Chapter 4

Tree

Two kinds of data structure

- Linear: list, stack, queue, string
- Non-linear: tree, graph

4.1 Tree

1.Definition:

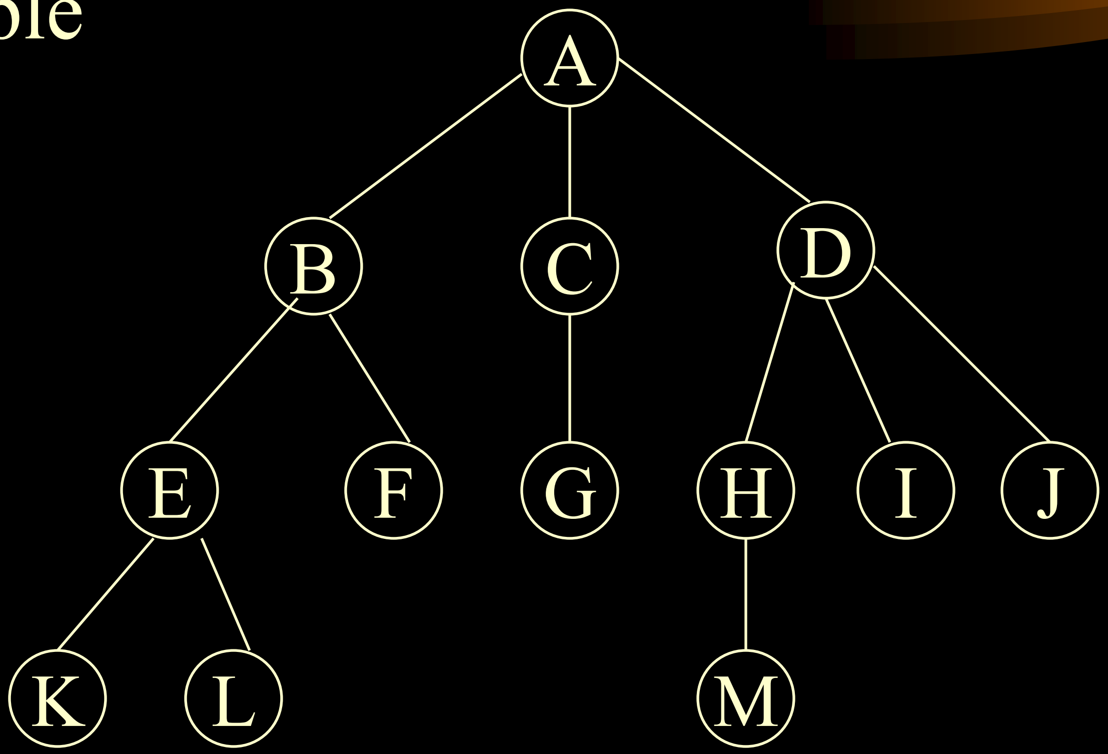
A tree T is a collection of nodes(element).

The collection can be empty;

otherwise, a tree consists of a distinguished node r , called the root, and zero or more nonempty(sub)trees T_1, T_2, ……, T_k

4.1 Tree

example



4.1 Tree

2.Terminology

Degree of an elememts(nodes): the number of children it has.

Degree of a tree: the maximum of its element degrees

Leaf: element whose degree is 0

Branch: element whose degree is not 0

4.1 Tree

Level:

the level of root is 0 (1)

the level of an element=

the level of its parent+1

Depth of a tree:

the maximum level of its elements

4.2 Binary Trees

1.Definition: A binary tree t is a finite (possibly empty) collection of elements.

When the binary tree is not empty:

- It has a root element
- The remaining elements(if any) are partitioned into two binary trees, which are called the left and right subtrees of t.

5种基本构型：

4.2 Binary Trees

2. The essential differences between a binary tree and a tree are:

1)Each element in a binary tree has exactly two subtrees(one or both of these subtrees may be empty).

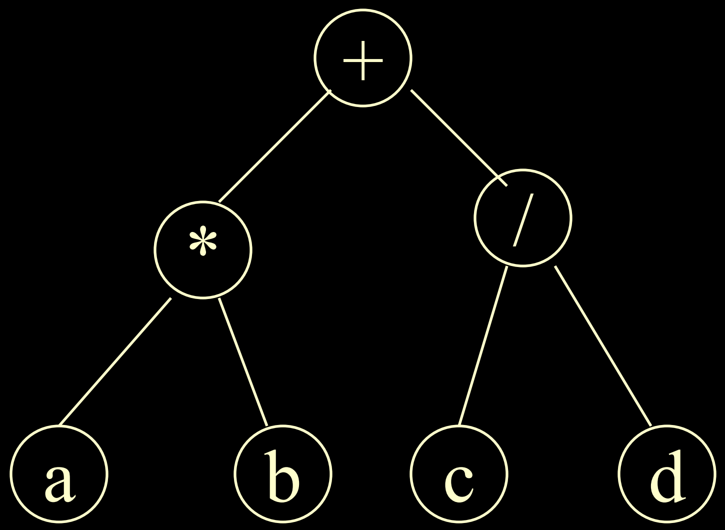
Each element in a tree can have any number of subtrees.

2) The subtrees of each element in a binary tree are ordered. That is, we distinguish between the left and the right subtrees.

The subtrees in a tree are unordered.

4.2 Binary Trees

Example of a binary tree



(a*b)+(c/d)

4.3 Properties of binary trees

Property 1. The drawing of every binary tree with n elements($n>0$)has exactly n-1 edges.

Property 2. The number of elements at level i is at most 2^i ($i>=0$).

4.3 Properties of binary trees

Property 3. A binary tree of height h, h>=0, has at least h+1 and at most $2^{h+1}-1$ elements in it.

proof of property 3:

$$\sum_{i=0}^{h} 2^i = 2^0 + 2^1 + \ldots\ldots + 2^h = 1*(1-2^{h+1})/(1-2) = 2^{h+1}-1$$

4.3 Properties of binary trees

Property 4. If number of leaves is n_0,and the number of the 2 degree elements is n_2, then $n_0=n_2+1$.

Proof:

设: 度为1的结点数是 n_1个

$$n = n_0 + n_1 + n_2$$

$n = B+1$ 这里B为分支数

$$n_0 + n_1 + n_2 = 1*n_1 + 2*n_2 + 1$$

$$n_0 = n_2 + 1$$

4.3 Properties of binary trees

Property 5.The height of a binary tree that contains n ($n>=0$) element is at most n-1 and at least $\lceil \log_2(n+1) \rceil -1$

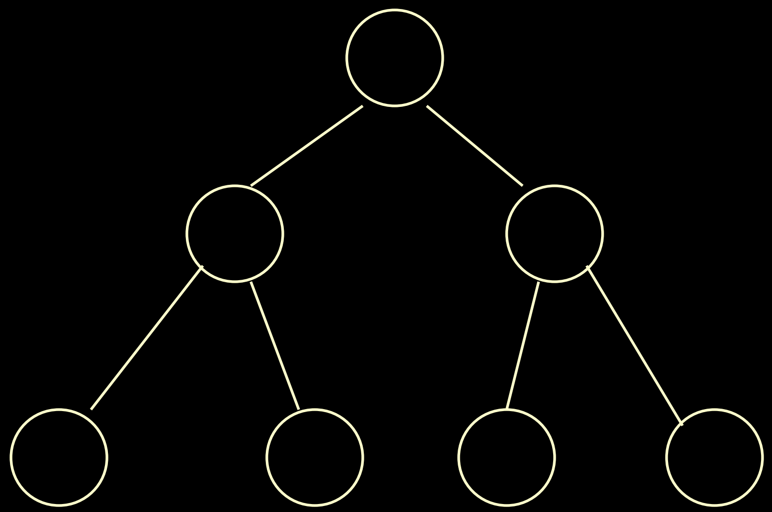
proof: Since there must be at least one element at each level, the height cannot exceed n-1.

From property 3,we know $n<=2^{h+1}-1$, so,$h>=\log_2(n+1)-1$, since h is an integer,we get $h=\lceil \log_2(n+1) \rceil -1$

4.3 Properties of binary trees

Definition of a full binary tree :

A binary tree of height h that contains exactly $2^{h+1}-1$ elements is called a full binary tree.



4.3 Properties of binary trees

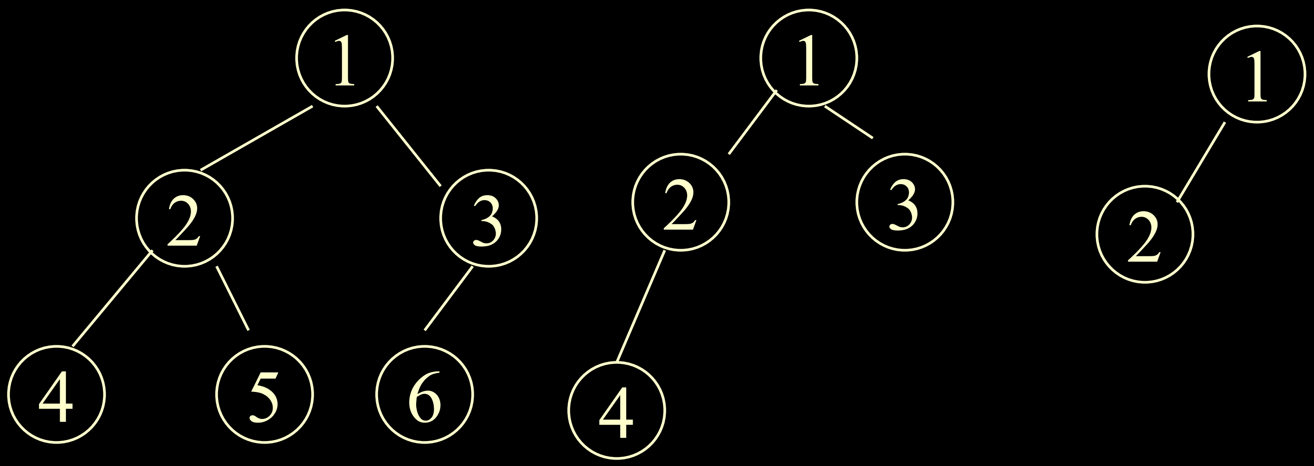
Definition of a complete binary tree:

Suppose we number the elements in a full binary tree of height h using the number 1 through 2^{h+1}-1.We began at level 0 and go down to level h.Within levels the elements are numbered left to right.

Suppose we delete the k elements numbered 2^{h+1}-i, 1<=i<=k,the resulting binary tree is called a complete binary tree.

4.3 Properties of binary trees

Example of complete binary trees



4.3 Properties of binary trees

Property 6. Let i, 0<=i<=n-1,be the number assigned to an element of a complete binary tree. The following are true.

1) if i=0, then this element is the root of the binary tree.

if i>0,then the parent of this element has been assigned the number $\lfloor (i-1)/2 \rfloor$

2) if 2*i+1>=n,then this element has no left child. Otherwise,its left child has been assigned the number 2*i+1.

4.3 Properties of binary trees

3) if 2*i+2>=n, then this element has no right child, Otherwise its right child has been assigned the number 2*i+2.

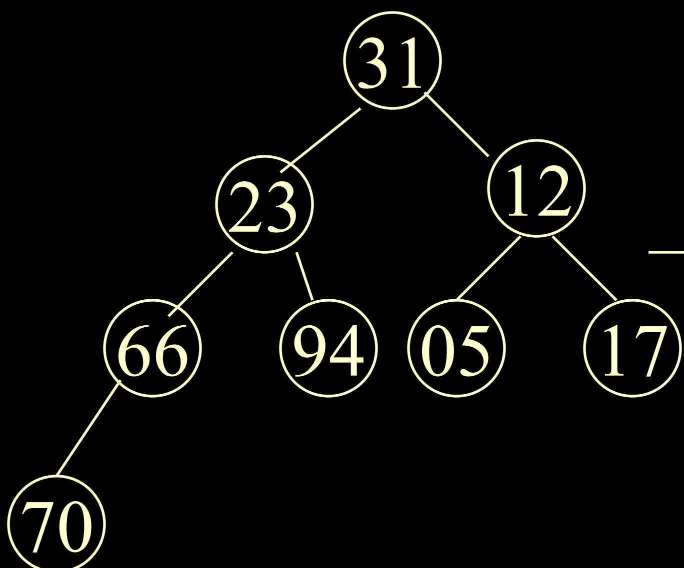
4.4 Representation of binary tree

1. Formula-Based Representation (array representation)

The binary tree to be represented is regarded as a complete binary tree with some missing elements.

4.4 Representation of binary tree

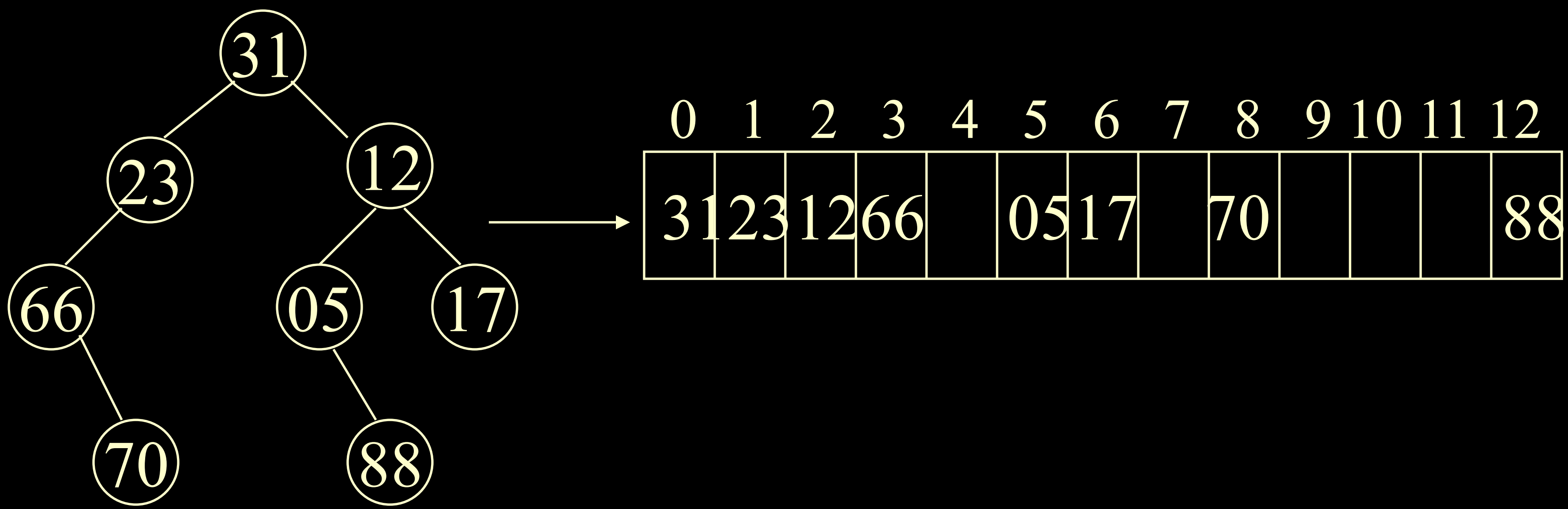
Example of a complete binary tree (array representation)



