Proton Motion in the Sun with PFSS Model

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Introduction of PFSS Model

Potential Field Source Surface Model is a semi analytical model allows us to calculate the magnetic field between the photosphere (which radius is R_0) and the source surface (which radius is R_s), which is a spherical surface, is assumed that all the magnetic field passing through it is open and radial. Based on the argument that in the lower steady-state corona the magnetic pressure dominates over the plasma thermal pressure, the PFSS model neglects existence of coronal currents. That is, the field there is purely potential, we have

$$\mathrm{B} = -\nabla\Psi \quad \Rightarrow \quad \nabla^2\Psi = 0$$

By solving the Laplace's equation, we obtain the following:

$$\Psi = R_0\sum_{l=0}^{\infty}\sum_{m=0}^{l} P_l^m(\cos\theta)\left\{g'_{lm}\cos m\phi\left[\left(\frac{R_0}{r}\right)^{l+1} + \frac{R_s}{R_0}\left(\frac{r}{R_s}\right)^{l} c_{lm}\right] + h'_{lm}\sin m\phi\left[\left(\frac{R_0}{r}\right)^{l+1} + \frac{R_s}{R_0}\left(\frac{r}{R_s}\right)^{l} d_{lm}\right]\right\},$$

where P is the associated Legendre function, g'_{lm}, h'_{lm}, c_{lm} and d_{lm} are unknown coefficients.

After some pain and suffering, we obtain the magnetic field on solar surface in the following:

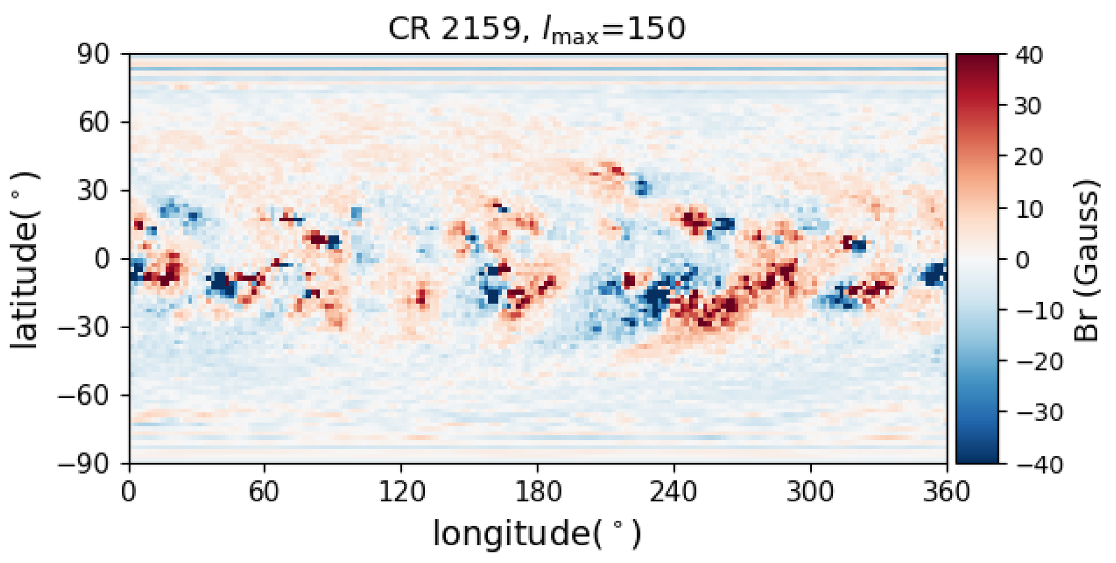
$$B_r(r,\theta,\phi) = -\frac{\partial\Psi}{\partial r} = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_l^m(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi)\times\left(\frac{R_0}{r}\right)^{l+2}\left[l+1+l\left(\frac{r}{R_s}\right)^{2l+1}\right]\bigg/\left[l+1+l\left(\frac{R_0}{R_s}\right)^{2l+1}\right]$$

$$B_\theta(r,\theta,\phi) = -\frac{1}{r}\frac{\partial\Psi}{\partial\theta} = -\sum_{l=0}^{\infty}\sum_{m=0}^{l}\frac{\partial P_l^m(\cos\theta)}{\partial\theta}(g_{lm}\cos m\phi + h_{lm}\sin m\phi)\times\left(\frac{R_0}{r}\right)^{l+2}\left[1-\left(\frac{r}{R_s}\right)^{2l+1}\right]\bigg/\left[l+1+l\left(\frac{R_0}{R_s}\right)^{2l+1}\right]$$

