥ 1 – 1/256 ≈ 99.5%

Proof: $\Pr\left[\exists k' \neq k : c = DES(k,m) = DES(k',m)\right] \leq$

$$\leq \sum_{k' \in \{0,1\}^{56}} \Pr\left[DES(k,m) = DES(k',m)\right] \leq 2^{56} \cdot \frac{1}{2^{64}} = \frac{1}{2^8}$$

Exhaustive Search for block cipher key

For two DES pairs $\big(m_1, c_1{=}\mathrm{DES}(k, m_1)\big)$, $\big(m_2, c_2{=}\mathrm{DES}(k, m_2)\big)$

unicity prob. $\approx 1 - 1/2^{71}$

For AES-128: given two inp/out pairs, unicity prob. $\approx 1 - 1/2^{128}$

$\Rightarrow$ two input/output pairs are enough for exhaustive key search.

DES challenge

msg = "`The unknown messages is: XXXX …` "

CT = c_1 c_2 c_3 c_4

Goal: find $k \in \{0,1\}^{56}$ s.t. $DES(k, m_i) = c_i$ for $i=1,2,3$

1997: Internet search -- **3 months**

1998: EFF machine (deep crack) -- **3 days** (250K $)

1999: combined search -- **22 hours**

2006: COPACOBANA (120 FPGAs) **-- 7 days** (10K $)

⇒ 56-bit ciphers should not be used !! (128-bit key ⇒ 2^{72} days)

Strengthening DES against ex. search

Method 1: **Triple-DES**

- Let $E : K \times M \longrightarrow M$ be a block cipher
- Define **3E**: $K^3 \times M \longrightarrow M$ as

$$\mathbf{3E}\big((k_1,k_2,k_3), m\big) = E(k_1, D(k_2, E(k_3, m)))$$

$k_1 = k_2 = k_3 \Rightarrow$ single DES

For 3DES: key-size = 3×56 = 168 bits. 3×slower than DES.

(simple attack in time $\approx 2^{118}$)

Why not double DES?

- Define $\quad 2E\big((k_1,k_2), m\big) = \; E\big(k_1 , E(k_2 , m) \big)$

key-len = 112 bits for DES

$m \to E(\mathbf{k_2},\cdot) \to E(\mathbf{k_1},\cdot) \to c$

Find (k_1,k_2) s.t.
$E(k_1, E(k_2,M)) = C$
Equivalently:
$E(k_2,M) = D(k_1,C)$

Attack: $\quad M = (m_1,\dots, m_{10}) \; , \quad C = (c_1,\dots,c_{10})$.

- step 1: build table.

  sort on 2nd column

$k^0 = 00\dots00$	$E(k^0 , M)$
$k^1 = 00\dots01$	$E(k^1 , M)$
$k^2 = 00\dots10$	$E(k^2 , M)$
$\vdots$	$\vdots$
$k^N = 11\dots11$	$E(k^N , M)$

2^{56} entries

Meet in the middle attack

$m \to E(\mathbf{k_2},\cdot) \to E(\mathbf{k_1},\cdot) \to c$

Attack: $M = (m_1,\ldots, m_{10})$, $C = (c_1,\ldots,c_{10})$

$k^0 = 00\ldots00$	$E(k^0, M)$
$k^1 = 00\ldots01$	$E(k^1, M)$
$k^2 = 00\ldots10$	$E(k^2, M)$
$\vdots$	$\vdots$
$k^N = 11\ldots11$	$E(k^N, M)$

- step 1: build table.

