University of Nottingham
School of Mathematical Sciences

Project 3, Problem 5, Modelling Traffic flow

1 Outline brief

It is now standard practice on some motorways to reduce the speed limit at busy times so as to keep the traffic moving. The reason for this is that at lower speeds, stopping distances are reduced and so more cars can be accommodated on the road. This allows for an orderly, higher traffic density and so a higher throughput of traffic (or flow rate) is possible, even though each car is moving more slowly. In a more general situation, the speed and relative motion between cars is often influenced by the distance behind the leading car and possibly other factors.

There are several approaches to the mathematical modelling of traffic flow. A classical method defines averaged quantities such as traffic density, flow rate etc. and then obtains continuum kinematic equations (partial differential equations) which describe these averaged quantities change, and hence predict flow behaviour, see, for example, Howison [2] and Whitham [6].

More recently, discrete approaches have been proposed: these are based on the behaviour of individual cars, the attributes of the car driver and the surrounding circumstances. One approach uses cellular automata, in which space, time and velocity are all discretised, with the car's position at a sequence of distinct snapshots in time, updated by deterministic evolution rules, as proposed by Nagel and Schreckenberg [3, 5].

Another model uses dynamical information from the car ahead (it's speed and the size of the gap) together with the reaction time of the driver to give an ordinary differential equation (ODE) for the car's motion. Applying these rules to a whole line of cars generates a system of coupled ODEs, and allows the large-scale simulation of traffic. These are known as 'car-following models', see Wilson [7].

Your task is to evaluate at least two (possibly all three) approaches. You should understand the model formulations and underlying assumptions; and show how the models can be used to predict realistic traffic flow under a variety of conditions (eg low traffic density, high traffic density, the stability of traffic flow to perturbations caused by a driver braking, see [4] for an illustration).

2 Task

Propose one or more Cellular Automaton models of traffic flow, justifying each of the rules you include (start with a simple set of rules, and later introduce additional complexity to make your model more realistic).

Implement your model, simulate numerically a stream of traffic; investigate the behaviour of cars as the speed of the leading car increases, decreases or oscillates. Investigate the behaviour of traffic flow at a range of densities, on an infinitely long road, or on a closed loop.

Propose one or more 'car-following' models for the behaviour of a driver following a vehicle - you can start with a simple set of rules, and later introduce additional complexity to make your model more realistic. Assume that the speed of a lead driver follows some prescribed function and that all the other drivers adjust their speed instantaneously according to the relative speed between themselves and the car in front. In the car-following approach, this will give a model based on a set of coupled (differential) equations governing the relative car motions and guided by the behaviour of the lead car. Assuming all drivers follow this behaviour, simulate numerically a stream of traffic; investigate the behaviour of cars as the speed of the leading car increases, decreases or oscillates. If possible, investigate the stabilty of uniform traffic flow analytically.

Explore the behaviour of your models as parameter values are changed, or stochastic effects are introduced into the models to model drivers having different responses to the road conditions. Whilst the simplest models are based on the assumption that cars and drivers react identically, your model could be made more realistic by allowing differences in how drivers respond to the car ahead (for example, by making small changes in parameters or introducing delays into the response function). It would also be possible to consider the case of a series of cars on a circuit.

Investigate solutions of the continuum (PDE) model of traffic flow, which relies on a relationship between average traffic speed and average local density. Compare and contrast the ability of the continuum approach with the cellular automata and/or car-following approaches to describe realistic phenomena in traffic flow. If possible, derive a speed-density relationship from your cellular automaton model and/or car-following approach (to show that the models could be used in conjunction with each other).

3 Deliverables

You should start by making simple assumptions to create tractable mathematical models, before considering the effects of adding in various complicating factors. The task deliverables should illustrate the group competence in mathematical modelling, in using theoretical and computational tools at the level expected of a recent mathematics graduate as well as identify further developments.

The Group should provide: (i) a substantial but concise technical report on your solution to the task, which includes output from working software (implemented in a package or computer language) which illustrates the model's analytic and predictive capabilities; (ii) an oral presentation. The report is expected to be a maximum of 25 pages of text (i.e., not counting figures, tables, appendices or table of contents, it should be single-spaced in 12pt font). The oral presentation is maximum of 20 minutes (15 minutes report plus 5 minutes for questions/discussion).

You are required to provide an oral report supported by a Powerpoint presentation of your project outcomes during the Workshop Session, Friday 15th March 2024 from 9am, and be prepared to answer questions. Electronic copies of your powerpoint presentation, your software and the written report, together with a completed submission sheet, are required to be submitted via moodle by 3pm on Thursday 14th March.

References

[1] Haberman, R, Mathematical Models – Mech. Vibrations, Pop. Dynamics and Traffic Flow, (1977).

[2] Howison, S, Practical Applied Mathematics, CUP, Cambridge, (2005).

[3] Nagel, K & Schreckenberg, M, A cellular automaton model for freeway traffic, *Journal de Physique I*, **2**, 2221–2229, (1992).

[4] NewScientist, Shockwave traffic jams recreated for first time, (2008) https://www.youtube.com/watch?v=Suugn-p5C1M (last accessed 27/1/2022)

[5] Rickert, M, Nagel K, Schreckenberg, M & Latour, A, Two Lane Traffic Simulations using Cellular Automata, LANL Report No. LA-UR 95-4367, arXiv:cond-mat/9512119v1 (1995).

[6] Whitham, G. Linear and Nonlinear Waves, Wiley, New York, (1974).

[7] Wilson, RE, An analysis of Gipps' car-following model of highway traffic. *IMA Journal of Applied Mathematics*, **66**, 509–537, (2001).