$$B_\phi(r,\theta,\phi) = -\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\phi} = \sum_{l=0}^{\infty}\sum_{m=0}^{l}\frac{mP_l^m(\cos\theta)}{\sin\theta}(g_{lm}\sin m\phi - h_{lm}\cos m\phi)\times\left(\frac{R_0}{r}\right)^{l+2}\left[1-\left(\frac{r}{R_s}\right)^{2l+1}\right]\bigg/\left[l+1+l\left(\frac{R_0}{R_s}\right)^{2l+1}\right],$$

$$\text{where} \quad g_{lm} = g'_{lm}\left[l+1+l\left(\frac{R_0}{R_s}\right)^{2l+1}\right], \quad h_{lm} = h'_{lm}\left[l+1+l\left(\frac{R_0}{R_s}\right)^{2l+1}\right].$$

g'_{lm}, h'_{lm} depend on R_S. However, as the radius of source surface is not a constant, we reconstruct the PFSS model after every measurement of magnetic field in each Carrington Rotation (individual rotation of the Sun).



Using orthogonal property of the associated Legendre function,

$$\int d\Omega P_l^m(\cos\theta)^{\cos}_{\sin} m\phi P_{l'}^{m'}(\cos\theta)^{\cos}_{\sin} m'\phi = \frac{4\pi}{2l+1}\delta_{ll'}\delta_{mm'}$$

The PFSS model coefficients above can be obtained with the input map!

With an HMI synoptic map (observational data), one can compute the magnetic field components between R_0 < r < R_S with the PFSS model!

Relativistic Proton Motion in Magnetic Field

As the protons are moving very fast, we need to consider relativistic Lorentz force!

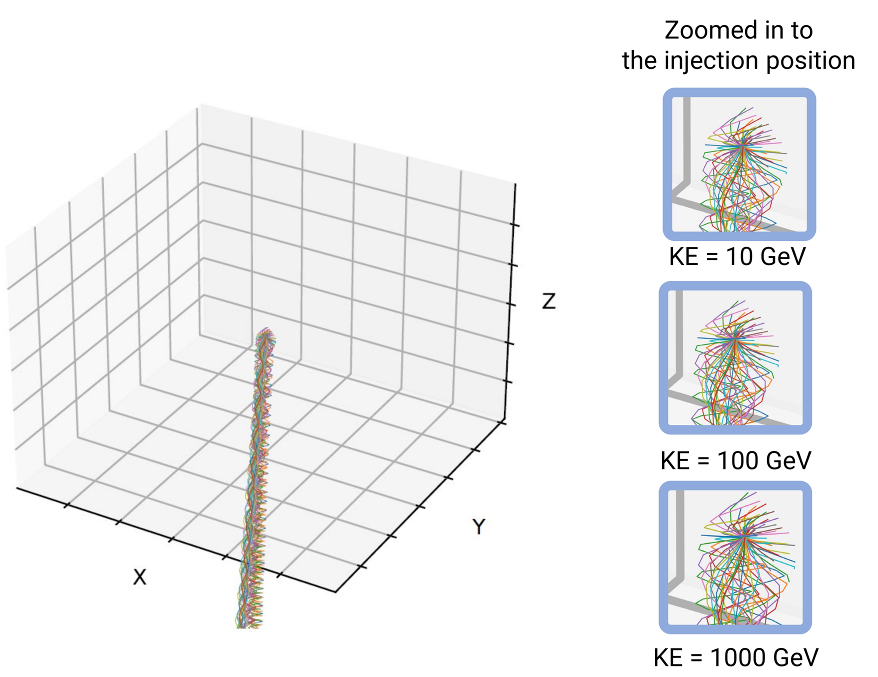
$$\frac{d\mathbf{p}}{dt} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$, where E is considered to be 0

$$\mathbf{p} = \gamma m\boldsymbol{v}$$, in which γ is constant due to zero work done!