- Step 2: for all $k \in \{0,1\}^{56}$ do:

  test if $D(k, C)$ is in 2nd column.

  if so then $E(k^i,M) = D(k,C) \Rightarrow (k^i,k) = (k_2,k_1)$

Meet in the middle attack

$$m \rightarrow E(\mathbf{k_2},\cdot) \rightarrow E(\mathbf{k_1},\cdot) \rightarrow c$$

Time = $\underbrace{2^{56}\log(2^{56})}_{\text{build + sort table}} + \underbrace{2^{56}\log(2^{56})}_{\text{search in table}} < 2^{63} \ll 2^{112}$, space $\approx 2^{56}$

Same attack on 3DES: Time = 2^{118} , space $\approx 2^{56}$

$$m \rightarrow E(\mathbf{k_3},\cdot) \rightarrow E(\mathbf{k_2},\cdot) \rightarrow E(\mathbf{k_1},\cdot) \rightarrow c$$

Method 2: DESX

$E : K \times \{0,1\}^n \longrightarrow \{0,1\}^n$ a block cipher

Define EX as $\boxed{EX\big((k_1,k_2,k_3), m\big) = k_1 \oplus E(k_2, m \oplus k_3)}$

For DESX: key-len = 64+56+64 = 184 bits

… but easy attack in time $2^{64+56} = 2^{120}$ (homework)

Note: $k_1 \oplus E(k_2, m)$ and $E(k_2, m \oplus k_1)$ does nothing !!

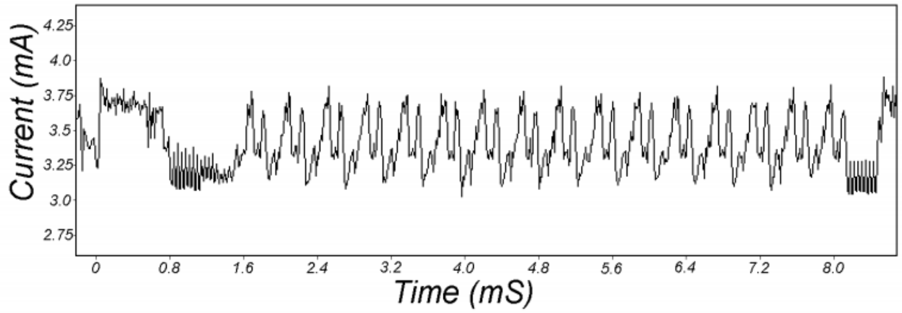
Block ciphers

More attacks on block ciphers

Attacks on the implementation

1. Side channel attacks:
   – Measure **time** to do enc/dec,   measure **power** for enc/dec

smartcard



[Kocher, Jaffe, Jun, 1998]

2. Fault attacks:
   – Computing errors in the last round expose the secret key k

⇒ do not even implement crypto primitives yourself …

Linear and differential attacks [BS'89,M'93]

Given *many* inp/out pairs, can recover key in time less than 2^{56} .

<u>Linear cryptanalysis</u> (overview) : let c = DES(k, m)

Suppose for random k,m :

$$\Pr\Big[\ \underbrace{m[i_1]\oplus\cdots\oplus m[i_r]}_{\text{subset of msg bits}} \ \bigoplus\ \underbrace{c[j_j]\oplus\cdots\oplus c[j_v]}_{\text{subset of ciphertext bits}} = \underbrace{k[l_1]\oplus\cdots\oplus k[l_u]}_{\text{subset of key bits}}\ \Big] = \tfrac{1}{2} + \varepsilon$$

For some ε. For DES, this exists with $\varepsilon = 1/2^{21} \approx 0.000000477$

Linear attacks

$$\Pr\left[\ m[i_1] \oplus \cdots \oplus m[i_r]\ \bigoplus\ c[j_j] \oplus \cdots \oplus c[j_v]\ =\ k[l_1] \oplus \cdots \oplus k[l_u]\ \right] = ½ + \varepsilon$$

Thm: given $1/\varepsilon^2$ random $\big(m,\ c=DES(k, m)\big)$ pairs then

$$k[l_1,\ldots,l_u] = \mathrm{MAJ}\left[\ m[i_1,\ldots,i_r]\ \bigoplus\ c[j_j,\ldots,j_v]\ \right]$$

with prob. $\geq 97.7\%$

$\Rightarrow$ with $1/\varepsilon^2$ inp/out pairs can find $k[l_1,\ldots,l_u]$ in time $\approx 1/\varepsilon^2$.