0	1	2	3	4	5	6	7
31	23	12	66	94	05	17	70

4.4 Representation of binary tree

Example of a common binary tree(array representation)



4.4 Representation of binary tree

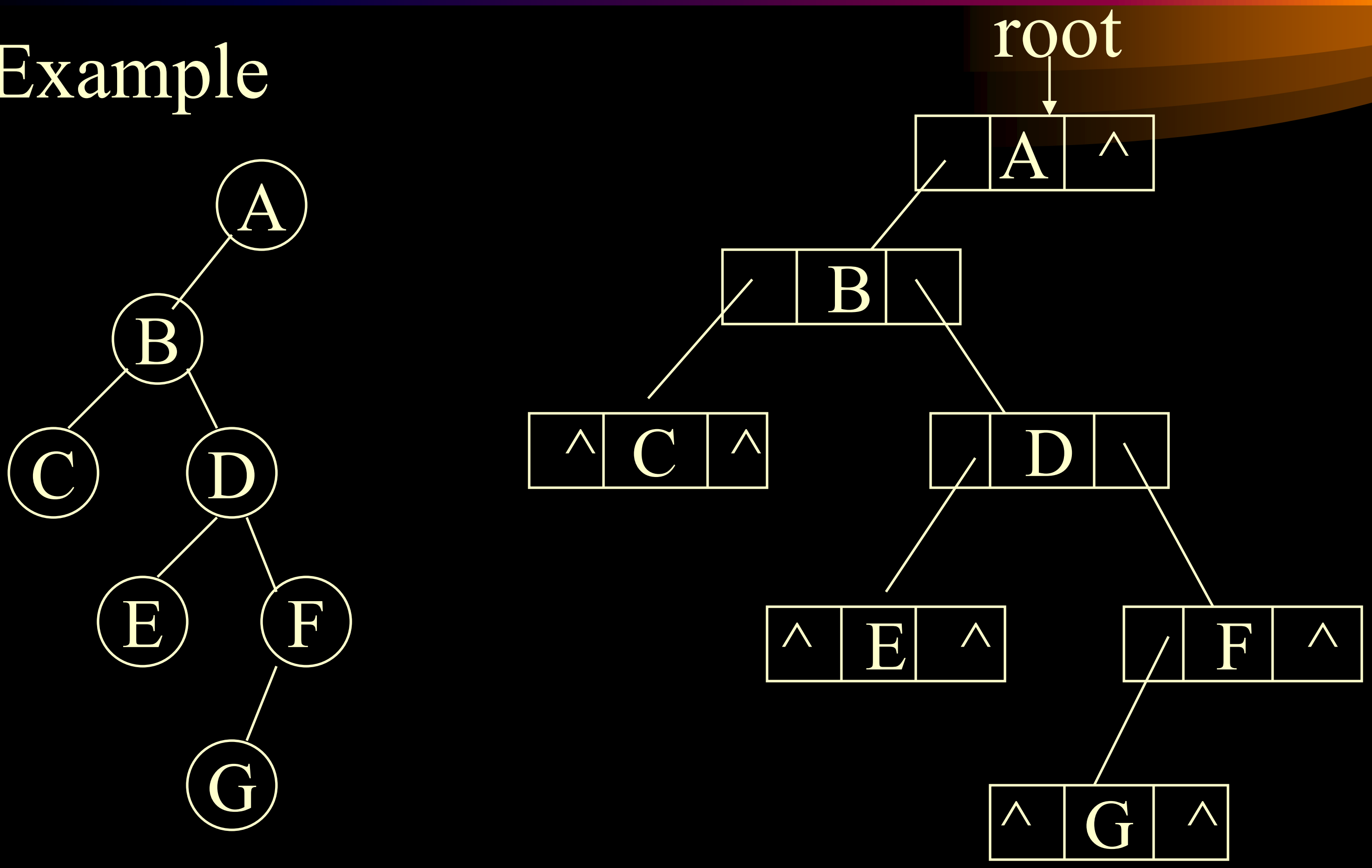
2. Linked representation(also called L-R linked storage)

The node structure:

LeftChild	data	RightChild

4.4 Representation of binary tree

Example



4.4 Representation of binary tree

3. Represented by Cursor

	data	leftchild	rightchild
0	A	1	-1
1	B	2	3
2	C	-1	-1
3	D	4	5
4	E	-1	6
5	F	-1	-1
6	G	-1	-1

4.4 Representation of binary tree

Node class for linked binary trees

```
class BianryNode
{
    BinaryNode(){Left=Right=0;}
    BinaryNode(Object e)
            {element=e; Left=Right=0;}
    BinaryNode(Object e,  BinaryNode l, BinaryNode r)
            {element=e;  Left=l;  Right=r; }

     Object element;
     BinaryNode  left;    //left subtree
     BinaryNode  right;   //right subtree
};
```

4.5 Common binary tree operations

the abstract data type binary tree

- Create()
- IsEmpty()
- Root(x)
- MakeTree(root, left, right)
- BreakTree(root, left, right)
- PreOrder
- InOrder
- PostOrder
- LevelOrder

4.5 Common binary tree operations

The Class BinaryTree

1.Binary tree class

```
template<class T>class BinaryTree
{  public:
       BinaryTree(){root=0;};
       ~BinaryTree(){};
       bool IsEmpty()const
           {return ((root)?false:true);}
       bool Root(T& x)const;
       void MakeTree(const T& data,
             BinaryTree<T>& leftch, BinaryTree<T>& rightch);
```

4.5 Common binary tree operations

```
void BreakTree(T& data , BinaryTree<T>& leftch,
        BinaryTree<T>& rightch);
void PreOrder(void(*visit)(BinaryNode<T>*u))
        {PreOrder(visit, root);}
void InOrder(void(*visit)(BinaryNode<T>*u))
        {InOrder(visit, root);}
void PostOrder (void(*visit)(BinaryNode<T>*u))
        {PostOrder(visit, root);}
void LevelOrder
        (void(*visit)(BinaryNode<T> *u));
```

4.5 Common binary tree operations

```
private:
    BinaryNode<T>* root;
    void PreOrder(void(*visit)(BinaryNode<T> *u),
                                    BinaryNode<T>*t);
    void InOrder(void(*visit)(BinaryNode<T> *u),
                                    BinaryNode<T>*t);
    void PostOrder(void(*visit) (BinaryNode<T> *u),
                                    BinaryNode<T>*t);
};
```

4.5 Common binary tree operations

- In this class ,we employ a linked representation for binary trees.
- The function visit is used as parameter to the traversal methods,so that different operations can be implemented easily

4.5 Common binary tree operations

2.Implementation of some member functions

```
Template<class T>
  void BinaryTree<T>::MakeTree(const T& data,
          BinaryTree<T>& leftch,   BinaryTree<T>& rightch)
  {  root=new  BinaryNode<T>(data, leftch.root,
                                          rightch.root);
     leftch.root=rightch.root=0;
  }
```

4.5 Common binary tree operations

```
template<class T>
void BinaryTree<T>::BreakTree(T& data,
      BinaryTree<T>& leftch,  BinaryTree<T>& rightch)
{  if(!root)throw BadInput(); //tree empty
   data=root. element;
   leftch.root=root. Left;
   rightch.root=root. Right;
   delete root;
   root=0;
}
```

4.5 Common binary tree operations

3. Application of class BinaryTree(Create BinaryTree)

```
#include<iostream.h>
#include "binary.h"
int count=0;    BinaryTree<int>a,x,y,z;

template<class T>
void ct(BinaryTreeNode<T>*t){count++;}

void main(void)
{  a.MakeTree(1,0,0);
   z.MakeTree(2,0,0);
   x.MakeTree(3,a,z);
   y.MakeTree(4,x,0);
   y.PreOrder(ct);
   cout<<count<<endl;
}
```

4.6 Binary Tree Traversal

Each element is visited exactly once

V-----表示访问一个结点

L------表示访问V的左子树

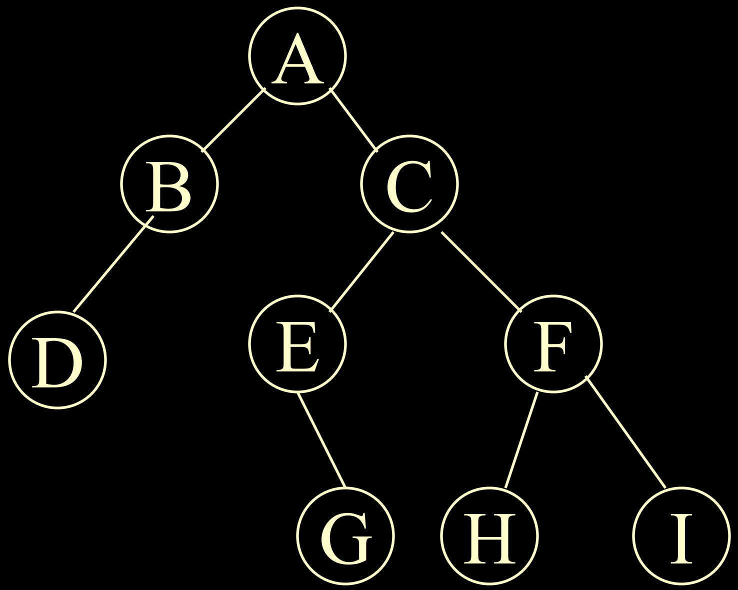
R------表示访问V的右子树

VLR LVR LRV VRL RVL RLV

- Preorder
- Inorder
- Postorder
- Level order

4.6 Binary Tree Traversal

Example of binary tree traversal



Preorder :ABDCEGFHI

Inorder : DBAEGCHFI

Postorder :DBGEHIFCA

Level order: ABCDEFGHI

4.6 Binary Tree Traversal

Preorder traversal

```
template<class T>
void PreOrder(BinaryNode<T>* t)
{// preorder traversal of *t.
   if(t){   visit(t);
            PreOrder(t→Left);
            PreOrder(t→Right);
        }
}
```

4.6 Binary Tree Traversal

Inorder traversal

```
template<class T>
void InOrder(BinaryNode<T>* t)
{ if(t){  InOrder(t→Left);
          visit(t);
          InOrder(t→Right);
        }
}
```

4.6 Binary Tree Traversal

Postorder traversal

```
template<class T>
void PostOrder(BinaryNode<T>* t)
{ if(t){
        PostOrder(t→Left);
        PostOrder(t→Right);
        visit(t);
    }
}
```

4.6 Binary Tree Traversal

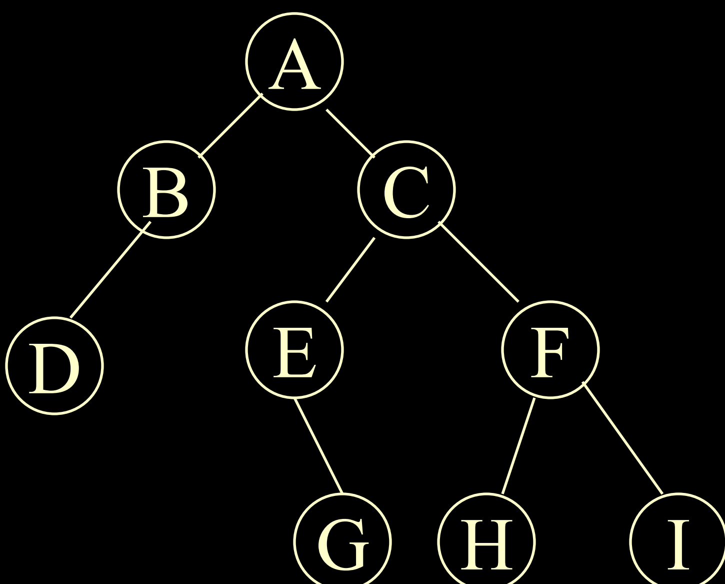
Example:

Inorder traversal:

4.6 Binary Tree Traversal

Level order

it is a non-recursive function and a queue is used.



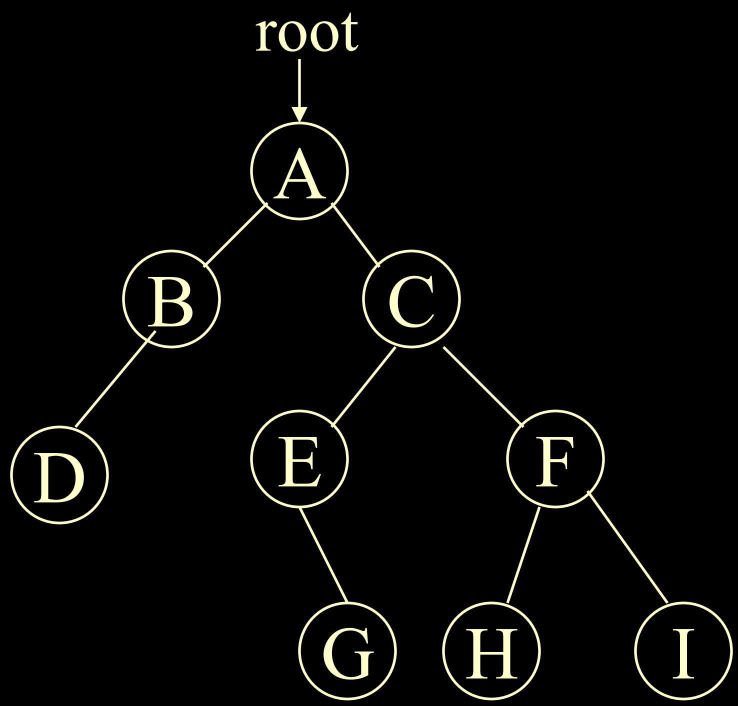
4.6 Binary Tree Traversal

Level order

```
 template<class T>
 void LevelOrder(BinaryNode<T>* t)
{  LinkedQueue<BinaryNode<T>*> Q;
   while(t){
       visit(t);    //visit t
        if(t→Left) Q.Add(t→Left);
        if(t→Right) Q.Add(t→Right);
        try{Q.Delete(t);}
        catch(OutOfBounds){return;}
   }
}
```

