Project: Kinematics Pick & Place

Kinematic Analysis

1. **Run the forward_kinematics demo and evaluate the kr210.urdf.xacro file to perform kinematic analysis of Kuka KR210 robot and derive its DH parameters.**

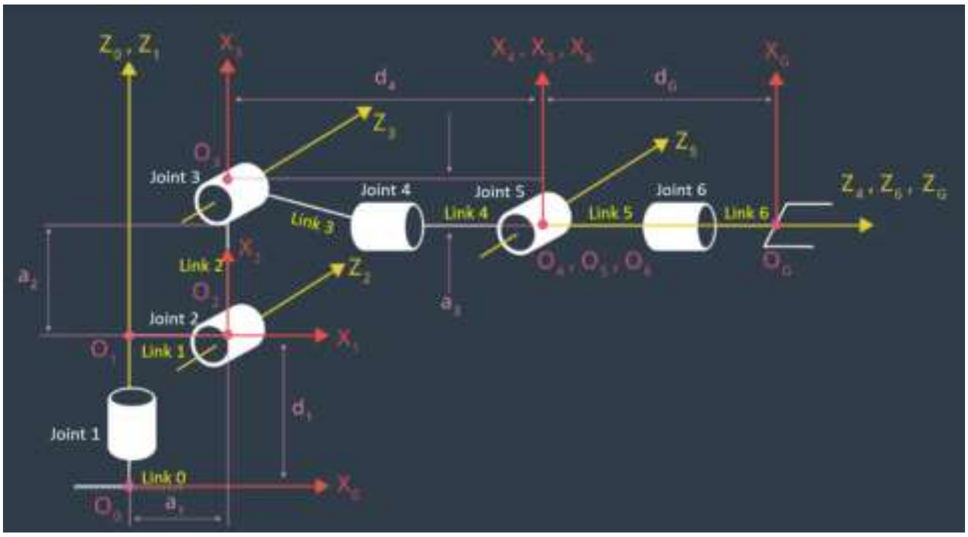

Figure 1. robotic arm joints

The Figure 1 shows joints of robotic arm in demo where all joints are labeled from 1 to 6 and all links are labeled from 0 to 6. The reference frames are also assigned to each link, respectively. Then the DH parameters are summarized as follows. The red color denotes default joint angles. Since all joints in this model are revolute, only joint angles are variables.

i	α_{i-1}	a_{i-1}	d_i	θ_i
T_1^0	0	0	0.75	0
T_2^1	-90	0.35	0	-90
T_3^2	0	1.25	0	0
T_4^3	-90	-0.054	1.5	0
T_5^4	90	0	0	0
T_6^5	-90	0	0	0
T_G^6	0	0	0.303	0

2. **Using the DH parameter table you derived earlier, create individual transformation matrices about each joint. In addition, also generate a generalized homogeneous transform between base_link and gripper_link using only end-effector (gripper) case.**

Given a group of parameters such as α_{i-1}, a_{i-1}, d_i and θ_i, the homogeneous transform ${}^{i-1}_{i}T$ is defined as,

$$ {}^{i-1}_{i}T = R_x(\alpha_{i-1})T_x(a_{i-1})R_z(\theta_i)T_z(d_i) \quad (1) $$

or

$$ {}^{i-1}_{i}T = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 & a_{i-1} \\ sin\theta_i cos\alpha_{i-1} & cos\theta_i cos\alpha_{i-1} & -sin\alpha_{i-1} & -dsin\alpha_{i-1} \\ sin\theta_i sin\alpha_{i-1} & cos\theta_i sin\alpha_{i-1} & cos\alpha_{i-1} & dcos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2) $$

Where $R_x(\alpha_{i-1})$ is rotation about the x_{i-1} axis by α_{i-1}, $T_x(a_{i-1})$ is translation along x_{i-1} axis by a_{i-1}, $R_z(\theta_i)$ is rotation about the z_i axis by θ_i, and $T_z(d_i)$ is translation along z_i axis by d_i.