Linear attacks

For DES, $\varepsilon = 1/2^{21} \Rightarrow$

with 2^{42} inp/out pairs can find $k[l_1,\dots,l_u]$ in time 2^{42}

Roughly speaking: can find 14 key "bits" this way in time 2^{42}

Brute force remaining $56-14=42$ bits in time 2^{42}

Total attack time $\approx 2^{43}$ ($<< 2^{56}$) with 2^{42} random inp/out pairs

Lesson

A tiny bit of linearly in S_5 lead to a 2^{42} time attack.

$\Rightarrow$ don't design ciphers yourself !!

Quantum attacks

Generic search problem:

Let $f: X \longrightarrow \{0,1\}$ be a function.

Goal: find $x \in X$ s.t. $f(x)=1$.

Classical computer: best generic algorithm time = $O(|X|)$

Quantum computer [Grover '96] : time = $O(|X|^{1/2})$

Can quantum computers be built: unknown

Quantum exhaustive search

Given m, c=E(k,m) define

$$f(k) = \begin{cases} 1 & \text{if } E(k,m) = c \\ 0 & \text{otherwise} \end{cases}$$

Grover $\Rightarrow$ quantum computer can find k in time $O(|K|^{1/2})$

DES: time $\approx 2^{28}$, AES-128: time $\approx 2^{64}$

quantum computer $\Rightarrow$ 256-bits key ciphers (e.g. AES-256)

