Homework 4: Due Fri 09-14-2018
Total Points (40 pts)

1. (10 pts) (Gradient Vector) Consider the function

$$y = f(x_1, x_2) = x_1^2 + 2x_1x_2 + 5x_2^2.$$

(a) Let $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Determine a matrix $\mathbf{A}$ such that

$$f(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x}.$$

(b) The gradient vector of $y = f(\mathbf{x})$ is

$$\frac{dy}{d\mathbf{x}} = \begin{pmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} \end{pmatrix}^\top.$$

Compute $\dfrac{dy}{d\mathbf{x}}$.

(c) The gradient vector of $f(\mathbf{x})$ is also given by

$$\frac{dy}{d\mathbf{x}} = \left(2\mathbf{x}^\top \mathbf{A}\right)^\top = 2\mathbf{A}^\top \mathbf{x}.$$

Show that the matrix expression given above for the gradient vector gives the same result as the expression in part (b).

2. (10 pts) (Optimization) Determine $\mathbf{x}_{\text{opt}}$ that minimizes

$$y = f(\mathbf{x}) = f(x_1, x_2) = x_1^2 + 2x_1x_2 + 5x_2^2 - 6x_1 - 14x_2 + 13$$

in the following two ways:

(a) Set the partial derivatives $\dfrac{\partial y}{\partial x_1}$, $\dfrac{\partial y}{\partial x_2}$ equal to zero and solve.

(b) Express $y = f(\mathbf{x})$ in the form $\mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b}^\top \mathbf{x} + c$ where $\mathbf{A}$ is a matrix, $\mathbf{b}$ is a vector, and c is a scalar. Then set $\dfrac{dy}{d\mathbf{x}}$ equal to zero and solve the resulting matrix equation using `numpy`.

3. (10 pts) (Gradient Vector) The depth of water (in feet) at the point (x, y) on a certain lake is given by

$$z = f(x, y) = 300 - 2x^2 - 3y^2.$$

(a) A boy at $(4, 9)$ is frightened by something moving on the bottom of the lake. In which direction on the surface of the lake should the boy swim to *decrease* the depth of water under him the fastest? Show your work.

(b) A diver is at the bottom of the lake at $(4, 9, 25)$. In which direction in three dimensions should the diver move to get off of the bottom of the lake as fast as possible? Show your work.

Hint: Define $F(x, y, z) = z - f(x, y)$. (continued on back side)

4. (10 pts) (Predicting Stock Prices) Can past stock prices be used to predict future stock prices? The file `AdjustedClosingPrices.csv`[1] contains data on 467 U.S. stocks over the course of 4173 days. Train a linear regression neural network that uses today's stock prices to predict tomorrow's stock prices. (You will have multiple targets, one per stock.) Normalize the data by computing log price ratios as shown below:

$$\text{log price ratio } = \ln\left(\frac{\text{price today}}{\text{price yesterday}}\right).$$

Train using log price ratios instead of stock prices. Evaluate the accuracy of your trained neural network.

[1] Courtesy Dylan Vener. The data was downloaded from Yahoo Finance.