According to the DH tables and equation (1) or (2), the individual transformation matrices about each joint are then listed as bellows,

$$ {}^{0}_{1}T = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ -sin\theta_2 & -cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & 1.25 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}_{4}T = \begin{bmatrix} cos\theta_4 & -sin\theta_4 & 0 & -0.054 \\ 0 & 0 & 1 & 1.5 \\ -sin\theta_4 & -cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T = \begin{bmatrix} cos\theta_5 & -sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sin\theta_5 & cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{5}_{6}T = \begin{bmatrix} cos\theta_6 & -sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\theta_6 & -cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{6}_{G}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The homogeneous transform between the base_link and the gripper_link is ${}^{G}_{0}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T\,{}^{6}_{G}T$. Alternatively, if we know the position and orientation of the end-effector, we can write directly ${}^{G}_{0}T$ as

$$ {}^{G}_{0}T = \begin{bmatrix} R_{rpy} & r_{ee} \\ 0 & 1 \end{bmatrix} \quad (3) $$

where r_{ee} represents the position of end-effector with respective to base_link and $R_{rpy} = R_z(\gamma)R_y(\beta)R_x(\alpha)$ represents the rotation part. Here α , β and γ is roll-pitch-yaw angles of the end-effector.

3. Decouple Inverse Kinematics problem into Inverse Position Kinematics and inverse Orientation Kinematics; doing so derive the equations to calculate all individual joint angles.

As Figure 1 shows, the last three joints in the robotic arm are revolute and their joint axes intersect at a single point (O_4), we have the case of wrist center being the common intersection point. This allows us to kinematically decouple the inverse kinematics problem into inverse position and inverse orientation problems. First let's calculate the position of wrist center, r_{wc} which satisfies

$$ r_{ee} = r_{wc} + T_{rpy} \begin{bmatrix} 0 \\ 0 \\ d_G \\ 1 \end{bmatrix} $$

Then we have,

$$r_{wc} = r_{ee} - T_{rpy} \begin{bmatrix} 0 \\ 0 \\ d_G \\ 1 \end{bmatrix}$$

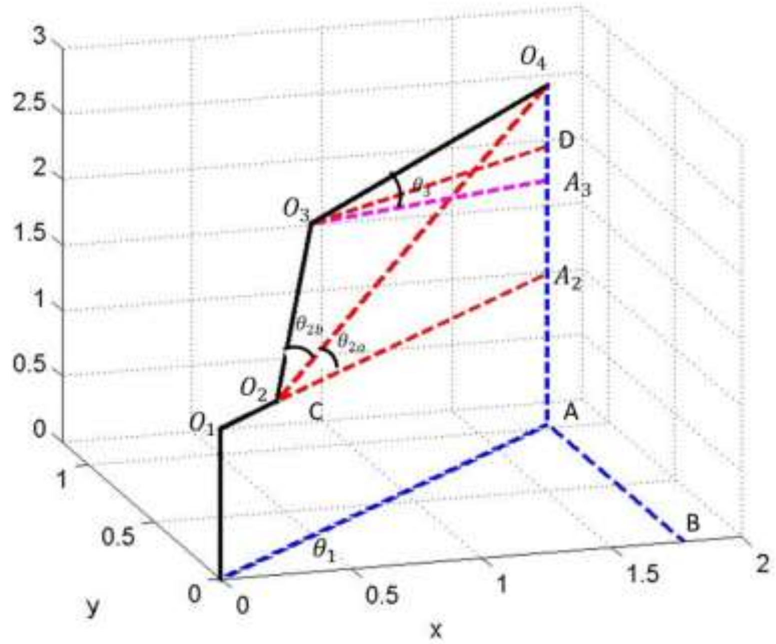

Figure 2. The geometric analysis of first three joints.

Next let's see the first three joints first. As show in Figure 2, $O_4(x_c, y_c, z_c)$ is the wrist center, $OO_4 = r_{wc}$, and we already know

$$|O_1O_2| = \mathrm{a}_1$$
$$|O_2O_3| = \mathrm{a}_2$$
$$|O_3O_4| = \sqrt{a_3^2 + d_4^2}$$
$$|OA| = |O_1A_2| = \sqrt{x_c^2 + y_c^2}$$
$$|OO_1| = |AA_2| = \mathrm{d}_1$$
$$|AO_4| = z_c$$