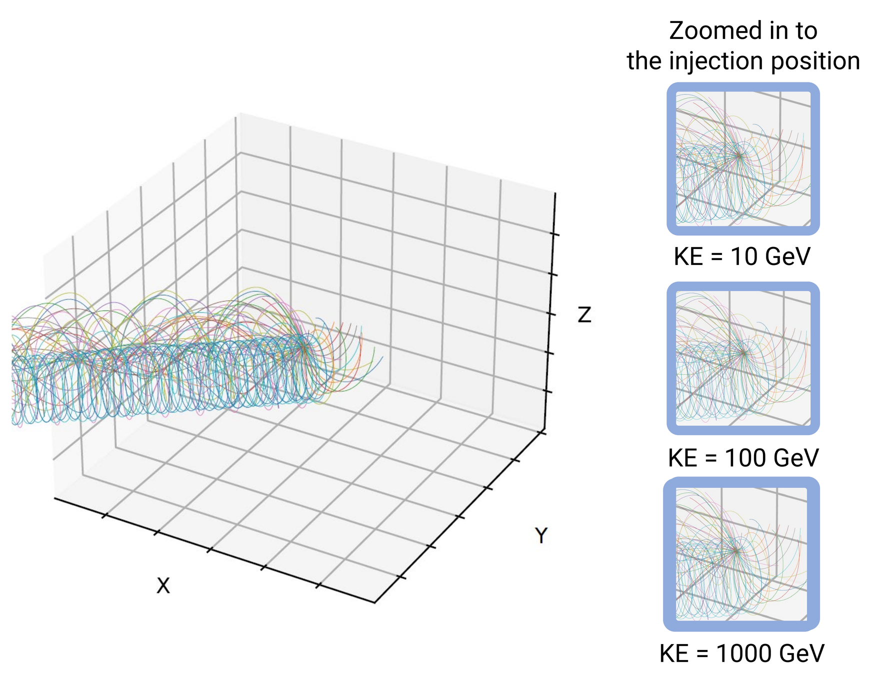
Numerical solution of the path can be obtained by solving six first order ODE.

Proton Motion under PFSS Model

Trajectory of 100 protons (KE = 10 GeV)
(Using PFSS Model April 2014, R_s = 2.5 $R_\odot$)