Inorder, Postorder non-recursive algorithm

- Inorder non-recursive algorithm

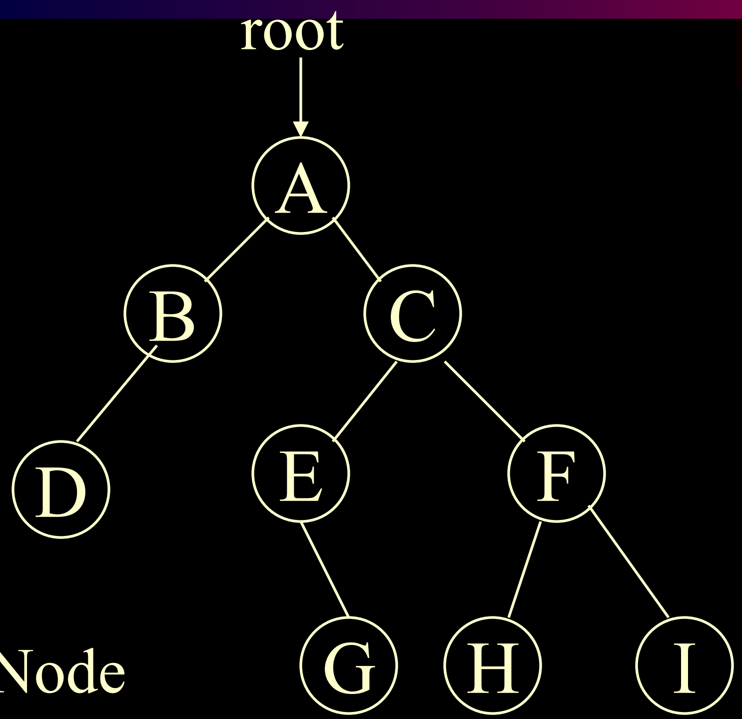


Inorder non-recursive algorithm

```
void Inorder(BinaryNode <T> * t)
{  Stack<BinaryNode<T>*> s(10);
   BinaryNode<T> * p = t;
   for (  ;  ;  )
   { 1) while(p!=NULL)
          { s.push(p);   p = p->Left; }
     2)  if (!s.IsEmpty( ))
          {  p = s.pop( );
             cout << p->element;
             p = p->Right;
          }
          else return;
   }
}
```

Inorder, Postorder non-recursive algorithm

- Postorder non-recursive algorithm




```
struct StkNode
    { BinaryNode <T> * ptr;
      int tag;
    }
```

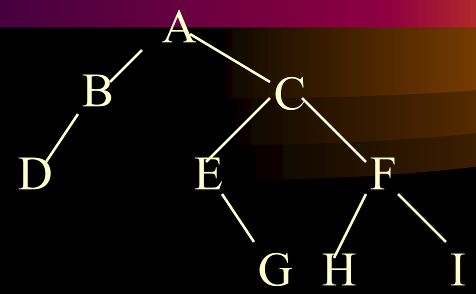
Postorder non-recursive algorithm

```
void Postorder(BinaryNode <T> * t)
{  Stack <StkNode<T>>s(10);
   StkNode<T> Cnode;
   BinaryNode<T> * p = t;
   for(  ;  ;  )
   {  1)while (p!=NULL)
         { Cnode.ptr = p;   Cnode.tag = 0;  s.push(Cnode);
           p = p->Left;
         }
      2)Cnode = s.pop( );   p = Cnode.ptr;
      3)while ( Cnode.tag = = 1)   //从右子树回来
           {  cout << p->element;
              if ( !s.IsEmpty( ))
                { Cnode = s.pop( );   p = Cnode.ptr;  }
              else  return;
           }
      4)Cnode.tag = 1;  s.push(Cnode);    p = p->Right;    //从左子树回来
   }//for
}
```

建立一棵二叉树的方法

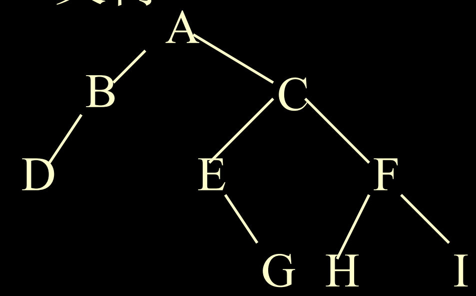
1. 利用MakeTree函数
2. 利用先序、中序唯一的构造一棵二叉树

先序：ABDCEGFHI
中序：DBAEGCHFI



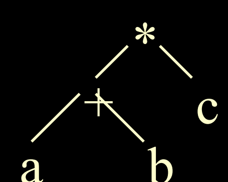
*3. 利用二叉树的广义表表示来构造一棵二叉树

A(B(D), C(E(,G), F(H,I)))



4. 利用二叉树的后缀表示来构造一棵二叉树

(a+b)*c ⟶ ab+c* ⟶



建立一棵二叉树的方法

4. 利用二叉树的后缀表示来构造一棵二叉树(习题）

$(a+b)*c \longrightarrow a\,b+\,c*$

$(\neg a+b)*c \longrightarrow a\neg b+c*$

$d+(-a+b)*c \longrightarrow d\,a\neg b+c*+$

2. 利用先序、中序唯一的构造一棵二叉树 string

1. 字符串的有关概念
   串的定义、术语、基本操作
2. 字符串的类说明
3. 部分成员函数的实现
4. 利用前序、中序序列建立一棵树

string

1.字符串（简称串）的定义以及一些术语

***串：是n（n>=0）个字符的一个有限序列，开头结尾用双引号“ ”括起来。**

例如： B=“ structure ” （B为串名）

***串的长度：串中所包含的字符个数n（不包括分界符‘ “ ’，也不包括串的结束符‘\0’）**

***空串：长度为0的串。或者说只包含串结束符‘\0’的串**

注意： “ \0 ” 不等于 “ \0 ” , 空串不等于空白串

***子串：串中任一连续子序列**

例子：B= “ peking ” 则 空串“ ” 、 “ ki ” 、 “ peking ” 都是B的子串

但“ pk ” 不是B的子串

string

*串的基本操作：

构造一个空串；

求串长；

两个串的连接（并置）；

取子串；

求一个子串在串中第一次出现的位置等。

string

Java与C/C++的不同处：

Java语言的字符串不是字符数组，所以不能以字符数组方式进行一些操作。 如， str[1] = “a” 是错误的，而只能通过方法（函数）来进行操作。

int length( )

boolean equals(Object obj)

char charAt(int index)

String substring (int beginIndex)

String substring (int beginIndex, int endIndex)

string

2. 字符串的类说明

```
const int maxlen=128;
class String
{ public:
    String(const String & ob);
    String(const char * init);
    String( );
    ~String( ) {delete[ ] ch;}
    int Length( )const {return curlen;}
    String & operator( )(int pos, int len);    //取子串
    int operator = =(const String & ob) const
        { return strcmp(ch, ob.ch)= =0;}    //判别相等否？
    int operator !=(const String &ob) const
        {  return strcmp(ch, ob.ch)!=0;}
    int operator ! ( ) const {return curlen= =0;}
```

string

```
    String & operator = (const String & ob);  //串赋值
    String & operator +=(const String & ob);  //并置运算
    char & operator[ ](int i);
    int Find(String pat) const;
  private:
     int curLen;
     char * ch;
}
```

string

3. 部分成员函数的实现

Taking Substring

```
String & String: :operator( )(int  pos, int len)
{   String *temp=new String;

    if (pos<0||pos+len-1>=maxlen ||len<0)
    {    temp->curLen=0; temp->ch[0]='\0';}

    else{  if (pos+len-1>=curLen) len=curLen-pos;

              temp->curLen=len;
              for(int i=0, j=pos; i<len; i++, j++)
                     temp->ch[i]=ch[j];
              temp->ch[len]='\0';
          }
     return *temp;
}
```

string

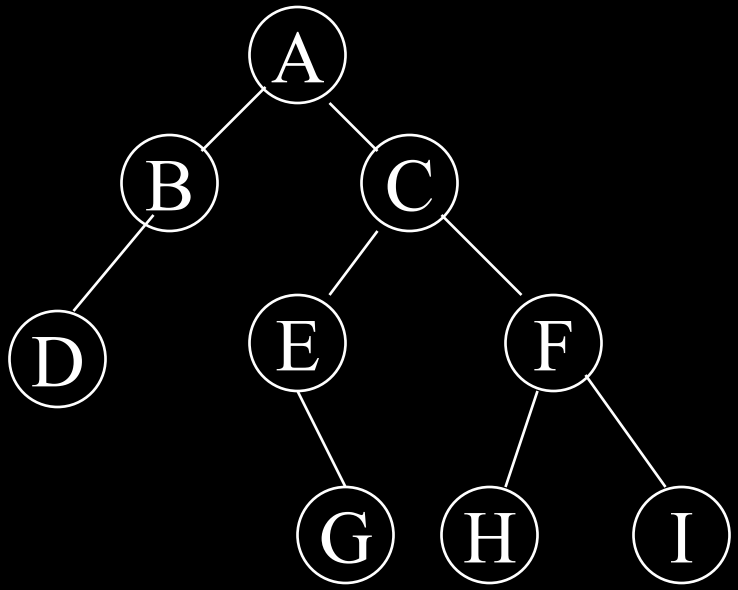
Assigning Operate

```
String & String ::operator=(const String &ob)
{ if (&ob!=this)
    {delete [ ] ch;
      ch=new char[maxLen+1];
      if(!ch) {cerr<< "Out Of Memory! \n";exit(1);}
      curLen=ob.curLen;
      strcpy(ch, ob.ch);
    }
  else cout<<"Attempted assignment of a String to itself!
   \n";
  return * this;
}
```

4.Create BinaryTree recursive algorithm

preorder:ABDCEGFHI

inorder: DBAEGCHFI



Create BinaryTree recursive algorithm 1

```
void CreateBT(String pres, ins ; BinaryNode <Type>* & t)
{   int inpos;
    String  prestemp, instemp ;
    if (pres.length( )= =0) t=NULL;

   else {  t=new BinaryNode;
           t->element=pres.ch[0];    inpos=0;
           while (ins.ch[inpos]!=t->element) inpos++;

           prestemp=pres(1,inpos);
           instemp=ins(0,inpos-1);
           CreateBT(prestemp, instemp, t->left);

           prestemp=pres(inpos+1, pres.length( )-1);
           instemp=ins(inpos+1, pres.length( )-1);
           CreateBT(prestemp, instemp, t->right);
        }
}
```

Create BinaryTree recursive algorithm 1

```
public:
    BinaryTree( string pre, string In )
    {   createBT( pre, In, root ) ;
    }
        .........
main( )
{   BinaryTree  t1( “ABHFDECKG”, “ HBDFAEKCG” ) ;
      .......
}
```

Create BinaryTree recursive algorithm 2

```
BinaryNode<Type>  * void CreateBT (String pres, ins )
{
    int inpos;   BinaryNode <Type>* temp;
    String  prestemp, instemp ;
    if (pres.length( )= =0) return NULL;
    else {   temp=new BinaryNode;
             temp->element=pres.ch[0];   inpos=0;
             while (ins.ch[inpos]!=temp->element) inpos++;

             prestemp=pres(1,inpos);
             instemp=ins(0,inpos-1);
             temp->left = CreateBT(prestemp, instemp);

             prestemp=pres(inpos+1, pres.length( )-1);
             instemp=ins(inpos+1, pres.length( )-1);
             temp->right = CreateBT(prestemp, instemp);
             return temp;
         }
}
```

Create BinaryTree recursive algorithm 2

```
public :
    BinaryTree(string pre,  string In)
   {  root = createBT( pre, In);
   }
         …….
 main( )
 {  string s1("ABHFDECKG");
    string s2("HBDFAEKCG");
    BinaryTree t1( s1, s2 );
     ……
    preorder(t1);  Inorder(t1);
    postorder(t1);
     ……..
 }
```

如果已知后序与中序，能否唯一构造一棵二叉树呢?

后序：　**DBGEHIFCA**

中序：　**DBAEGCHFI**

如果已知先序与后序呢?

4.7 ADT and Class Extensions

- **PreOutput():output the data fields in preorder**
- **InOutput():output the data fields in inorder**
- **PostOutput():output the data fields in postorder**
- **LevelOutput():output the data fields in level order**
- **Delete():delete a binary tree,freeing up its nodes**
- **Height():return the tree height**
- **Size():return the number of nodes in the tree**

4.7 ADT and Class Extensions

The height of the tree is determined as:

max{hl, hr}+1

```
template<class T>
 int BinaryTree<T>::Height(BinaryNode<T> *t)const
{  if(!t) return 0;
   int hl=Height(t→Left);
   int hr=Height(t→Right);
   if(hl>hr)return ++ hl;
   else return ++hr;
}
```

4.8 Application

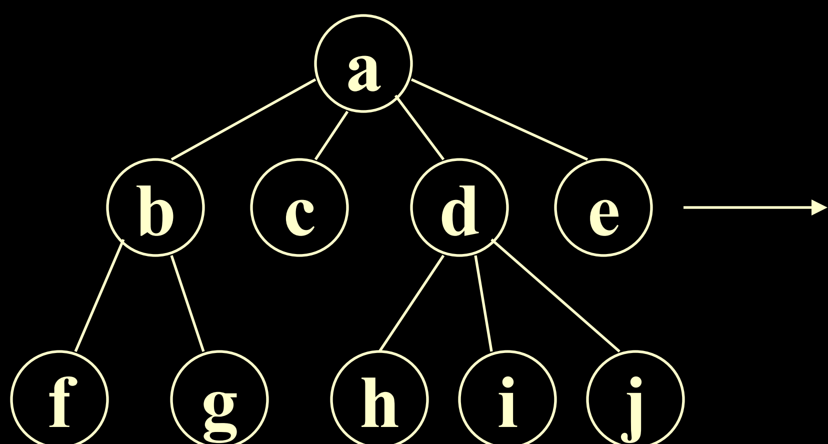
1.Binary-Tree Representation of a Tree

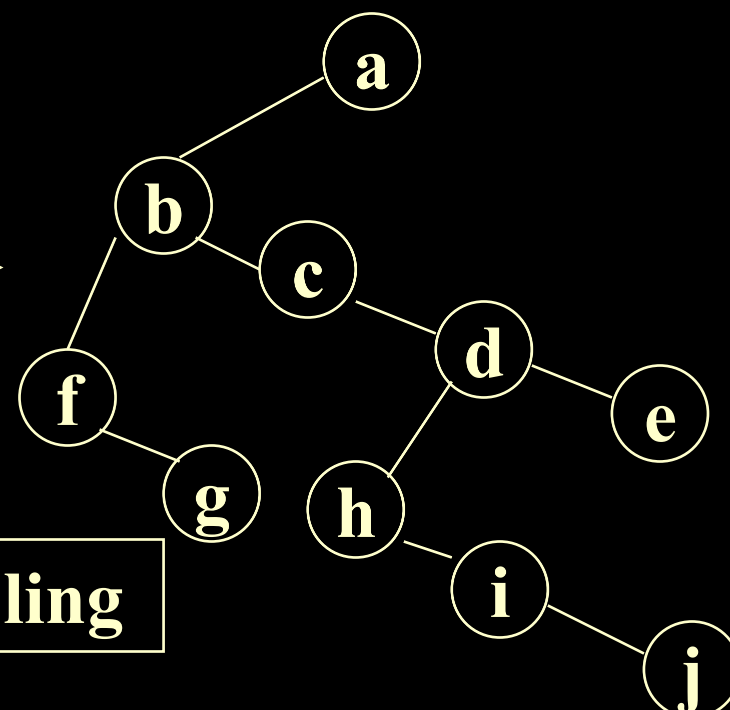
树的存储方式：三种

- 广义表表示：a(b(f,g),c,d(h,i,j),e)
- 双亲表示法
- 左子女—右兄弟表示法

	a	b	f	g	……….		
	0	1	2	2			

1) Take a tree as a binary tree





firstchild	data	nextsibling

4.8 Application

```
class TreeNode:
    T data;
    TreeNode *firstchild, *nextsibling;
class Tree:
    TreeNode * root,  *current;
```

