1 Formulas

Maxwell's Equations

- **Gauss's Law**

$$\nabla . D = \rho$$

$$\iint_S D.ds = \iiint_V \rho dV = Q$$

The divergence of electric flux at a point equals to the charge density at this point.

- **Gauss's Law for Magnetic Fields**

$$\nabla . B = 0$$

$$\iint_S BdS = 0$$

There're no sink or source of B. B forms close loop.

- **Farady's Law**

$$\nabla \times E = -\frac{\delta B}{\delta t}$$

$$\oint EdL = -\int EdL = V = -\iint_S \frac{\delta B}{\delta t} dS$$

Electric field E arise due to a time changing magnetic flux density.

- **Ampere's Law**

$$\nabla \times H = J_C + \frac{\delta D}{\delta t}$$

$$\int_L HdL = \iint (J + \frac{\delta D}{\delta t})dS = I$$

Magnetic field can arise due to conduction current density J or displacement current current density.

Electric Field

$$E = \frac{Q}{4\pi\epsilon r^2}$$

, where $\epsilon = \epsilon_0 \epsilon_r$, $\epsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$

$$E = -grad(V)$$

$$E = \frac{Q_1}{4\pi\epsilon_0} \frac{(x - x_1, y - y_1, z - z_1)}{((x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2)^{\frac{3}{2}}}$$

, where x_1, y_1, z_1 are the coordinates for the charge

Electric Flux

$$\Psi = \iint \epsilon E ds = \iint Dds$$

Electric Flux Density

$$D = \frac{\Psi}{A}$$

Capacitor

- $C = \frac{\epsilon A}{d}$
- $E = \frac{1}{2} CV^2$

Magnetic Flux

$$\Phi = \iint \mu H ds = \iint Bds$$

Magnetic Flux Density

$$B = \frac{\Phi}{A} = \mu H$$

Resistivity

$$\rho = \frac{RA}{l}$$

Drift Velocity

$$U_d = \mu_m E$$

Transmission Line

- **Shunt Admittance**: $Y = G + j\omega C$
- **Series Impedance**: $Z = R + j\omega L$
- **Propagation Constant**: $\sqrt{ZY}$
- **Attenuation Constant**:$Re\sqrt{ZY}$
- **Phase Constant**:$Im\sqrt{ZY}$
- **Characteristic Impedance**:$Z_{line} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$
- **Propagation Speed**:$\frac{1}{\sqrt{LC}}$
- **Attenuation(in dB)**: $20log_{10}exp(\alpha \times L)$
- **Voltage Standing-Wave Ratio(VSWR)**: $VSWR = \frac{1+\Gamma_v}{1-\Gamma_v}$
- **Reflection coefficient for V**:$\Gamma_v = \frac{Z_L - Z_0}{Z_L + Z_0}$
- **Reflection coefficient for I**:$\Gamma_i = -\frac{Z_L - Z_0}{Z_L + Z_0} = -\Gamma_v$
- **Reflected Voltage**: $V_r = V_i \times \Gamma_v$
- **Reflected Current**: $I_r = I_i \times \Gamma_i$

Poynting Vector

Table 1: Poynting Vector

Property	Circuit	Field
Voltage	V	E
Current	I	H
Impedance	R	$Z = \sqrt{\frac{\mu}{\epsilon}}$
Average Power 1	$\frac{1}{2}Re(VI^*)$	$\frac{1}{2}Re(EH^*)$
Average Power 2	$\frac{1}{2} \times \frac{V^2}{R}$	$\frac{1}{2} \times \frac{E^2}{Z}$
Average Power 3	$\frac{1}{2}I^2R$	$\frac{1}{2}H^2Z$

2 Definitions

- **Gauss's Law**: Total electric flux over a volumn is equal to the charge enclosed by that volumn.
- **Electric Field**: E at at a point in a Electric field is the force act on the unit charge at this point.
- **Absolute Potential**: The work move a unit charge from infinity to a radial distance r1.
- **Electric Flux**: Electric Flux through a surface is the integral of normal component of electric field multiplied by ϵ.
- **Electric Flux Density**: Electric flux divided by A.
- **Permittivity**:Permittivity of vacuum multiplied by relative permittivity.
- **Drift Velocity**: The drift velocity is the flow velocity that a particle, such as an electron, attains in a material due to an electric field.
- **Magnatic Flux Density**: B equals to Magnetic flux Φ divided by area A.
- **Relative Permeability**: Ratio of effective permeability to absolute permeability.
- **Permeability**: The degree of magnetization of a material in response to a magnetic field.
- **Transmission Line**: Guide electromagnetic energy or info from one point to another.
- **Application of Transmission Lines**: Telephone, coaxial cables, micro strip tracks on a PCB ......
- **AC Circuit Theory**: $l << \lambda$
- **Permittivity**: Measures the resistance encountered when forming an electric field.

3 Tao Lu

3.1 Know D, find ρ

1. $\iint Dds = \rho$
2. Determine if the ρ from last step is what we want.
3. If isn't, for example, we want the ρ of a line, but we have ρ in a volumn, then find the ρ we want.

3.2 Magnetic Flux Between Strips

1. $H = \frac{I}{W}$, where W is the width of the strip.
2. $\Phi = \mu HA$

3.3 Find EMF

1. Find EMF caused by change of B, $EMF = -\frac{d\Phi}{dt} = -\frac{AdB}{dt}$
2. Find EMF caused by $\int (v \times B)dL$
3. Add them together.

3.4 Wave Equation From Gauss's Law

1. We know $\nabla \times E = -\frac{dB}{dt}$
2. Calculate curl for both side. $\nabla \times \nabla \times E = \nabla \times \nabla \times -\frac{dB}{dt}$
3. Substitute $\nabla \times B = \mu_0 \epsilon_0 \frac{dE}{dt}$ into the equation obtained before
4. $\nabla \times \nabla \times E = \nabla(\nabla.E) - \nabla^2 E$, where $\nabla.E$ is 0 in vacuum
5. $\nabla^2 E = \mu_0 \epsilon_0 \frac{dE}{dt}$

3.5 Finding Attenuation

1. Find α
2. Calculate $20log_{10}(exp(\alpha \times x))$, where x is 1km.
3. The Unit is dB/km
4. If the Unit needs to be dB, then the α should be multiplied by the distance

3.6 Laplace Equation From Gauss's Law

1. $\nabla.D = \rho$
2. $D = \epsilon E = -\epsilon \nabla V$
3. $\nabla^2 V = \frac{\rho}{\epsilon}$
4. In free space, it is 0.