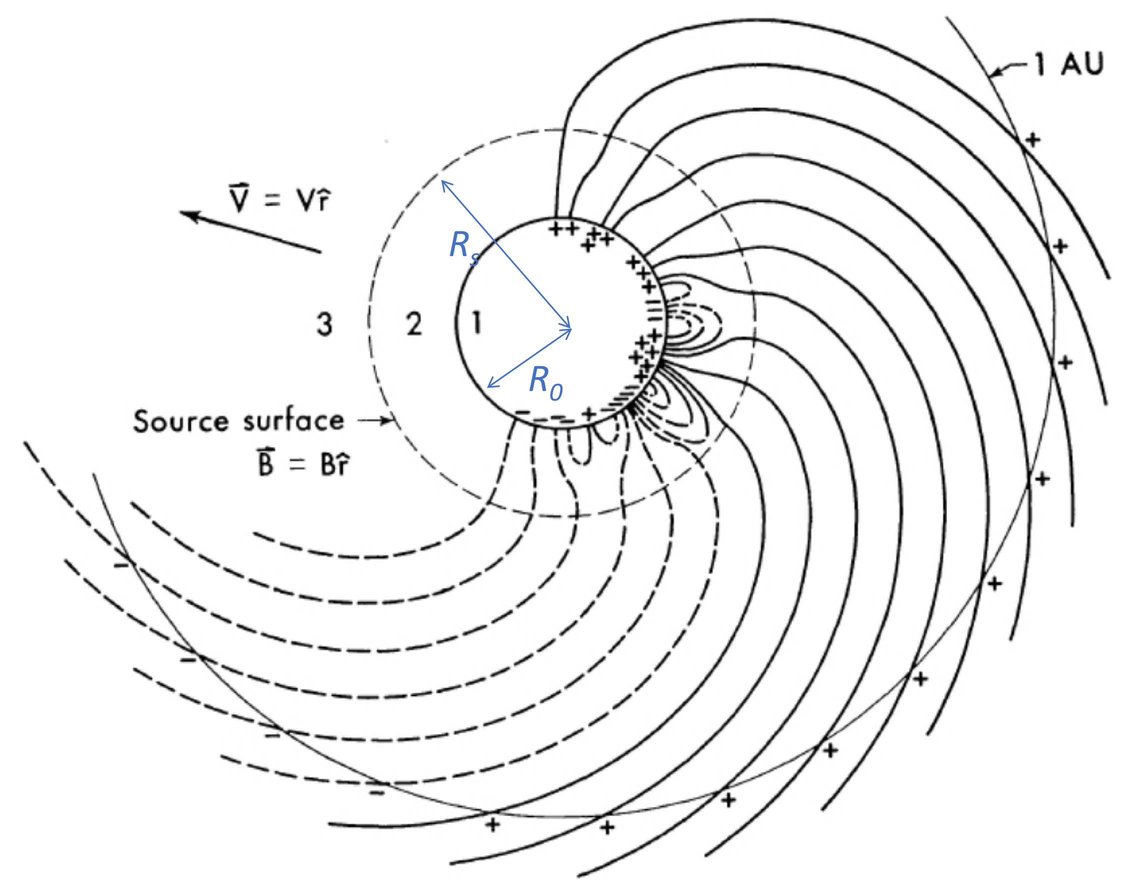


Trajectory of 100 protons (KE = 10 GeV)
(Using PFSS Model May 2019, R_s = 1.6 $R_\odot$)

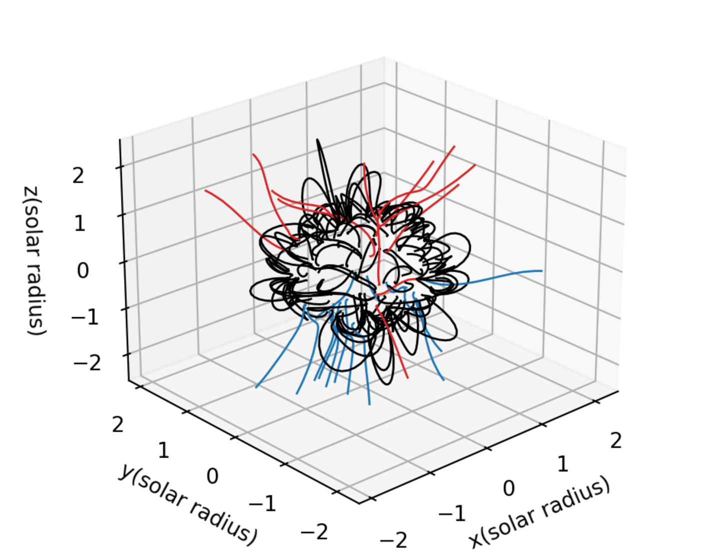


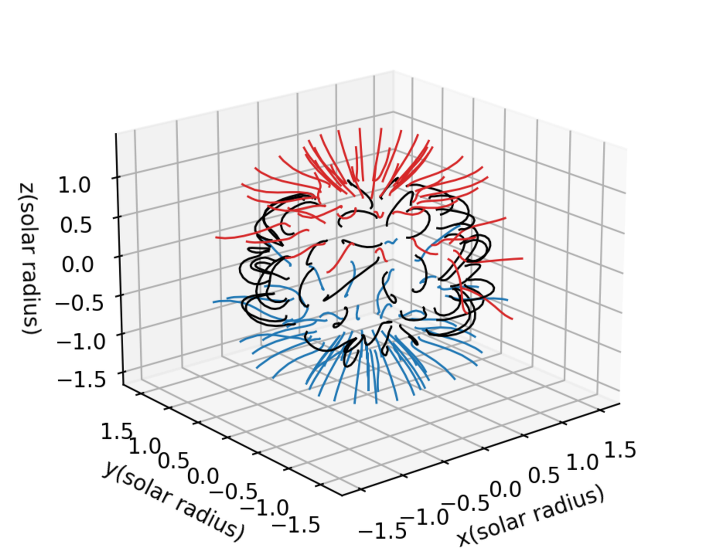
In the simulation, a total of 4050 protons were injected in $(R_0 + 3000,\ 0.5\pi,\ 0.5\pi)$, with direction of initial velocity uniformly distributed in 0 ≤ θ ≤ π, 0 ≤ φ ≤ 2π. For each proton, its $KE = 10$ GeV. To visualize the result, the graphs indicate the trajectories of 100 protons (direction of initial velocity distribute in 0 ≤ θ ≤ π, 0 ≤ φ ≤ 2π uniformly). Once the proton escaped the sphere of $r = R_0 + 3000$ km, its trajectory was not traced. The graphs indicated the proton trajectories in a cube with length = 20 km, where the center of the cube is the position of the proton injections.