4.8 Application

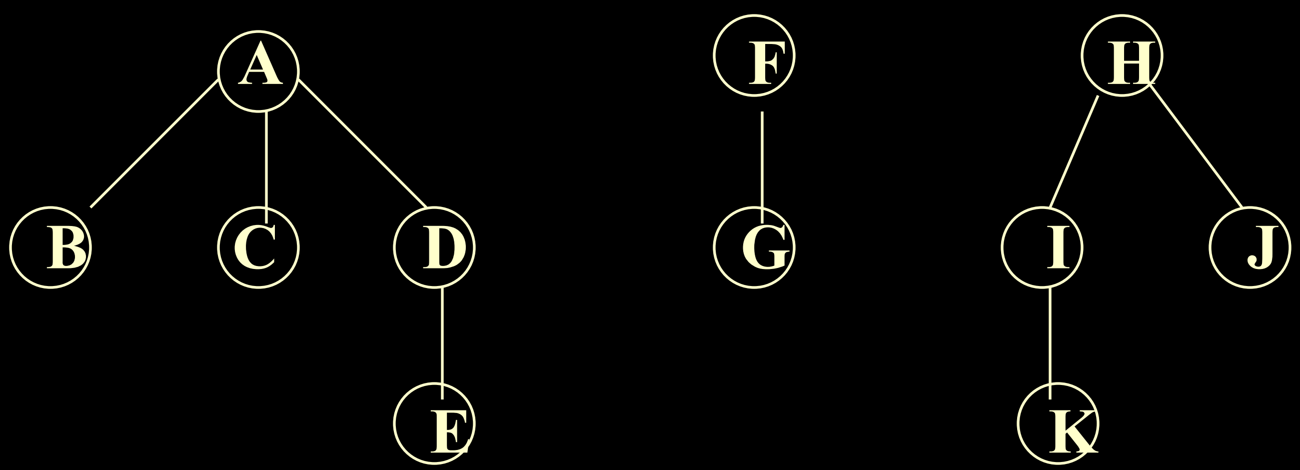
insert new node in tree

```
template <class T> void Tree <T>::Insertchild(T value)
{  TreeNode<T>*newnode = new TreeNode<T>(value);
    if(current->firstchild = = NULL)
                    current->firstchild = newnode;
   else
      {  TreeNode<T>*p = current->firstchild;
          while ( p->nextsibling!=NULL) p = p->nextsibling;
          p->nextsibling = newnode;
      }
}
```

