Homework 1 Yuanka Wang

Question 1: c) $\mathbb{1}^T\mathbb{1} = \underbrace{(1\ 1\ \cdots\ 1)}_{n\ 1\text{'s}} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \Big\} n\ 1\text{'s} = \underbrace{1\times1 + 1\times1 + \cdots + 1\times1}_{n\ 1\times1\text{'s}} = n$

b) $y^T\mathbb{1} = (y_1\ y_2\ \cdots\ y_n)\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = y_1\times1 + y_2\times1 + \cdots + y_n\times1 = y_1 + y_2 + \cdots + y_n = \sum_{i=1}^{n} y_i$

c) $X^TXw = X^Ty$

$\mathbb{1}^T\mathbb{1}w = \mathbb{1}^Ty$

$nw = \sum_{i=1}^{n} y_i$

$w = \frac{1}{n}\sum_{i=1}^{n} y_i$

$b = \bar{y}$

Question 2: a) $\frac{dy}{dx} = \begin{pmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_1 + 5 + 2x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$

b) $\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} \frac{f(x_1+\varepsilon, x_2) - f(x_1-\varepsilon, x_2)}{2\varepsilon} \\ \frac{f(x_1, x_2+\varepsilon) - f(x_1, x_2-\varepsilon)}{2\varepsilon} \end{pmatrix} = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$

Exactly the same.