Homework 4 Yuanlai Wang

Question 1:

a) $A = \begin{bmatrix} \text{coeff } x_1^2 & \text{coeff } x_1x_2 \\ \text{coeff } x_1x_2 & \text{coeff } x_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

b) $\frac{dy}{dx} = \begin{pmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 2x_1 + 2x_2 \\ 2x_1 + 10x_2 \end{pmatrix}$

c) $\frac{dy}{dx} = d(x^TAx) = d(x \cdot Ax) = d(Ax \cdot x) = (dx)^TAx + x^TA\,dx = (dx)^TAx + (dx)^TA^Tx$

~~$dx^TAx + x^TA\,dx$~~ $= (dx)^T(A + A^T)x$

~~$dx^TA^Tx^T + x^TA\,dx$~~ $\because A$ is symmetric

$\therefore A + A^T = 2A$

$\therefore \frac{dy}{dx} = 2A^Tx$

Question 2: a) $\frac{\partial y}{\partial x_1} = 2x_1 + 2x_2 - 6 = 0$

$\frac{\partial y}{\partial x_2} = 2x_1 + 10x_2 - 14 = 0$

$\begin{cases} x_1 = 2 \\ x_2 = 1 \end{cases}$

b) $y = x^T\begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}x + \begin{bmatrix} -6 \\ -14 \end{bmatrix}x + 13$

~~get x^{-1}~~

$\frac{dy}{dx} = 2Ax + B = \begin{bmatrix} 2 & 2 \\ 2 & 10 \end{bmatrix}x + \begin{bmatrix} -6 \\ -14 \end{bmatrix} = 0$

$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Question 3: a) $\begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} -4x \\ -6y \end{pmatrix}$ when $x=4, y=5$ $\boxed{(-16, -34)}$

b) $F(x,y,z) = 2x^2 + 3y^2 + z - 300$

$\begin{pmatrix} \frac{\partial F(x,y,z)}{\partial x} \\ \frac{\partial F(x,y,z)}{\partial y} \\ \frac{\partial F(x,y,z)}{\partial z} \end{pmatrix} = \begin{pmatrix} 4x \\ 6y \\ 1 \end{pmatrix} = \begin{pmatrix} 16 \\ 34 \\ 1 \end{pmatrix}$ It should decrease the depth

$\therefore \boxed{(-16, -34, -1)}$