Block ciphers

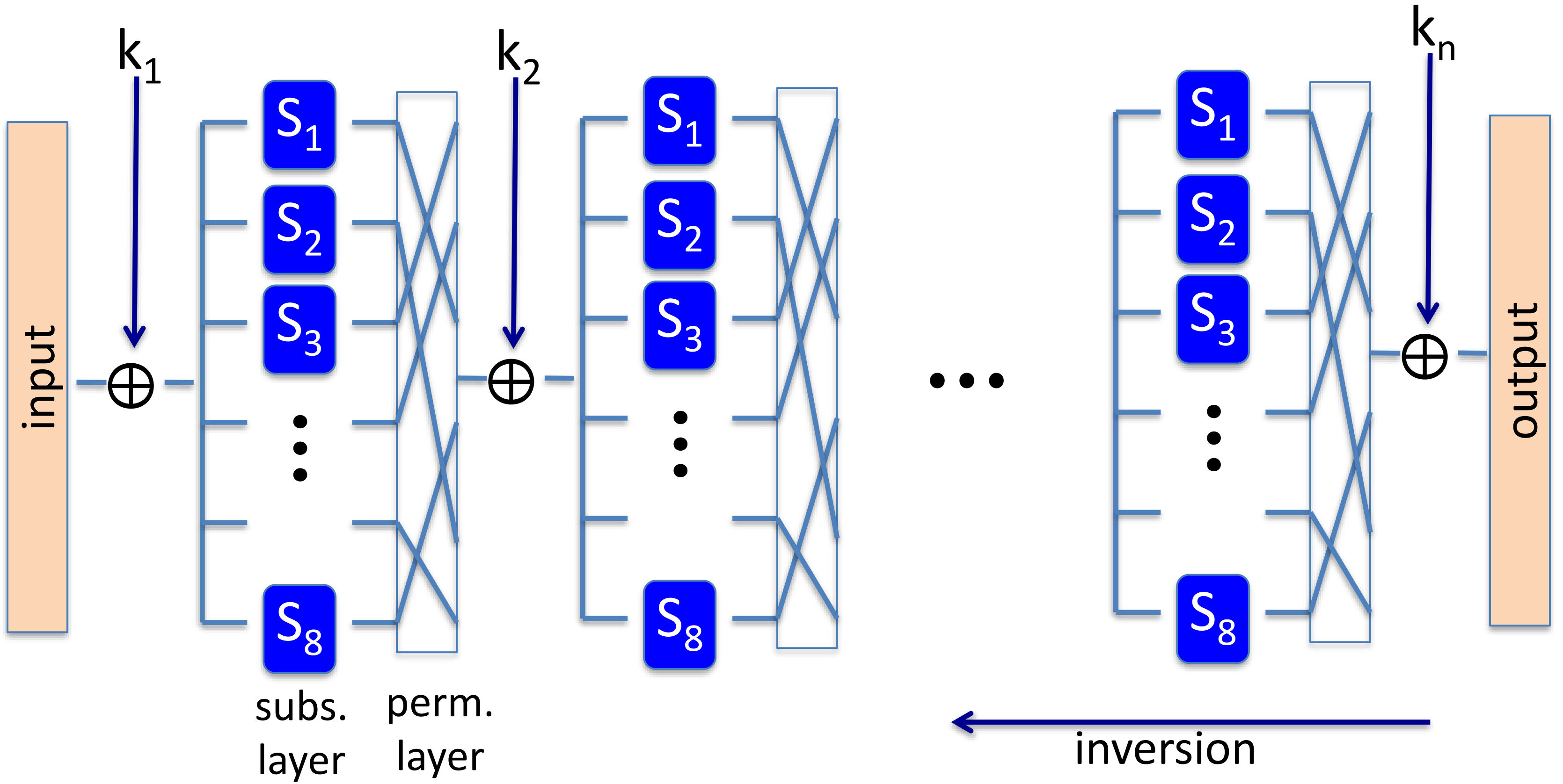
The AES block cipher

The AES process

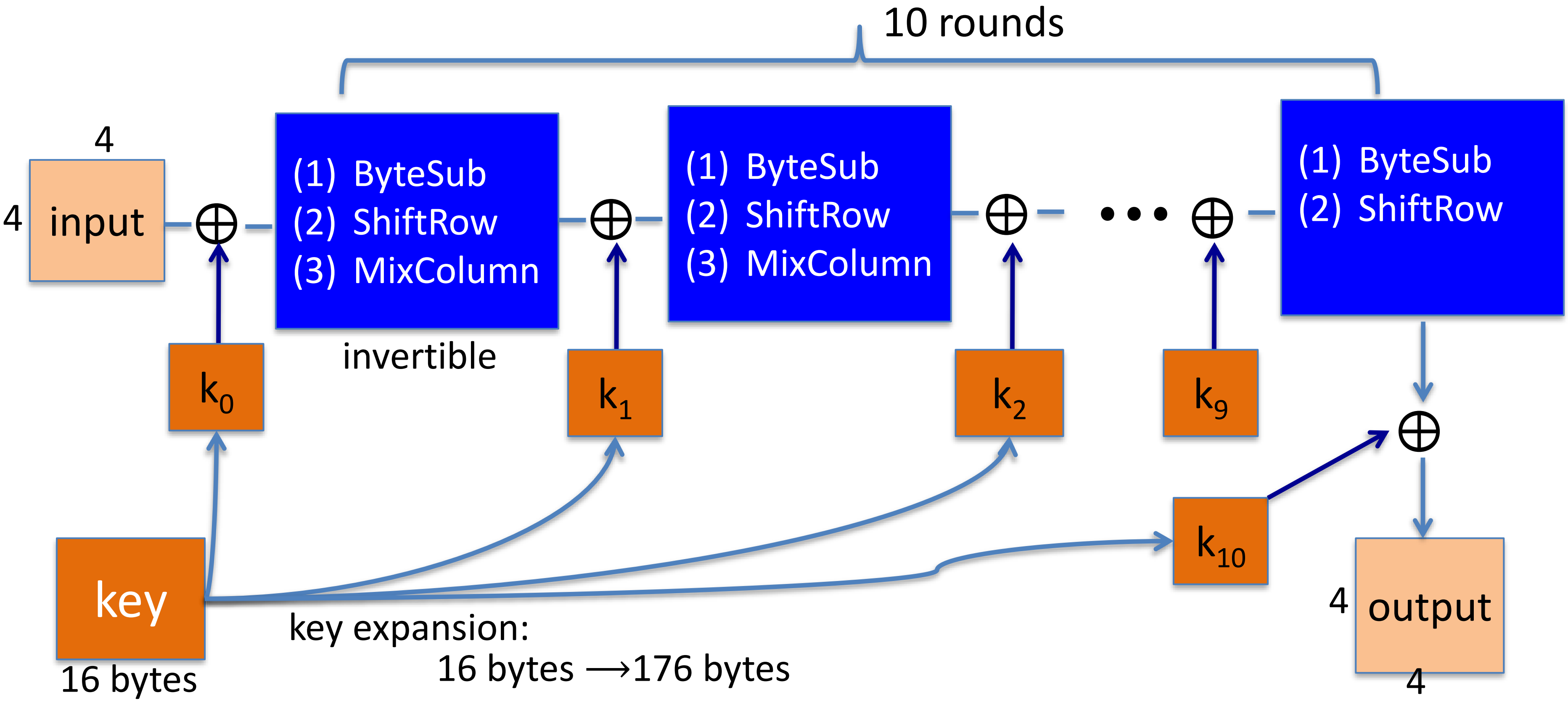
- 1997: NIST publishes request for proposal
- 1998: 15 submissions. Five claimed attacks.
- 1999: NIST chooses 5 finalists
- 2000: NIST chooses Rijndael as AES (designed in Belgium)