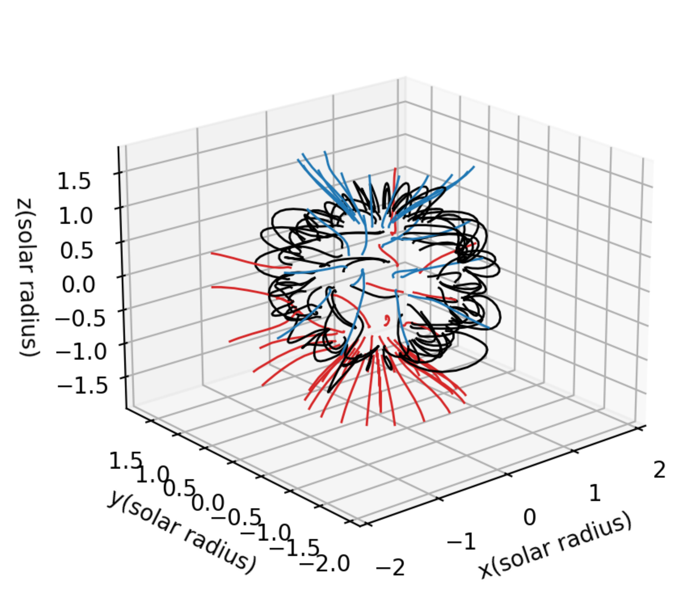
Visualization of Source Surface



Visualization of PFSS Solution







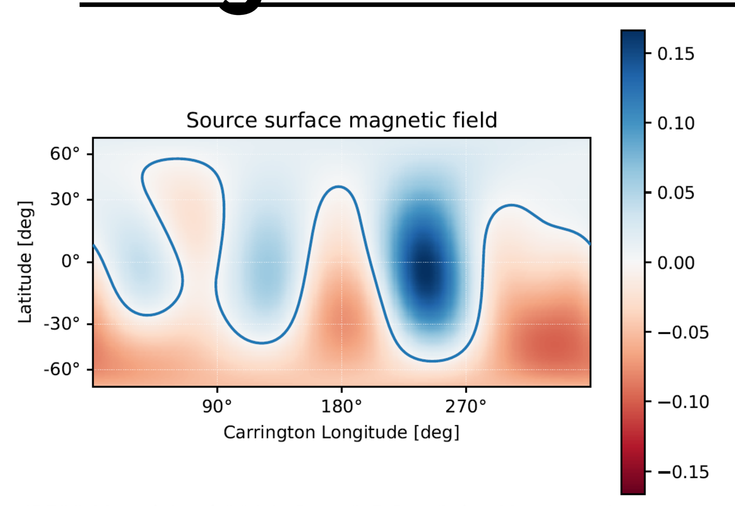
The graphs indicate the field lines of the solar magnetic field. The colors denote three kinds of magnetic field lines:

1. Red: open field going out
2. Blue: open field going in
3. Black: closed field

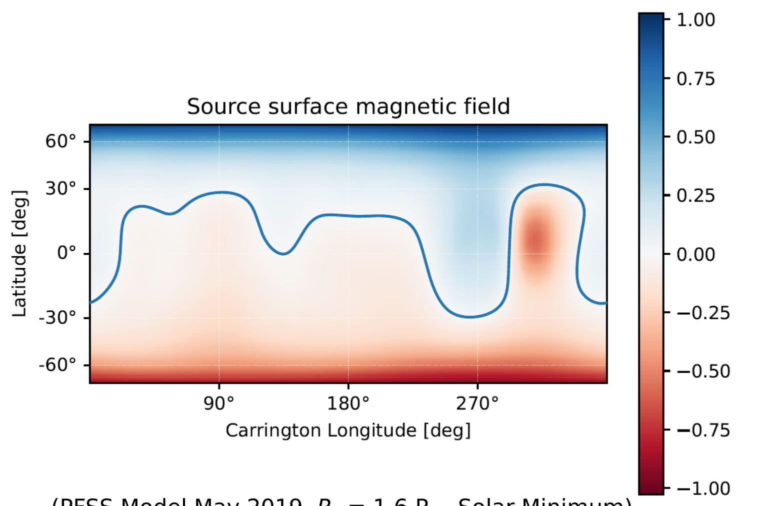
The source surface is not necessarily a spherical surface.

With the synoptic map labelled by Carrington rotation numbers, which , it allows us to compute the magnetic field components of the PFSS solution under coordinate R_0 < r < R_S, 0 < θ < π, 0 < φ < 2π.

Magnetic field on source surface



(PFSS Model April 2014, R_s = 2.5 $R_\odot$, Solar Maximum)



(PFSS Model May 2019, R_s = 1.6 $R_\odot$, Solar Minimum)

The graphs indicated the magnetic field on the source surface and the polarity inversion line. The solar magnetic field is highly variable. The magnetic pole could be inverted after a period of time.

Reference
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