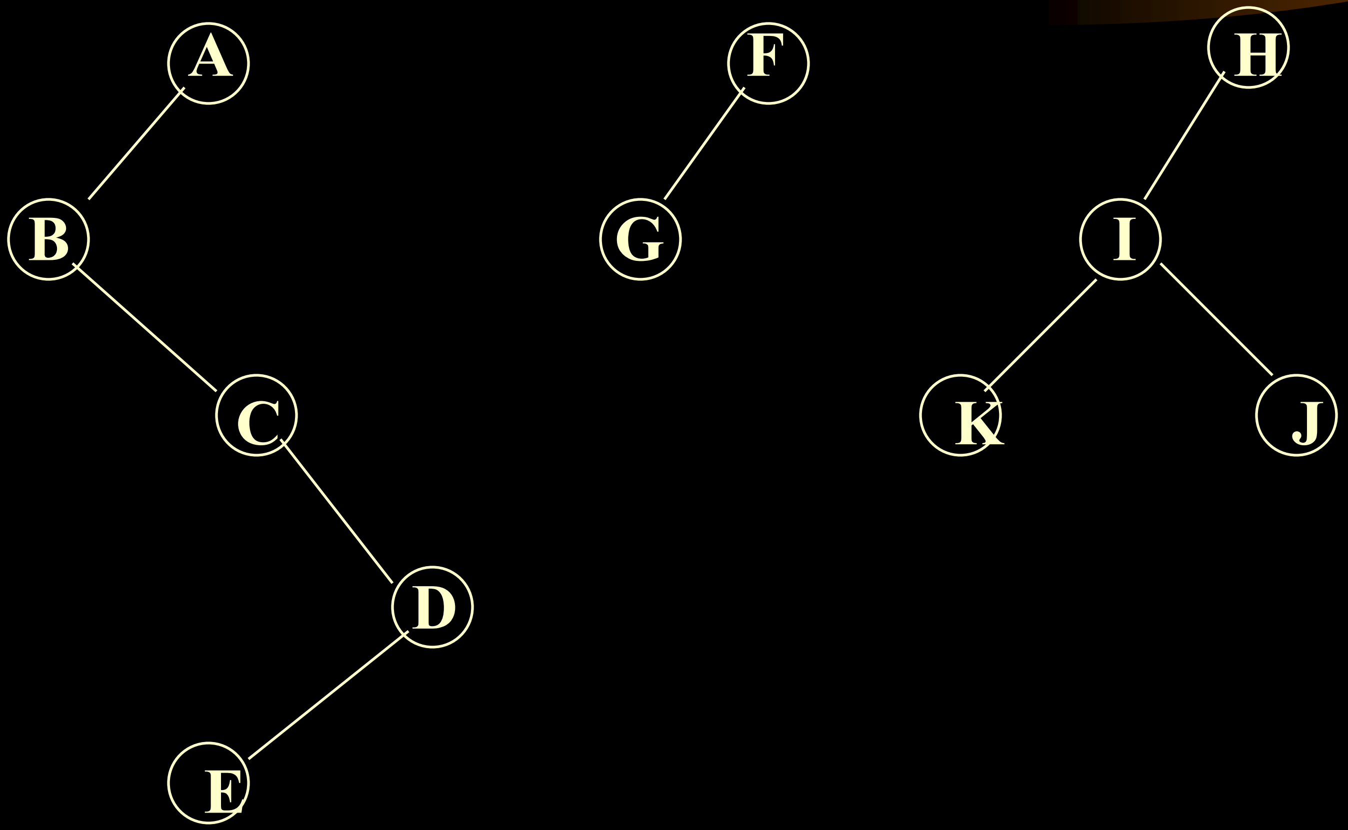
4.8 Application

2) Forest ⟷ Binary tree

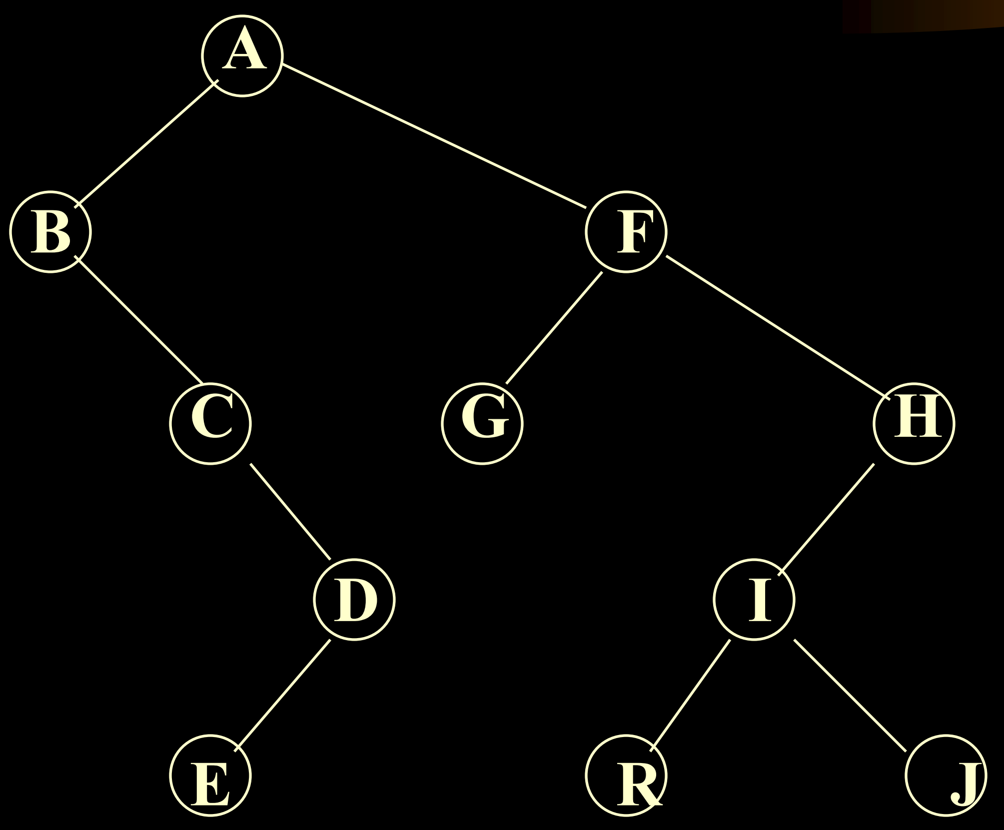
- Forest ⟶ Binary tree



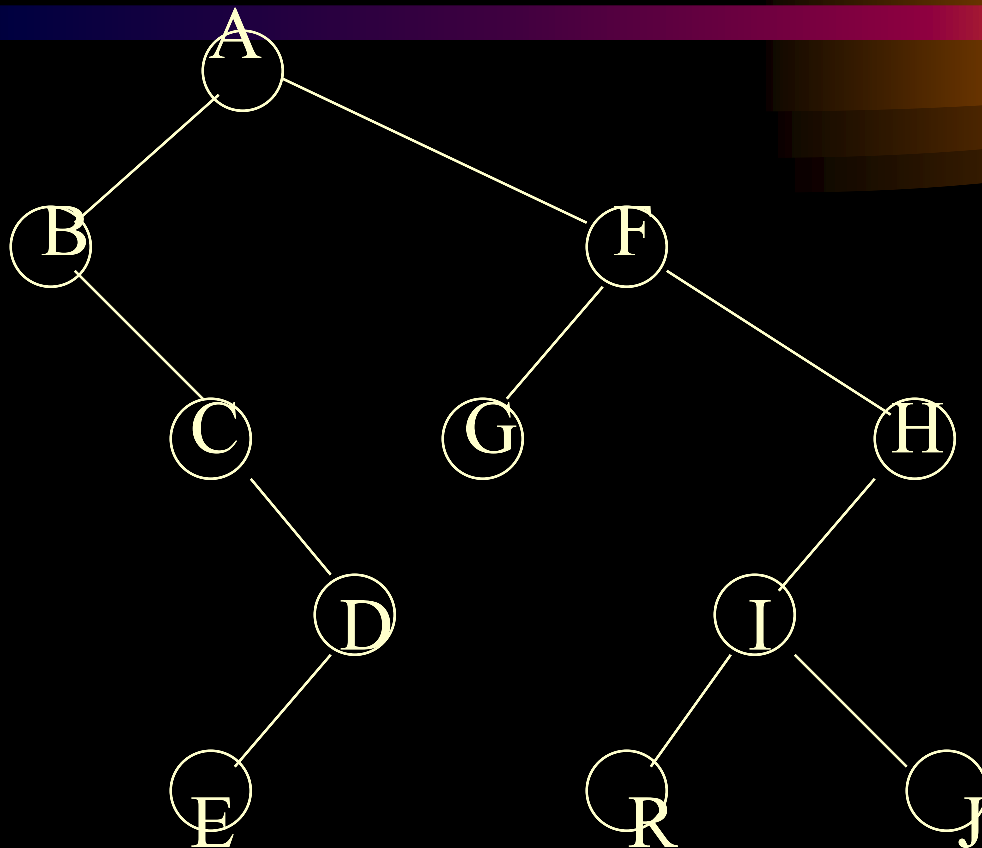
每棵树转为二叉树



把每棵二叉树根用右链相连



- **Binary tree ⟶ Forest**



4.8 Application

3) 树的遍历：深度优先遍历，广度优先遍历

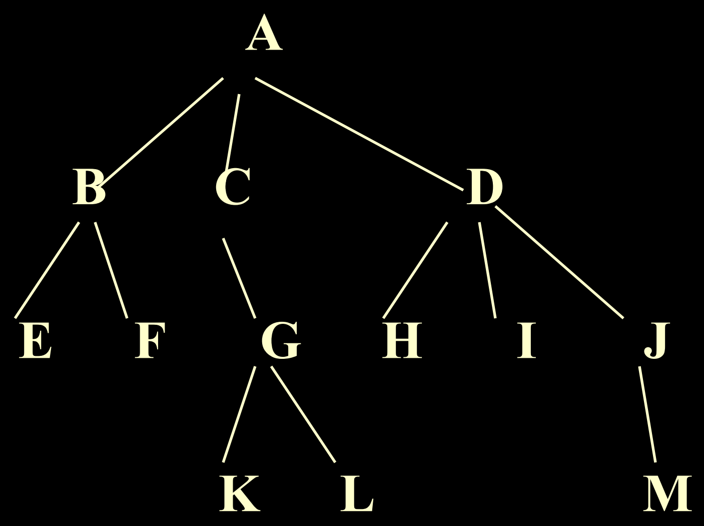
- 深度优先遍历

先序次序遍历（先序）

访问树的根 → 按先序遍历根的第一棵子树，第二棵子树，……等。

后序次序遍历（后序）

按后序遍历根的第一棵子树，第二棵子树，……等 → 访问树的根。

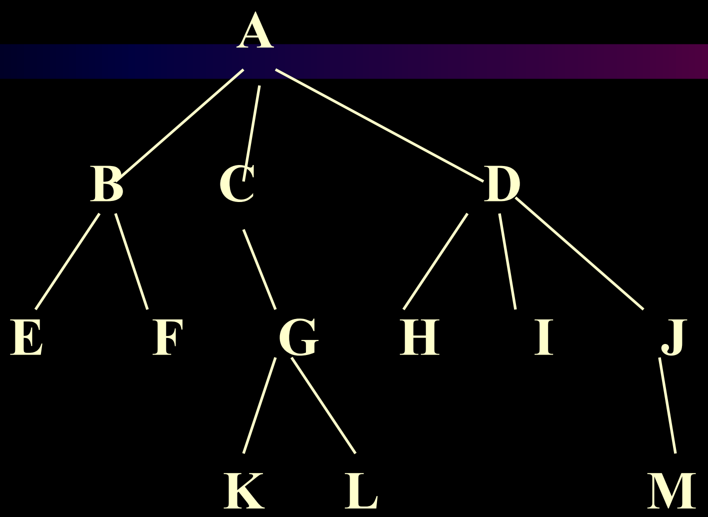


先根：ABEFCGKLDHIJM与对应的二叉树的先序一致

后根：EFBKLGCHIMJDA与对应的二叉树的中序一致

4.8 Application

- 广度优先遍历



分层访问：ABCDEFGHIJKLM

4) 森林的遍历

深度优先遍历

* 先根次序遍历

访问F的第一棵树的根
按先根遍历第一棵树的子树森林
按先根遍历其它树组成的森林

二叉树的先序

* 中根次序遍历

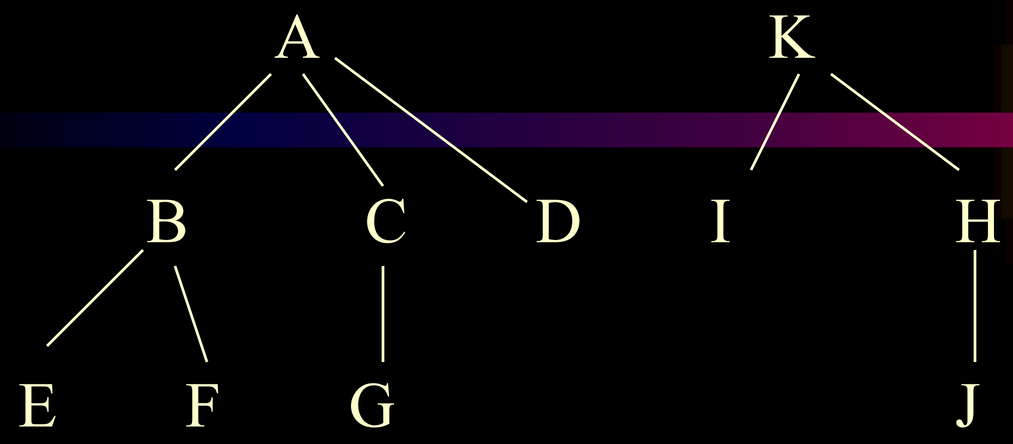
按中根遍历第一棵树的子树森林
访问F的第一棵树的根
按中根遍历其它树组成的森林

二叉树的中序

* 后根次序遍历

按后根遍历第一棵树的子树森林
按后根遍历其它树组成的森林
访问F的第一棵树的根

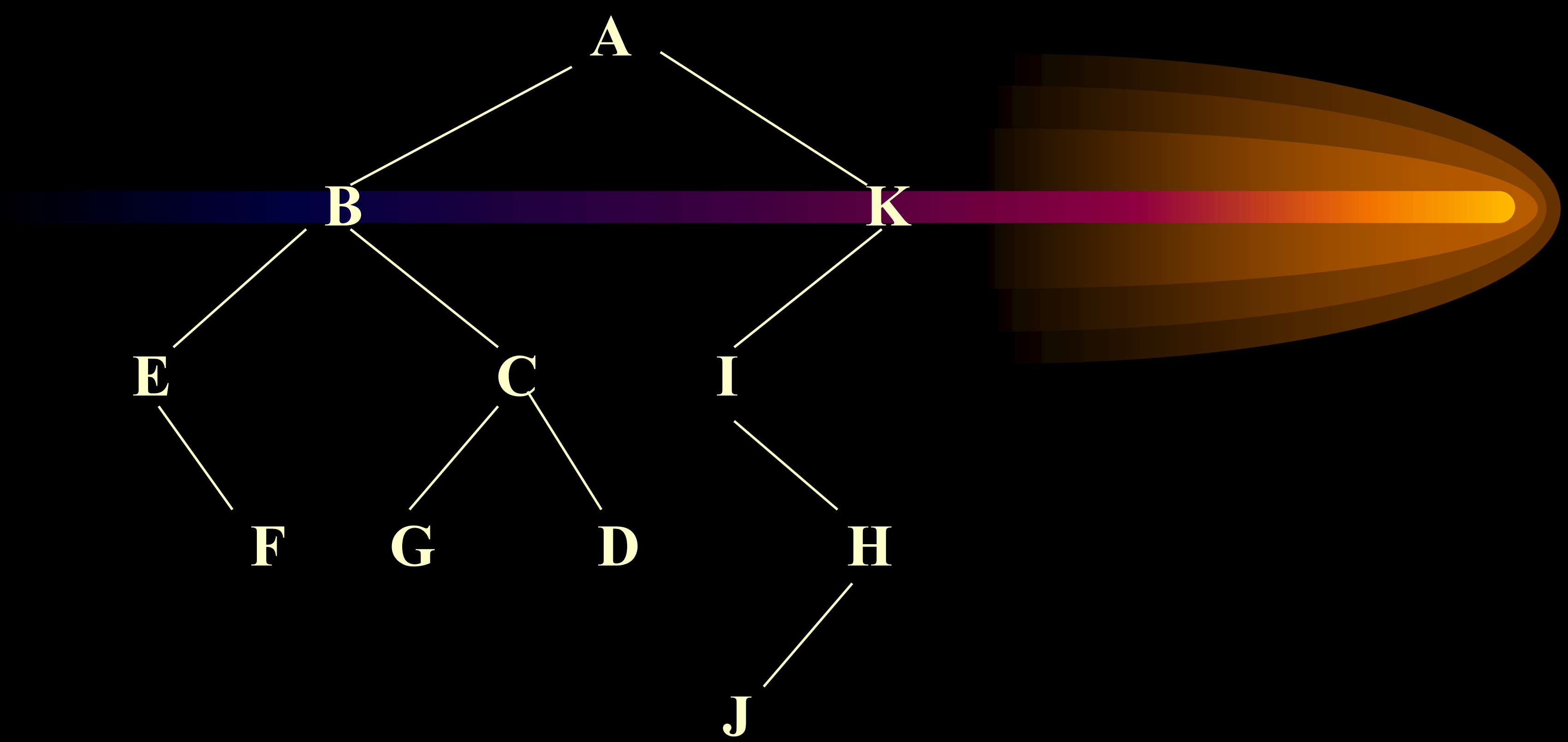
二叉树的后序



先根：ABEFCGDKIHJ

中根：EFBGCDAIJHK

后根：FEGDCBJHIKA



后序: FEGDCBJHIKA

广度优先遍历（层次遍历）

AKBCDIHEFGJ

Thread Tree

1.Purpose:

2. Thread Tree Representation

left Thread Tree and right Thread Tree

3.Thread Tree class

1.Purpose:

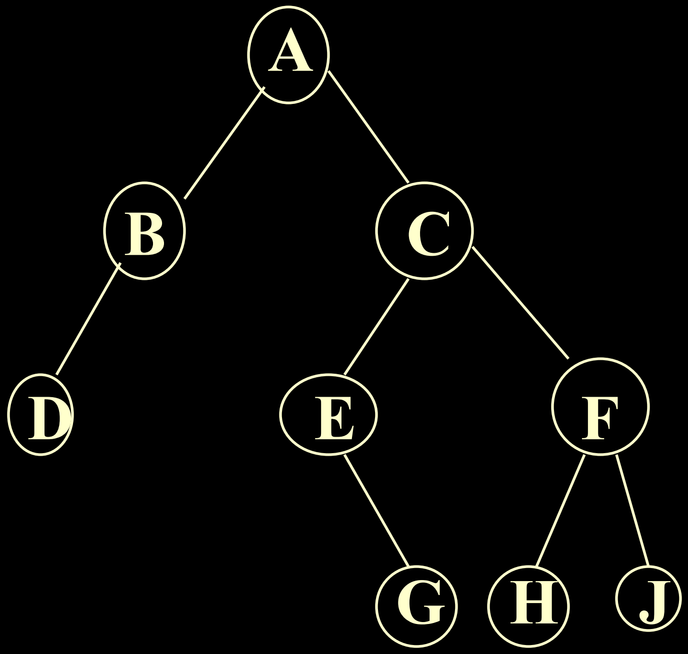
n个结点的二叉树有2n个链域，

其中真正有用的是n – 1个，其它n + 1个都是空域。

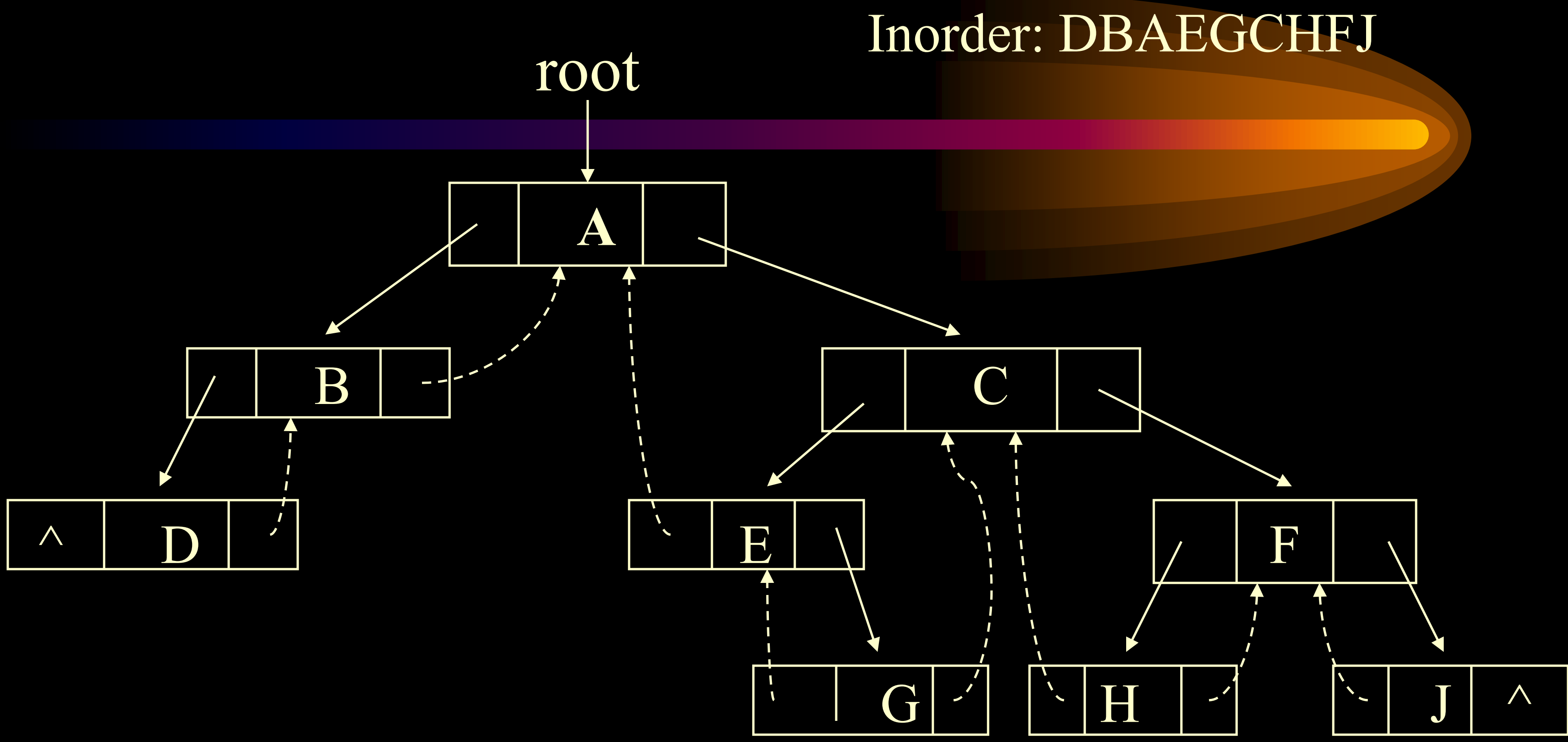
为了充分利用结点中的空域，使得对某些运算更快，如前驱或后继等运算。

Thread Tree

Example:



Thread Tree



2. 机内如何存储

一个结点增加两个标记域:

leftchild	leftthread	data	rightthread	rightchild

leftThread = = { 0 leftchild 指向左子女; 1 leftchild 指向前驱（某线性序列） }

rightThread = = { 0 rightchild 指向右子女; 1 rightchild 指向后继 }

3. 线索化二叉树的类声明。

```
template< class Type> class ThreadNode
   {
        friend class ThreadTree;
        private:
            int leftThread, rightThread;
            ThreadNode<Type>* leftchild, *rightchild;
            Type data;
        public:
            ThreadNode(const Type item): data(item), leftchild(0),
                    rihgtchild(0), leftThread(0), rihgtThread(0) { }
   };
```

```
template< class Type> class ThreadTree
{
    public:
        //  线索二叉树的公共操作
    private:
        ThreadNode<Type> * root;
        ThreadNode<Type> *current
};
```

讨论线索化二叉树的几个算法

1) 按中序遍历中序线索树

遍历算法（以中序为例）：

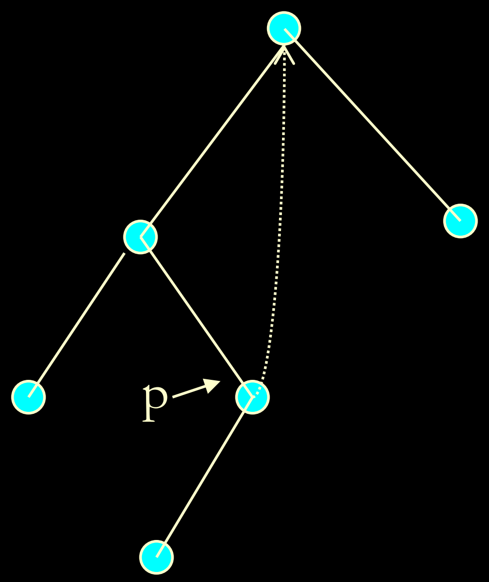
递归，非递归（需用工作栈）。

这里前提是中序线索树，所以既不要递归，也不要栈。

遍历算法：* 找到中序下的第一个结点（first）

* 不断找后继（Next）

如何找后继？

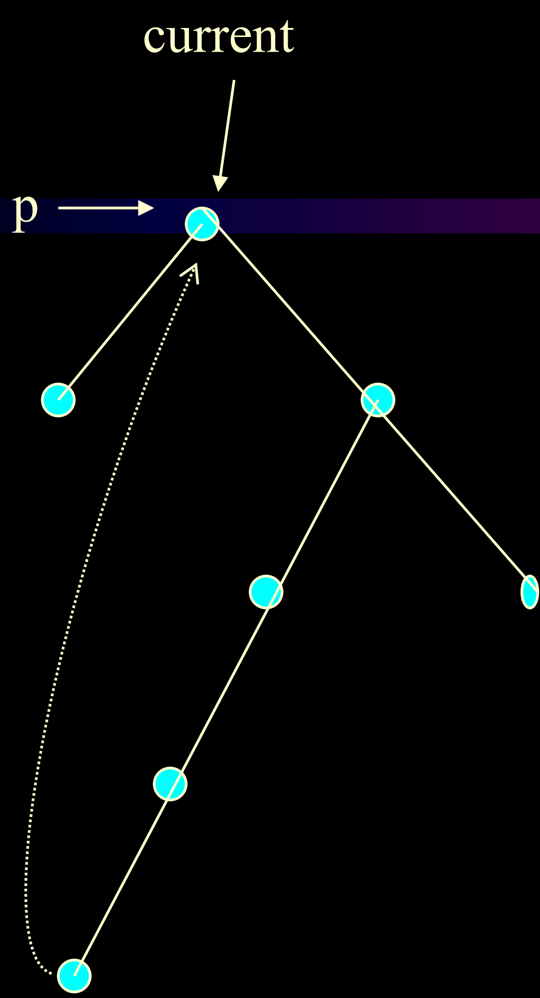


p指结点没有右子树

（p->rightthread= =1）

则p=p->rightchild

（右链就是后继）



p有右子树

（p->rightThread= =0)