Key sizes: 128, 192, 256 bits. Block size: 128 bits

AES is a Subs-Perm network (not Feistel)



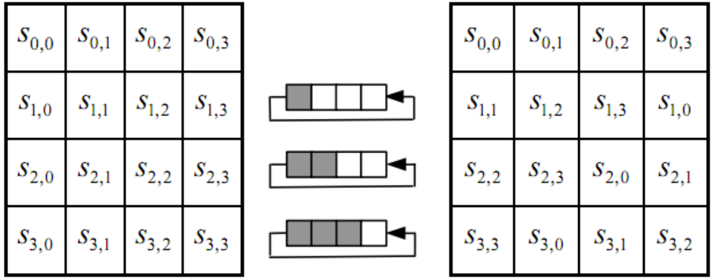
AES-128 schematic



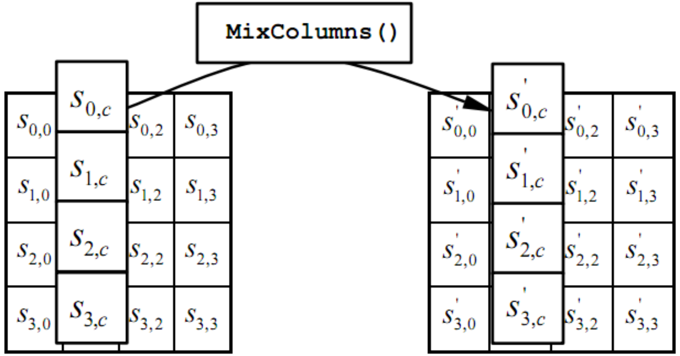
The round function

- **ByteSub**: a 1 byte S-box. 256 byte table (easily computable)

- **ShiftRows**:



- **MixColumns**:



Code size/performance tradeoff

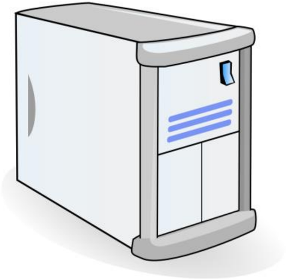
	Code size	Performance
Pre-compute round functions (24KB or 4KB)	largest	fastest: table lookups and xors
Pre-compute S-box only (256 bytes)	smaller	slower
No pre-computation	smallest	slowest

Example: Javascript AES

AES in the browser:

AES library (6.4KB)

no pre-computed tables



Prior to encryption:
    pre-compute tables

Then encrypt using tables

http://crypto.stanford.edu/sjcl/

AES in hardware

AES instructions in Intel Westmere:

- **aesenc, aesenclast**: do one round of AES

  128-bit registers: xmm1=state, xmm2=round key

  aesenc xmm1, xmm2 ; puts result in xmm1

- **aeskeygenassist**: performs AES key expansion

- Claim 14 x speed-up over OpenSSL on same hardware

Similar instructions on AMD Bulldozer

Attacks

Best key recovery attack:

four times better than ex. search [BKR'11]