For the first joint,

$$\theta_1 = tan^{-1}\frac{y_c}{x_c}.$$

For the second joint,

$$\theta_2 = \theta_{2a} + \theta_{2b} = \angle A_2O_2O_4 + \angle O_3O_2O_4$$

where

$$\theta_{2a} = tan^{-1}\frac{|O_4A_2|}{|O_2A_2|} = tan^{-1}\frac{z_c - \mathrm{d}_1}{\sqrt{x_c^2 + y_c^2} - \mathrm{a}_1}$$
$$\theta_{2b} = cos^{-1}\frac{|O_2O_3|^2 + |O_2O_4|^2 - |O_3O_4|^2}{2|O_2O_3||O_2O_4|} = cos^{-1}\frac{a_2^2 + \mathrm{w}^2 - a_3^2 - d_4^2}{2\mathrm{a}_2\mathrm{w}}$$

$$\mathrm{w} = |O_2O_4| = \sqrt{|O_4A_2|^2 + |O_2A_2|^2} = \sqrt{(z_c - \mathrm{d}_1)^2 + \left(\sqrt{x_c^2 + y_c^2} - \mathrm{a}_1\right)^2}$$

For the third joint,

$$\theta_3 = \angle O_2O_3A_3 - \angle O_2O_3O_4 = \left(\frac{\pi}{2} - \angle A_3O_3D\right) - \angle O_2O_3O_4 = \frac{\pi}{2} - \theta_{3a} - \theta_{3b}$$

Where

$$\theta_{3a} = tan^{-1}\frac{|\mathrm{a}_3|}{\mathrm{d}_4}$$

$$\theta_{3b} = cos^{-1}\frac{|O_2O_3|^2 + |O_3O_4|^2 - |O_2O_4|^2}{2|O_2O_3||O_3O_4|} = cos^{-1}\frac{a_2^2 + a_3^2 + d_4^2 - \mathrm{w}^2}{2\mathrm{a}_2\sqrt{a_3^2 + d_4^2}}$$

Since θ_1, θ_2 and θ_3 have been obtained from above formulations, we can calculate ${}^0_3R$ right away. Next from the equation,

$${}^0_6R = {}^0_3R\,{}^3_6R = \mathrm{R}_{rpy}$$

We compute ${}^3_6R$ by

$${}^3_6R = {}^0_3R^{-1}\mathrm{R}_{rpy}$$

Since

$${}^3_6R = \begin{bmatrix} cos\theta_4 cos\theta_5 cos\theta_6 - sin\theta_4 sin\theta_6 & -cos\theta_4 cos\theta_5 sin\theta_6 - sin\theta_4 cos\theta_6 & -cos\theta_4 sin\theta_5 \\ sin\theta_5 cos\theta_6 & -sin\theta_5 sin\theta_6 & cos\theta_5 \\ -sin\theta_4 cos\theta_5 cos\theta_6 - cos\theta_4 sin\theta_6 & sin\theta_4 cos\theta_5 sin\theta_6 - cos\theta_4 cos\theta_6 & sin\theta_4 sin\theta_5 \end{bmatrix}$$

We have

$$\theta_4 = -tan^{-1}\frac{{}^3_6R[2,2]}{{}^3_6R[0,2]}$$

$$\theta_5 = cos^{-1}\,{}^3_6R[1,2]$$

$$\theta_6 = -tan^{-1}\frac{{}^3_6R[1,1]}{{}^3_6R[1,0]}$$

Now we get all joint angles. Note actually there are actually a few sets of solutions for these joint angles. In above formulation we only consider one of them.

Project Implementation

1. **Fill in the IK_server.py file with properly commented python code for calculating Inverse Kinematics based on previously performed Kinematic Analysis. Your code must guide the robot to successfully complete 8/10 pick and place cycles. Briefly discuss the code you implemented and your results.**

For the code, at the beginning of the function handle_calulate_IK(), I define some symbolic variables representing DH parameters, and create individual homogeneous transformation matrices between each neighboring links. Inside the for loop, I first obtain the position and orientation of the end-effector and then use them to calculate the position of wrist center. Here, because the urdf model does not follow the DH convention, I define a correction matrix to compensate for the difference when do the computing as the previous section. Next I can calculate the first three joint angles from wrist center, and further I get the last three joint angles.

I use IK_debug.py to check my code. The result shows the error of position of end effector and wrist center are almost close to 0, while the error of thetas some times are not zero. The non zero error of joint angles can be explained by the fact that the IK problem has not only one solution. In the future, I could improve it by consider the time continuity.