则 (1) p=p->rightchild

(2) while(p->leftThread==0)

p=p->leftchild;

```
template<class Type>
ThreadNode<Type>* ThreadInorderIterator<Type>::First( )
{
        while (current->leftThread= =0) current=current->leftchild;
        return current;
}

template<class Type>
ThreadNode<Type>* ThreadInorderIterator<Type>::Next( )
{
      ThreadNode<Type>* p=current->rightchild;
      if(current->rightThread= =0)
              while(p->leftThread= =0) p=p->leftchilld;
      current=p;
}
```

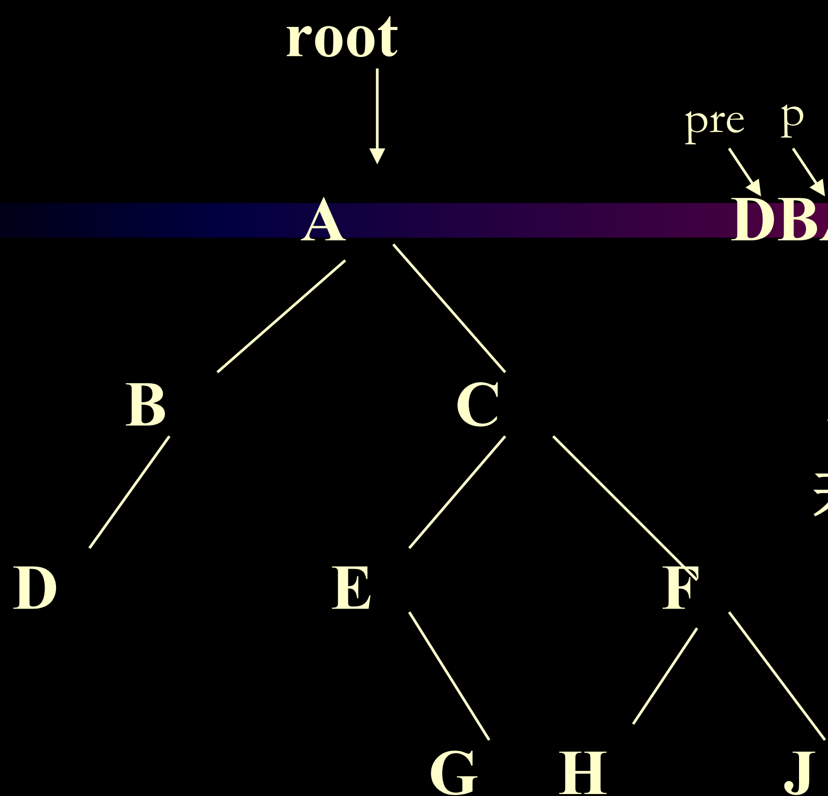
```
template<class Type>
 void ThreadInorderIterator<Type>:: Inorder( )
 {
     ThreadNode<Type> *p;
     for ( p=Frist( ); p!=NULL; p=Next( ))
          cout<< p->data<<endl;
  }
```

2) 构造中序线索树

对已存在的一棵二叉树建立中序线索树

分析： 与中序遍历算法差不多， 但是要填左空域右空域的前驱、后继指针。 所以除了流动指针p外，还要加一个pre指针，它总是指向遍历指针p的中序下的前驱结点。

例如：



即pre为p 的中序下的前驱，
若p的左链为空，则可填pre

即p为pre的中序下的后继
若pre的右链为空，则 可填p

Thread Tree

Create inorder threadTree:

```
Void Inthread(threadNode<T> * T)
{  stack <threadNode <T> *> s (10)
   ThreadNode <T>  * p = T ; ThreadNode <T> * pre = NULL;
   for (  ;  ;  )
    { 1.while (p!=NULL)
            { s.push(p); p = p ->leftchild; }
       2.if (!s.IsEmpty( ))
            {  1) p = s.pop;
               2) if (pre != NULL)
                     { if (pre ->rightchild = = NULL)
                          { pre ->rightchild = p;  pre ->rightthread = 1;}
                       if ( p -> leftchild = = NULL)
                          { p -> leftchild = pre ; p ->leftthread = 1;  }
                     }
               3) pre = p ;  p = p -> rightchild ;
            }
          else return;
    }//for
}
```

4.8 Application

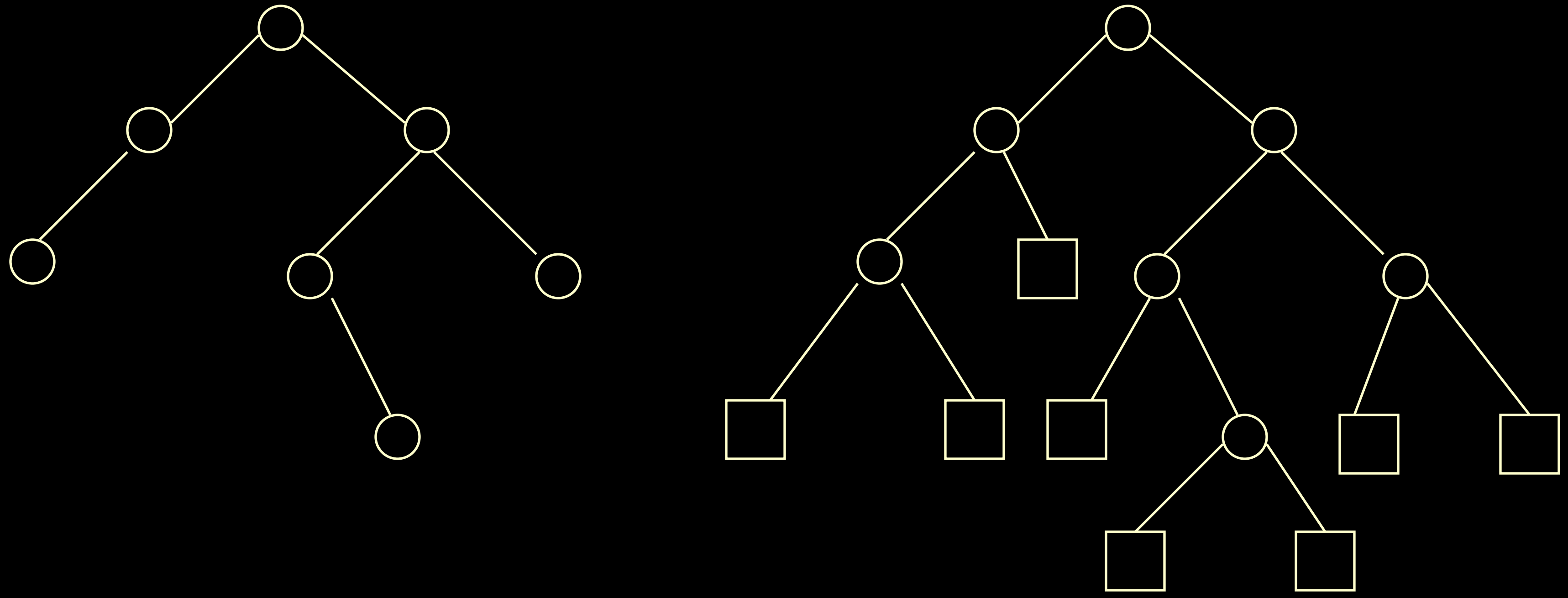
2. 霍夫曼树(Huffman Tree)

- 增长树的概念

1) 增长树

对原二叉树中度为1的结点， 增加一个空树叶

对原二叉树中的树叶， 增加两个空树叶



2) 外通路长度（外路径）E： 根到每个外结点（增长树的叶子）的路径长度的总和（边数）

$E=3+3+2+3+4+4+3+3=25$

3) 内通路长度（内路径）I： 根到每个内结点（非叶子）的路径长度的总和（边数）。

$I=2+1+0+3+2+2+1=11$

4) 结点的带权路径长度： 一个结点的权值与结点的路径长度的乘积。

5) 带权的外路径长度： 各叶结点的带权路径长度之和。

6)带权的内路径长度：各非叶结点的带权路径长度之和。

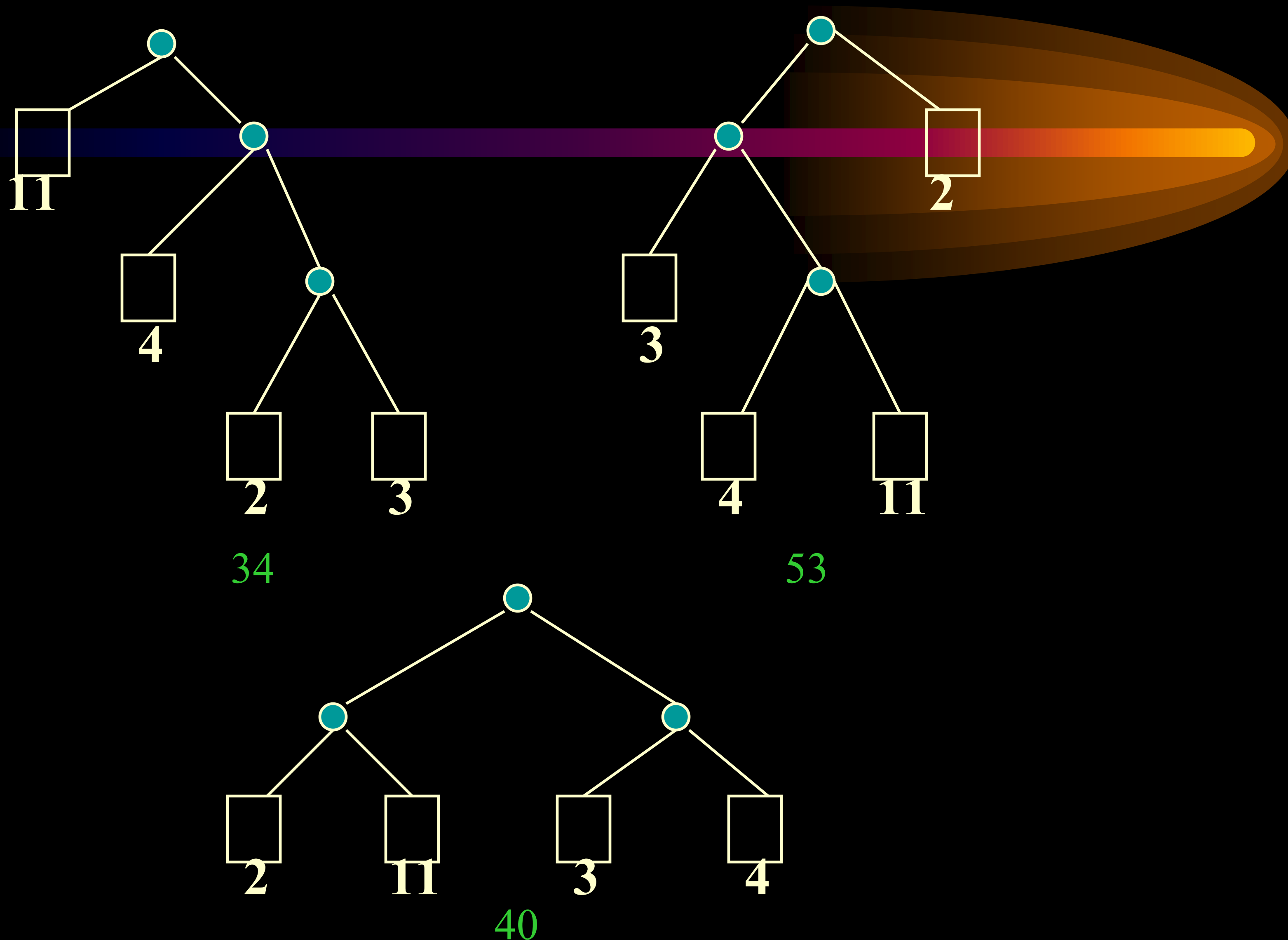
- 霍夫曼树

1) 给出m个实数W1，W2，…，Wm （m>=2） 作为m个外结点的权构造一棵增长树， 使得带权外路径长度

$$\sum_{i=1}^{m} W_i l_i \text{最小。}$$

其中l_i 为从根结点出发到具有权为w_i的外结点的通路长。

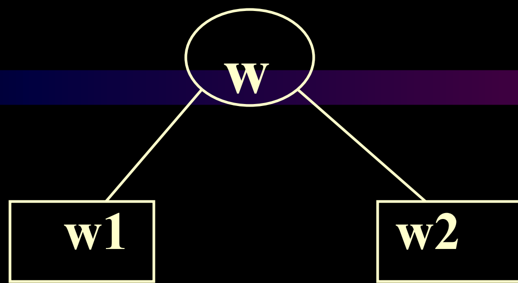
2) 例子： 外结点权为 2，3，4，11 则可构造



3) Huffman 算法

思想： 权大的外结点靠近根， 权小的远离根。

算法： 从m个权值中找出两个最小值W1，W2构成



W=W1+W2表通过该结点的频度。

然后对m-1个权值W， W3，W4，…Wm 经由小到大排序，求解这个问题。

例子：2， 3， 5， 7， 11， 13， 17， 19， 23， 29， 31， 37， 41

注意: 当内结点的权值与外结点的权值相等的情况下， 内结点应排在外结点之后。除了保证$\Sigma W_i l_i$最小外，还保证max I_j, ΣI_j也有最小值

例如：7, 8, 9, 15

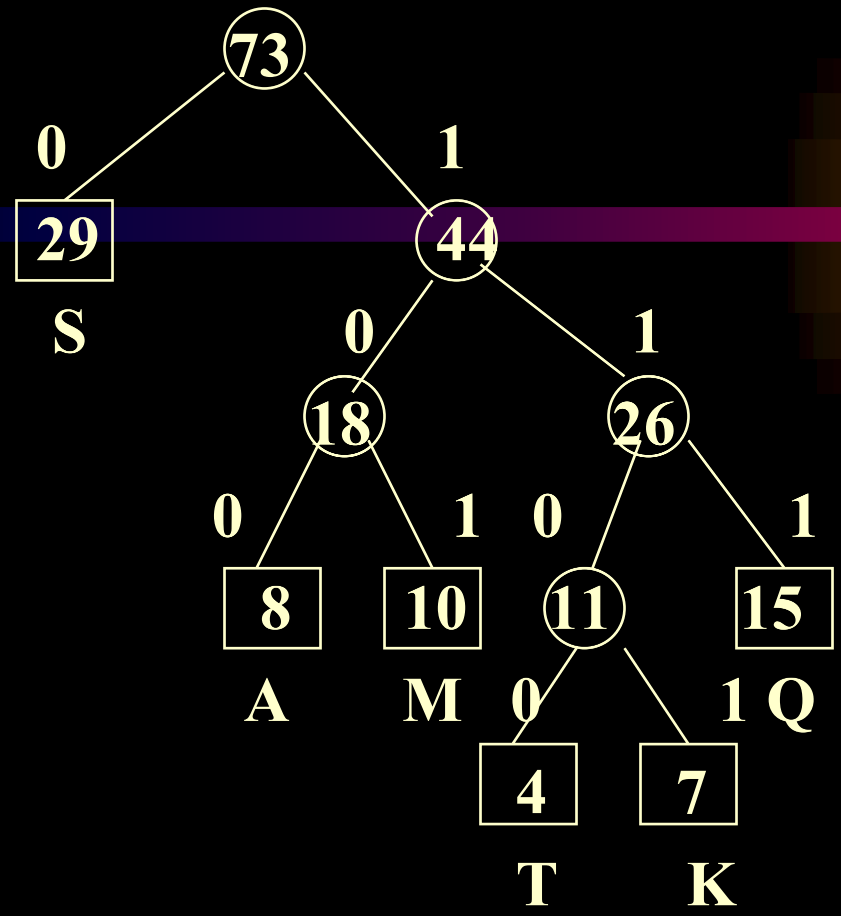
- 霍夫曼编码

是霍夫曼树在数据编码中一种应用。具体的讲用于通信的二进制编码中。设一电文出现的字符为D={M，S，T，A，Q，K}，每个字符出现的频率为W={10，29，4，8，15，7}，如何对上面的诸字符进行二进制编码，使得