Related key attack on AES-256: [BK'09]

Given 2^{99} inp/out pairs from **four related keys** in AES-256

can recover keys in time $\approx 2^{99}$

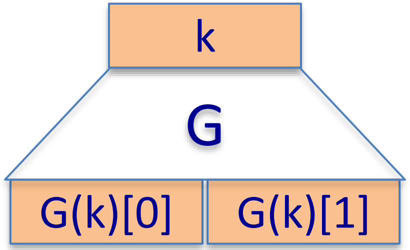
End of Segment

Block ciphers

Block ciphers from PRGs

Can we build a PRF from a PRG?

Let $G: K \longrightarrow K^2$ be a secure PRG



Define 1-bit PRF $F: K \times \{0,1\} \longrightarrow K$ as

$$F(k, x \in \{0,1\}) = G(k)[x]$$

Thm: If G is a secure PRG then F is a secure PRF

Can we build a PRF with a larger domain?

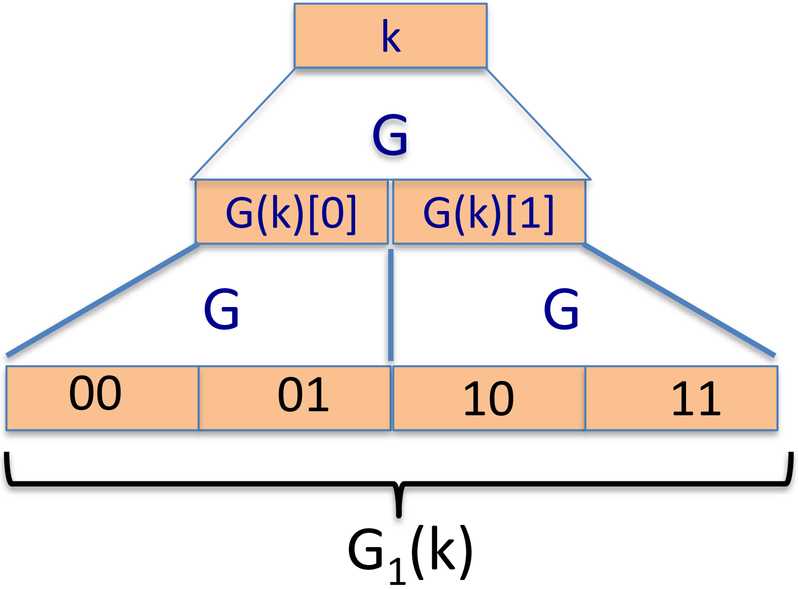
Extending a PRG

Let $G: K \longrightarrow K^2$.

