

## 1 Formula

### De Broglie Hypothesis

$$p = mv = \frac{h}{\lambda} = k \frac{h}{2\pi}$$

$$k = \frac{2\pi}{\lambda}$$

$$L = n \frac{\lambda}{2}$$

$$p = \frac{hn}{2L}$$

### Bragg

$$2d \sin \theta = n\lambda$$

### Mass Action Law

$$n_i^2 = np$$

### Conductivity

$$\sigma = q(\mu_h p + \mu_e n)$$

For intrinsic,  $\rho$  is several  $\Omega$ . For extrinsic, smaller than 1  $\Omega$

### Depletion Region Width

$$\epsilon E = q N_D x_p$$

$$\epsilon E = q N_A x_n$$

$$W = x_n + x_p = \frac{\epsilon E_{max}}{q} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)$$

$$V_{bi} = \frac{1}{2} W E$$

$$V_{bi} = \phi_{Bn} + \phi_{Bp}$$

### Speed of CMOS

$$\tau \propto \frac{L^2}{\mu}$$

### Band Gap

$$n_i^2 = N_v N_c \exp\left(-\frac{E_g}{k_B T}\right)$$

### Subthreshold Regime

$$I_D \propto \exp\left(\frac{q\phi_s}{kT}\right)$$

$$V_G = m\phi_s + C$$

$$m = 1 + \frac{C_s}{C_{ox}}$$

$C_s$ : Change Substrate doping

$C_o$ : Change Oxide thickness

## 2 Definition

- **Enhancement Type:** Gate voltage is needed for a channel.
- **Depletion Type:** Need a gate voltage to turn it off.

## 3 Tao Lu

### 3.1 Derive $I_D$

1. Mos capacitor:  $Q = CV = -C_0(V_G - V_T - V_x) = qndz$
2. Current definition  $I = qnvA$ , where  $v$  is given by  $v = \mu E$ ,  $A = Wdz$
3. Substitute,  $I = -C_0(V_G - V_T - V_x)\mu EW$
4.  $E = -\frac{dV_x}{dx}$
5.  $I = C_0(V_G - V_T - V_x)\mu\frac{dV_x}{dx}W$
6. Multiplied by  $dx$ , integration, find  $I_D$

### 3.2 Derive Threshold Voltage

1.  $V_{ox} = \frac{V_G - V_s}{t_{ox}}$ ,  $V_G = V_{ox}t_{ox} + \phi_s$
2. Apply Gauss's Law, get  $E_s = \sqrt{\frac{N_A t_s q}{\epsilon_0 \epsilon_s}}$
3.  $V$  is the area below  $E$   $t_s = \frac{2\phi_s}{E_s}$
4.  $E_s = \sqrt{2\frac{N_A q \phi_s}{\epsilon_0 \epsilon_s}}$
5.  $\epsilon_{ox} E_{ox} = \epsilon_s E_s$
6. Find  $V_G$

### 3.3 Find Depletion Region Width

1. Find the point where  $E = E_{max}$
2. Apply Gauss's Law  $\int DdA = Q = x_p q A N_A = x_n q A N_D$
3.  $D = \epsilon E_{max}$
4.  $V_{bi} = \frac{1}{2}(x_n + x_p)E_{max}$

### 3.4 Find Threshold Voltage of CMOS circuit

1. Remember  $V_{in} = V_{out}$  at threshold point(But not always equal to  $\frac{1}{2}V_{CC}$ !).
2. Determine the state of each MOSFET by comparing  $|V_{GS}| - |V_T|$  and  $|V_{DS}|$
3. Find the current for each transistor.
4. Apply KCL for the proper node.
5. Find the answer.