1）该电文的总长度最短。

2）为了译码，任一字符的编码不应是另一字符的编码的前缀

算法：利用Huffman算法，把{10，29，4，8，15，7}作为外部结点的权，构造具有最小带权外路径长度的扩充二叉树，把每个结点的左子女的边标上0，右子女标上1。这样从根到每个叶子的路径上的号码连接起来，就是外结点的字符编码。



编码： S：0　A：100　M：101　Q：111

T：1100　K：1101

已知二进制编码， 请译码。

方法是从根结点开始确定一条到达叶结点的路径。

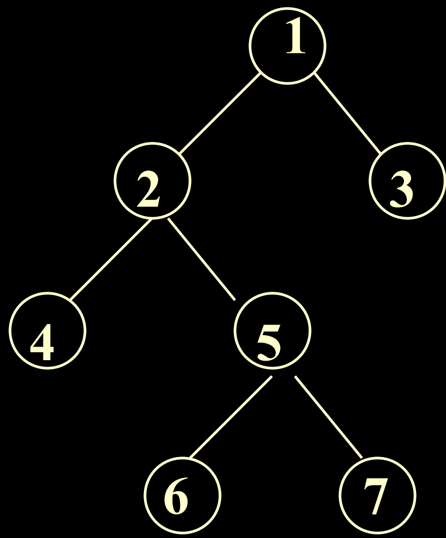
100	1100	101	0	111	0	1101	0	0
A	T	M	S	Q	S	K	S	S

Chapter 4

2009年统考题(单项选择):

3. 给定二叉树如下图所示. 设N 代表二叉树的根, L 代表二叉树的左子树, R 代表根结点的右子树. 若遍历后的结点序列为 3, 1, 7, 5, 6, 2, 4, 则其遍历方式是

A. LRN B. NRL C. RLN D. RNL



Chapter 4

2009年统考题(单项选择):

4. 已知一棵完全二叉树的第6 层(设根为第1层) 有8个叶结点，则该完全二叉树的结点个数最多是

A. 39　　B. 52　　C. 111　　D. 119

5. 将森林转换为对应的二叉树，若在二叉树中, 结点u是结点v的父结点的父结点，则在原来的森林中，u 和v 可能具有的关系是

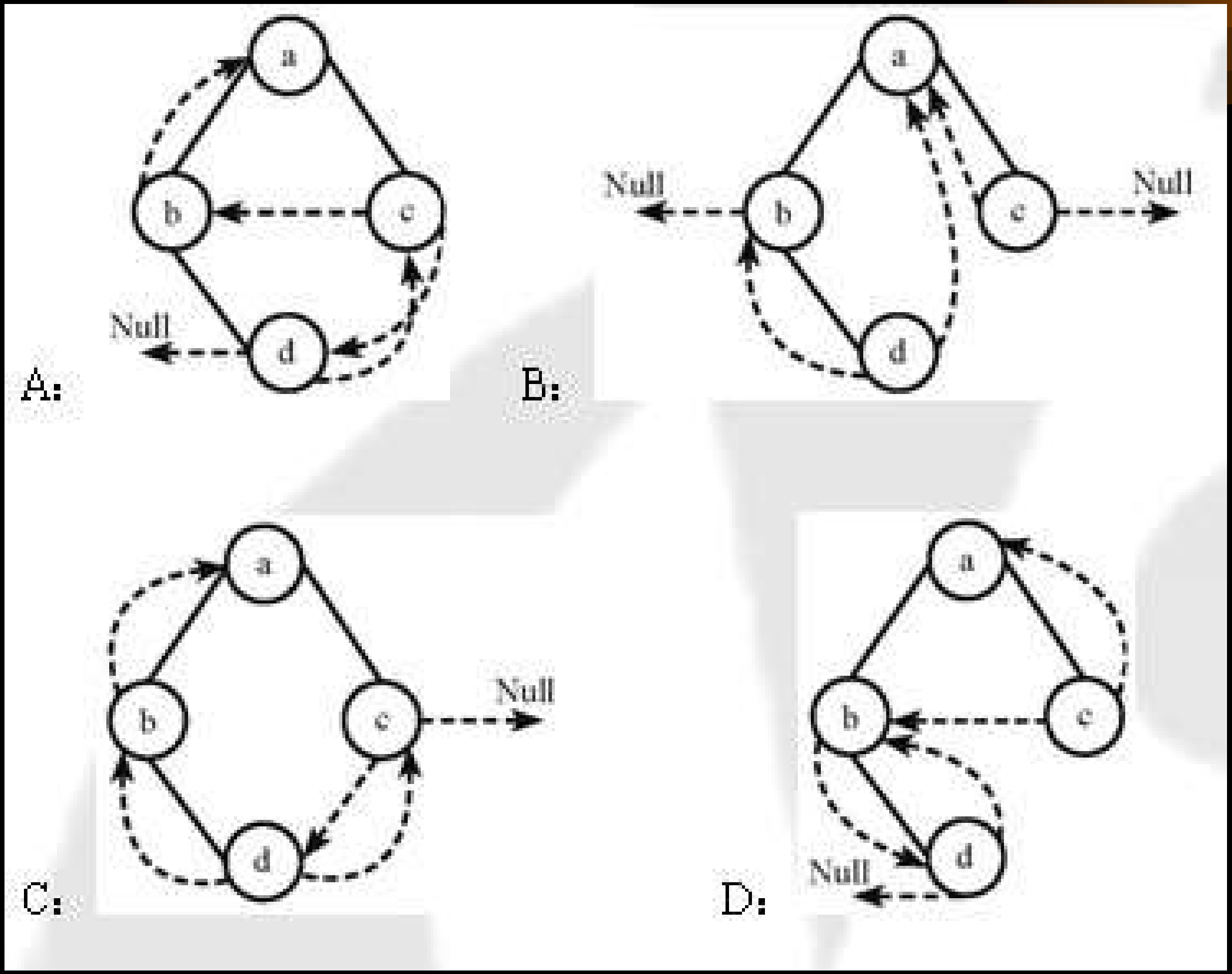
1) 父子关系　　2) 兄弟关系　　3) u 的父结点与v 的父结点是兄弟关系

A. 只有2)　　B. 1)和2)　　C. 1)和3)　　D. 1), 2)和3)

Chapter 4

2010年全国考研题

3、下列线索二叉树中（用虚线表示线索），符合后序线索树定义的是（ ）



Chapter 4

5、在一棵度为4的树T中，若有20个度为4的结点，10个度为3的结点，1个度为2的结点，10个度为1的结点，则树T的叶节点个数是（）

A：41　B：82　C：113　D：122

6、对n(n大于等于2)个权值均不相同的字符构成哈夫曼树，关于该树的叙述中，错误的是（）

A：该树一定是一棵完全二叉树

B：树中一定没有度为1的结点

C：树中两个权值最小的结点一定是兄弟结点

D：树中任一非叶结点的权值一定不小于下一任一结点的权值

Chapter 4

1. 给出如下各表达式的二叉树：

   1）(a+b)/(c-d*e)+e+g*h/a

   2) -x-y*z+(a+b+c/d*e)

   3) ((a+b)>(c-d))|| a<f &&(x<y || y>z)

2. 如果一棵树有n_1个度为1的结点，有n_2个度为2的结点，……，n_m个度为m的结点，试问有多少个度为0的结点？写出推导过程。

3. 分别找出满足以下条件的所有二叉树：

   1）二叉树的前序序列与中序序列相同

   2）二叉树的中序序列与后序序列相同

   3）二叉树的前序序列与后序序列相同

4. 若用二叉链表作为二叉树的存储表示，试对以下问题编写递归算法。

   1）统计二叉树中叶结点的个数。

   2）以二叉树为参数，交换每个结点的左子女和右子女

Chapter 4

5. 已知一棵二叉树的先序遍列结果是 ABECDFGHIJ，
中序遍列结果是 EBCDAFHIGJ
试画出这棵二叉树。

6. 编写一个Java函数， 输入后缀表达式， 构造其二叉树
表示。设每个操作符有一个或两个操作数。

7. 给定权值{ 15, 03, 14, 02, 06, 09, 16, 17 }, 构造相应的霍夫曼树，并
计算它的带权外路径长度。

8. c1, c2, c3, c4, c5, c6, c7, c8这八个字母的出现频率分别
{ 5,25,3,6,10,11,36,4,} 为这八个字母设计不等长的Huffman编码, 并
给出该电文的总码数.

实习题:

6. 建立一棵二叉树，并输出前序、中序、后序遍历结果。

** General Lists

1. 广义表的概念
2. 广义表的性质
3. 广义表的操作
4. 广义表的存储结构
5. 广义表的类声明
6. 输入二叉树的广义表表示来建立一棵树

** General Lists

1.广义表的概念(LS)

*定义为n(n>=0)个表元素a_0 , a_1, a_2,……a_{n-1}组成的有限序列，记作:

$LS=(a_0 , a_1, a_2,\ldots\ldots a_{n-1})$

其中每个表元素a_i可以是原子,也可以是子表.

原子: 某种类型的对象,在结构上不可分(用小写字母表示).

子表: 有结构的 .(用大写字母表示)

example:

L = (3, ( ), (‘b’, ‘c’), (((‘d’))))

*广义表的长度:表中元素的个数

** General Lists

*广义表的表头(head),表尾(tail)

head=a_0;

tail=(a_1, a_2,……a_{n-1})

*广义表的深度：表中所含括号的最大层数

1)A=( );

2)B=(6，2)

3)C=(‘a’，(5，3，‘x’)) 表头为‘a’,表尾为((5，3，‘x’))

4)D=(B，C，A)

5)E=(B，D)

6)F=(4，F) 递归的表

2. 广义表的性质(特点)