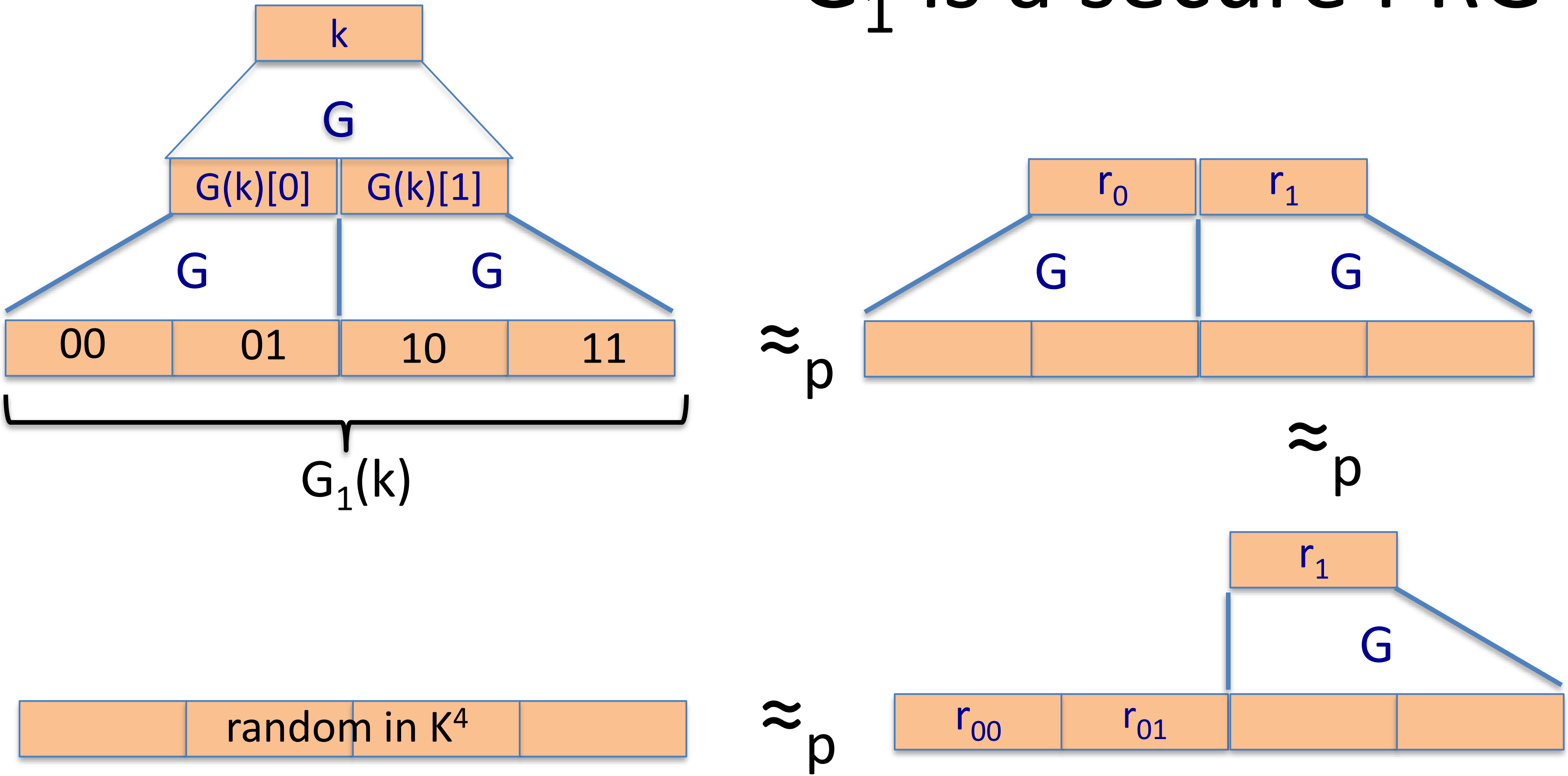
define $G_1: K \longrightarrow K^4$ as $G_1(k) = G\big(G(k)[0]\big) \,\|\, G\big(G(k)[1]\big)$

We get a 2-bit PRF:

