

# 1 Formulas

## Maxwell's Equations

- **Gauss's Law**

$$\nabla \cdot D = \rho$$
$$\iiint_S D \cdot ds = \iiint_V \rho dV = Q$$

The divergence of electric flux at a point equals to the charge density at this point.

- **Gauss's Law for Magnetic Fields**

$$\nabla \cdot B = 0$$
$$\iiint_S B \cdot dS = 0$$

There're no sink or source of B. B forms close loop.

- **Farady's Law**

$$\nabla \times E = -\frac{\delta B}{\delta t}$$
$$\oint E \cdot dL = - \int E \cdot dL = V = - \iint_S \frac{\delta B}{\delta t} dS$$

Electric field E arise due to a time changing magnetic flux density.

- **Ampere's Law**

$$\nabla \times H = J_C + \frac{\delta D}{\delta t}$$
$$\int_L H \cdot dL = \iint (J + \frac{\delta D}{\delta t}) \cdot dS = I$$

Magnetic field can arise due to conduction current density J or displacement current density.

## Electric Field

$$E = \frac{Q}{4\pi\epsilon r^2}$$

, where  $\epsilon = \epsilon_0\epsilon_r$ ,  $\epsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$

$$E = -grad(V)$$

$$E = \frac{Q_1}{4\pi\epsilon_0} \frac{(x - x_1, y - y_1, z - z_1)}{((x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2)^{\frac{3}{2}}}$$

, where  $x_1, y_1, z_1$  are the coordinates for the charge

## Electric Flux

$$\Psi = \iint \epsilon E \cdot ds = \iint D \cdot ds$$

## Electric Flux Density

$$D = \frac{\Psi}{A}$$

## Capacitor

- $C = \frac{\epsilon A}{d}$
- $E = \frac{1}{2} CV^2$

## Magnetic Flux

$$\Phi = \iint \mu H \cdot ds = \iint B \cdot ds$$

## Magnetic Flux Density

$$B = \frac{\Phi}{A} = \mu H$$

## Resistivity

$$\rho = \frac{RA}{l}$$

## Drift Velocity

$$U_d = \mu_m E$$

## Transmission Line

- **Shunt Admittance:**  $Y = G + j\omega C$
- **Series Impedance:**  $Z = R + j\omega L$
- **Propagation Constant:**  $\sqrt{ZY}$
- **Attenuation Constant:**  $Re\sqrt{ZY}$
- **Phase Constant:**  $Im\sqrt{ZY}$
- **Characteristic Impedance:**  $Z_{line} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$
- **Propagation Speed:**  $\frac{1}{\sqrt{LC}}$
- **Attenuation(in dB):**  $20\log_{10}exp(\alpha \times L)$
- **Voltage Standing-Wave Ratio(VSWR):**  $VSWR = \frac{1+\Gamma_v}{1-\Gamma_v}$
- **Reflection coefficient for V:**  $\Gamma_v = \frac{Z_L - Z_0}{Z_L + Z_0}$
- **Reflection coefficient for I:**  $\Gamma_i = -\frac{Z_L - Z_0}{Z_L + Z_0} = -\Gamma_v$
- **Reflected Voltage:**  $V_r = V_i \times \Gamma_v$
- **Reflected Current:**  $I_r = I_i \times \Gamma_i$

#### Poynting Vector

Table 1: Poynting Vector		
Property	Circuit	Field
Voltage	$V$	$E$
Current	$I$	$H$
Impedance	$R$	$Z = \sqrt{\frac{\mu}{\epsilon}}$
Average Power 1	$\frac{1}{2}Re(VI^*)$	$\frac{1}{2}Re(EH^*)$
Average Power 2	$\frac{1}{2} \times \frac{V^2}{R}$	$\frac{1}{2} \times \frac{E^2}{Z}$
Average Power 3	$\frac{1}{2}I^2R$	$\frac{1}{2}H^2Z$

## 2 Definitions

- **Gauss's Law:** Total electric flux over a volume is equal to the charge enclosed by that volume.
- **Electric Field:** E at a point in a Electric field is the force act on the unit charge at this point.
- **Absolute Potential:** The work move a unit charge from infinity to a radial distance r1.
- **Electric Flux:** Electric Flux through a surface is the integral of normal component of electric field multiplied by  $\epsilon$ .
- **Electric Flux Density:** Electric flux divided by A.
- **Permittivity:** Permittivity of vacuum multiplied by relative permittivity.
- **Drift Velocity:** The drift velocity is the flow velocity that a particle, such as an electron, attains in a material due to an electric field.
- **Magnetic Flux Density:** B equals to Magnetic flux  $\Phi$  divided by area A.
- **Relative Permeability:** Ratio of effective permeability to absolute permeability.
- **Permeability:** The degree of magnetization of a material in response to a magnetic field.
- **Transmission Line:** Guide electromagnetic energy or info from one point to another.
- **Application of Transmission Lines:** Telephone, coaxial cables, micro strip tracks on a PCB .....
- **AC Circuit Theory:**  $l \ll \lambda$
- **Permittivity:** Measures the resistance encountered when forming an electric field.

### 3 Tao Lu

#### 3.1 Know D, find $\rho$

1.  $\iint Dds = \rho$
2. Determine if the  $\rho$  from last step is what we want.
3. If isn't, for example, we want the  $\rho$  of a line, but we have  $\rho$  in a volumn, then find the  $\rho$  we want.

#### 3.2 Magnetic Flux Between Strips

1.  $H = \frac{I}{W}$ , where W is the width of the strip.
2.  $\Phi = \mu HA$

#### 3.3 Find EMF

1. Find EMF caused by change of B,  $EMF = -\frac{d\Phi}{dt} = -\frac{AdB}{dt}$
2. Find EMF caused by  $\int (v \times B)dL$
3. Add them together.

#### 3.4 Wave Equation From Gauss's Law

1. We know  $\nabla \times E = -\frac{dB}{dt}$
2. Calculate curl for both side.  $\nabla \times \nabla \times E = \nabla \times \nabla \times -\frac{dB}{dt}$
3. Substitute  $\nabla \times B = \mu_0\epsilon_0 \frac{dE}{dt}$  into the equation obtained before
4.  $\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$ , where  $\nabla \cdot E$  is 0 in vacuum
5.  $\nabla^2 E = \mu_0\epsilon_0 \frac{dE}{dt}$

#### 3.5 Finding Attenuation

1. Find  $\alpha$
2. Calculate  $20\log_{10}(\exp(\alpha \times x))$ , where x is 1km.
3. The Unit is dB/km
4. If the Unit needs to be dB, then the  $\alpha$  should be multiplied by the distance