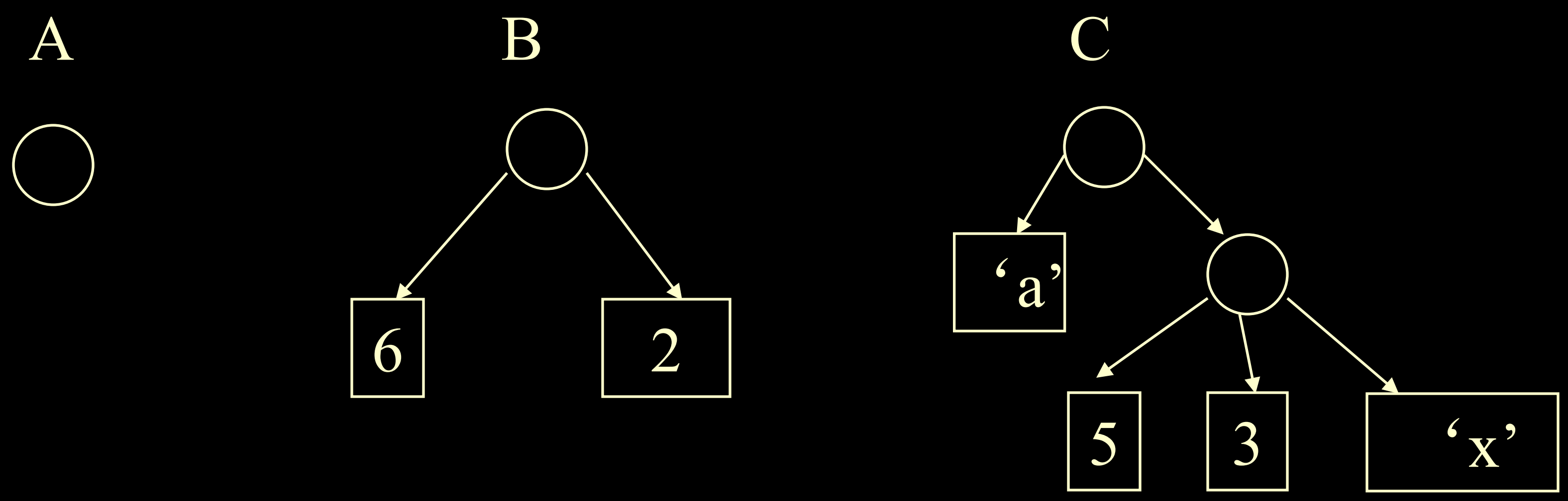
1)有序性

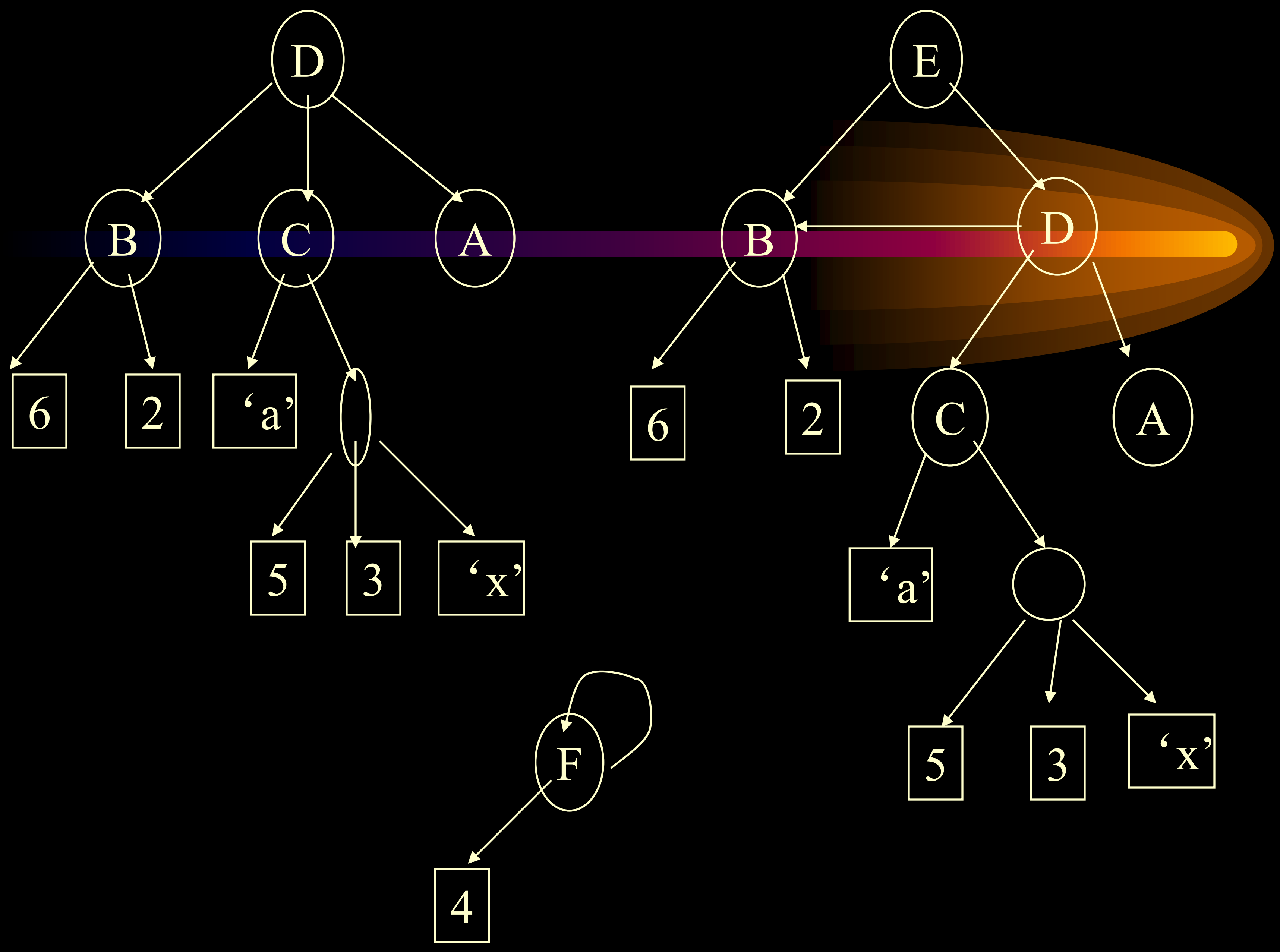
2)有长度,有深度

3)可递归,如上面例6

4)可共享,如E中B为E, D所共享

5)各种广义表都可用一种示意图来表示,用 ○ 表示表元素, 用 □ 表示原子




D
B
C
A
6
2
'a'
5
3
'x'
F
4
E
D
B
6
2
C
A
'a'
5
3
'x'

** General Lists

3. General Lists operate:

1) head (list)
2) tail (list)
3) first (list)
4) info (elem)
5) next (elem)
6) nodetype (elem)
7) push (list,x)
8) addon (list,x)
9) setinfo (elem,x)
10) sethead (list,x)
11) setnext (elem1,elem2)
12) settail (list1,list2)

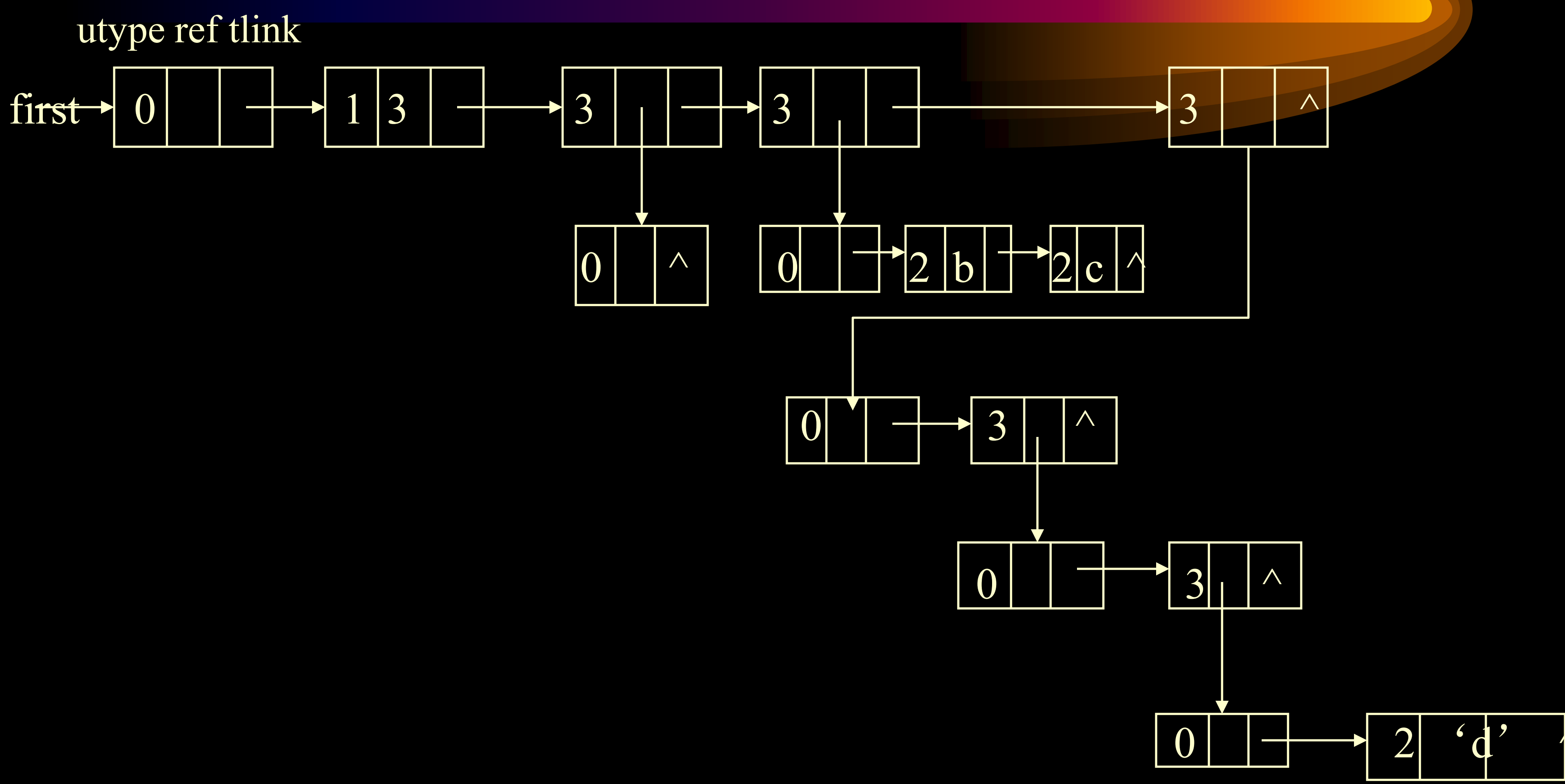
** General Lists

4. General Lists representation:

utype	value	tlink

utype=0	ref
utype=1	intgrinfo
utype=2	charinfo
utype=3	hlink

example: L=(3,( ),(‘b’ , ‘c’),(((‘d’))))



** General Lists

5. 广义表的类声明

```
#define HEAD 0
#define INTGR 1
#define CH 2
#define LST 3
class GenList;
class GenListNode
{ friend class GenList;
  private:
    int utype;
    GenListNode * tlink;
    union { int ref;
            int intgrinfo;
            char charinfo;
            GenListNode * hlink;
          } value;
```

** General Lists

```
 public:
    GenListNode & info (GenListNode * elem);
    int nodetype (GenListNode * elem) {return elem->utype;}
    void setinfo (GenListNode * elem,GenListNode & x);
};
class GenList
{ private:
   GenListNode * first;
   GenListNode * Copy (GenListNode * ls);
   int depth (GenListnode * ls);
   int equal (GenlistNode * s, Genlistnode * t);
   void Remove (GenlistNode * ls);
 public:
  GenList ( );
```

** General Lists

```
~GenList ( );
GenListNode & Head ( );
GenListNode * Tail ( );
GenlistNode * First ( );
GenlistNode * Next (GenListNode * elem);
void Push (GenListNode & x);
GenList & Addon ( GenList & list, GenListNode  & x);
void setHead (GenListNode  & x);
void setNext (GenlistNode  * elem1, GenlistNode  * elem2);
void setTail(GenList & list);
void Copy (const GenList & l);
int depth ( );
int Createlist (GenListNode  * ls, char * s);
}
```

6. 广义表的递归算法

递归算法:1)递归函数的外部调用-----公有函数　　界面

2)递归函数的内部调用-----私有函数　真正实现部分

1)求广义表的深度

广义表的深度为广义表中最大括号的重数

广义表 LS==(a_0 , a_1, a_2,……a_{n-1}),其中a_i(0<=I<=n-1)或者是原子或者是子表.

$$
\text{Depth(LS)}=\begin{cases} 0 & \text{当LS为原子时} \\ 1 & \text{当LS为空表时} \\ 1+\max\{\text{depth}(a_0),\text{depth}(a_1),\ldots\ldots\text{depth}(a_{n-1})\} \end{cases}
$$

递归结束条件

公共函数:

```
int GenList::depth( )
{  return depth(first);
}
```

私有函数:

```
 int  GenList::depth(GenListNode*ls)
{   if( ls-->tlink==NULL)  return 1;
    GenListNode*temp=ls-->tlink; int m=0;
    while( temp!=NULL)
   {   if( temp-->utype==LST)
       {     int n=depth(temp-->value.hlink);
              if(m<n)m=n;
       }
      temp=temp-->tlink;
   }
    return m+1;   }
```

2) 判断两个广义表相等否

相等的条件: 具有相同的结构

对应的数据元素具有相等的值

if(两个广义表都为空表)return相等

else if(都为原子^值相等)递归比较同一层的后面的表元素

else return 不相等.

```
int operator= =(const GenList&l,const GenList&m)//假设是友元
{  return equal(l.first, m.first);
}
```

```
int equal(GenListNode*s, GenListNode*t)//假设是友元
{   int  x;
    if(s-->tlink==NULL&&t-->tlink==NULL)return 1;
    if((s-->tlink!=NULL&&t-->tlink!=NULL&&s-->tlink-->utype==t-->tlink-->utype)
       {  if(s-->tlink-->utype==INTGR)
             if(s-->tlink-->value.intgrinfo== t-->tlink-->value.intgrinfo) x=1;
             else  x=0;
          else  if(s-->tlink-->utype==CH)
                 if(s-->tlink-->value.charinfo==t-->tlink-->value.charinfo)x=1;
                 else  x=0;
               else  x=equal(s-->tlink-->value.hlink, t-->tlink-->value.hlink);
          if(x)return equal(s-->tlink,  t-->tlink);
       }
    return 0;
}
```

3) 广义表的复制算法

分别复制表头,表尾,然后合成

前提是广义表不可以是共享表或递归表

公共函数:

```
void  GenList::copy(const GenList&l)
{first=copy(l.first);
}
```

私有函数:

```
GenListNode*GenList::copy(GenListNode*ls)
{
GenListNode*q=NULL;
if(ls!=NULL)
        {q=new GenListNode;
          q-->utype=ls-->utype;
```

```
Switch(ls-->utype)
{   case  HEAD:  q-->value.ref=ls-->value.ref; break;//表头结点
    case   INTGR: q-->value.intgrinfo=ls-->value.intgrinfo; break;
    case  CH:  q-->value.charinfo=ls-->value.charinfo; break;
    case  LST:  q-->value.hlink=ls-->value.hlink; break;
 }
 q-->tlink=copy(ls-->tlink);
}
  return q;
}
```

4) 广义表的析构函数--------- ~GenList( )

公共函数:

```
GenList::~GenList( )
{  remove(first); }
```

私有函数:

```
void GenList::remove(GenListNode*ls)
{  ls→value.ref--;
   if (!ls→value.ref)
      {  1) GenListNode*y=ls;
         2) while(y-->tlink!=NULL)
              {   y=y-->tlink;
                  if(y-->utype= =LST)remove(y-->value.hlink);
              }
          3) y-->tlink=av;  av=ls;        //回收顶点到可利用空间表中
      }
}
```

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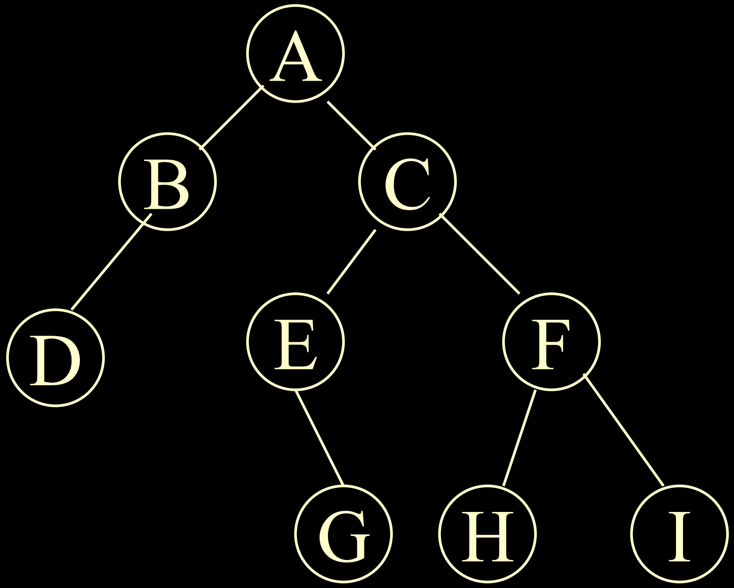
对广义表的回收算法如下, 请回答在回收广义表ls后, 在可利用空间表av中原广义表中的结点是按何种次序链接起来的(请用原广义表结点上方的字母组成一个序列来表示链接的顺序: 序列的头部是新的av所指向的地方, 序列的尾部连接原来av的空间).

```
GenList :: ~GenList( )
{   Remove(first);
}
Void GenList :: Remove(GenListNode *ls)
{   ls->value.ref--;
    if ( !ls -> value.ref)
      {  GenListNode * y = ls;
         while ( y ->tlink != NULL)
               {   y = y ->tlink;
                   if ( y -> utype = = LST)  Remove( y ->value.hlink);
               }
          y -> tlink = av;  av = ls;
      }
}
```

7. Create BinaryTree another method:

A(B(D), C(E(,G), F(H, I)))@ →



Create BinaryTree another method:

```
void CreateBTree(BTreeNode * & BT, char *a)
{  BTreeNode * s[10];
   int top = -1;
   BT = NULL;
   BTreeNode * p;
   int k;
   istrstream ins (a);
   char ch;
   ins >> ch;
```

Create BinaryTree another method:

```
 while( ch != '@')
 {  switch (ch)
     { case '(': top++; s[top ]= p; k = 1; break;
       case ')':top--; break;
       case ',': k = 2; break;
       default:
           p = new BTreeNode;
           p->data = ch;  p->left = p->right = NULL;
           if(BT = = NULL) BT = p ;
           else { if (k = = 1) s[top]->left = p;
                  else s[top]->right = p;
                }
     }//switch end
   ins >>ch;
 }//while end
}
```