$F(k, x \in \{0,1\}^2) = G_1(k)[x]$



G_1 is a secure PRG

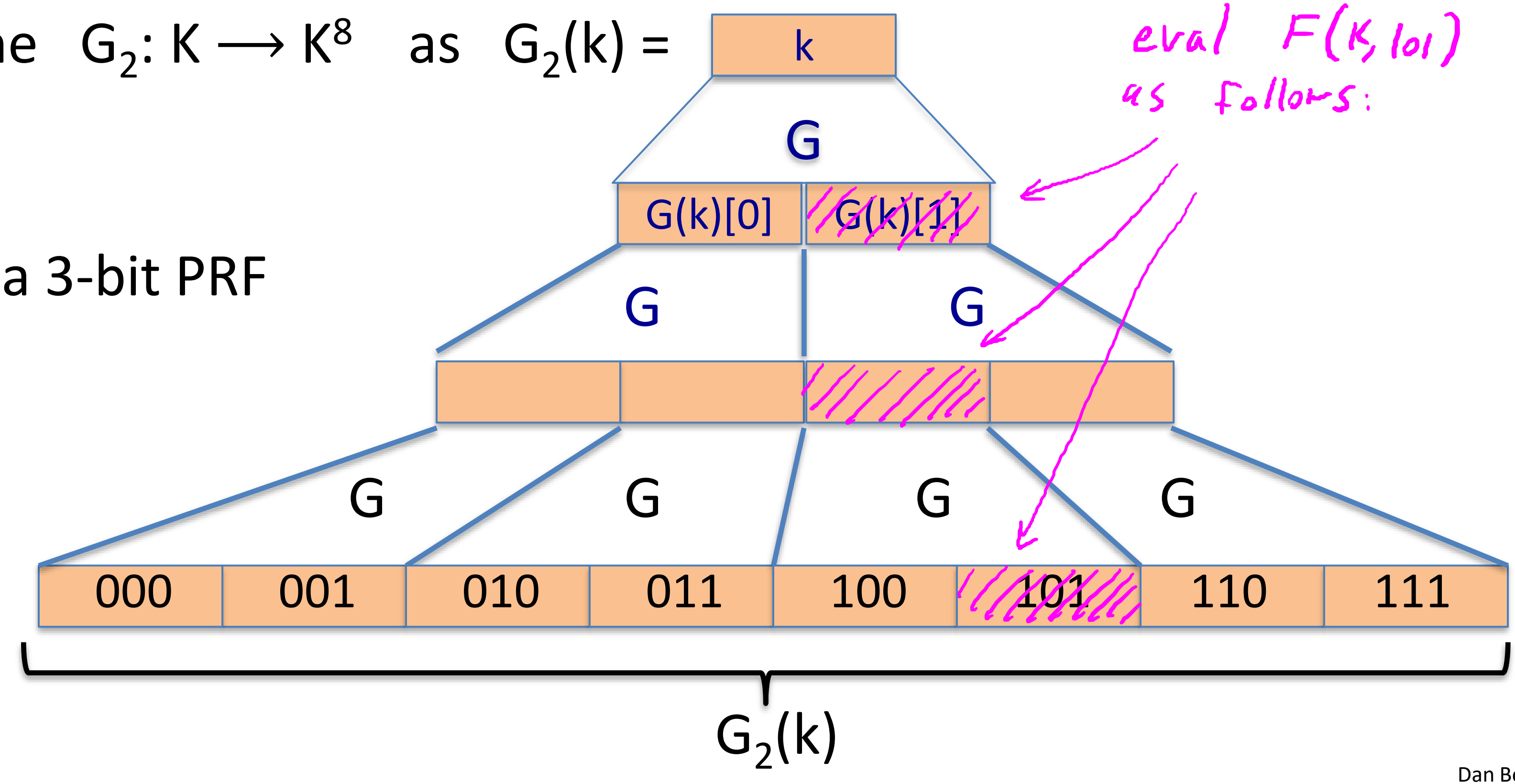


Extending more

Let $G: K \longrightarrow K^2$.

define $G_2: K \longrightarrow K^8$ as $G_2(k) =$

We get a 3-bit PRF



Extending even more: the GGM PRF

Let $G: K \longrightarrow K^2$. define PRF $F: K \times \{0,1\}^n \longrightarrow K$ as

For input $x = x_0\, x_1 \ldots x_{n-1} \in \{0,1\}^n$ do:

$$k \xrightarrow{\mathbf{G(k)[x_0]}} k_1 \xrightarrow{\mathbf{G(k_1)[x_1]}} k_2 \xrightarrow{\mathbf{G(k_2)[x_2]}} k_3 \quad \bullet\bullet\bullet \quad \xrightarrow{\mathbf{G(k_{n-1})[x_{n-1}]}} k_n$$

Security: G a secure PRG $\Rightarrow$ F is a secure PRF on $\{0,1\}^n$.

Not used in practice due to slow performance.

Secure block cipher from a PRG?

Can we build a secure PRP from a secure PRG?

- No, it cannot be done
- Yes, just plug the GGM PRF into the Luby-Rackoff theorem
- It depends on the underlying PRG
-

End of Segment