Chapter 6

Priority Queues

6.1 Introduction

- A priority queue is a collection of zero or more elements. Each element has a priority or value.

6.1 Introduction

- In a min priority queue the find operation finds the element with minimum priority, while the delete operation delete this element.
- In a max priority queue, the find operation finds the element with maximum priority, while the delete operation delete this element.

6.1 Introduction

ADT of a max priority queue

```
AbstractDataType  MaxPriorityQueue
{
  instances
      finite collection of elements,each has a priority
  operations
      Create(): create an empty priority queue
      Size():  return number of element in the queue
      Max(): return element with maximum priority
      Insert(x): insert x into queue
      DeleteMax(x):delete the element with largest priority
                 from  the queue; return it in x;
}
```

6.2 Linear List Representation

- Use an unordered linear list
- Insertions are performed at the right end of the list, $\theta(1)$
- A deletion requires a search for the element with largest priority, $\theta(n)$

6.3 Heaps

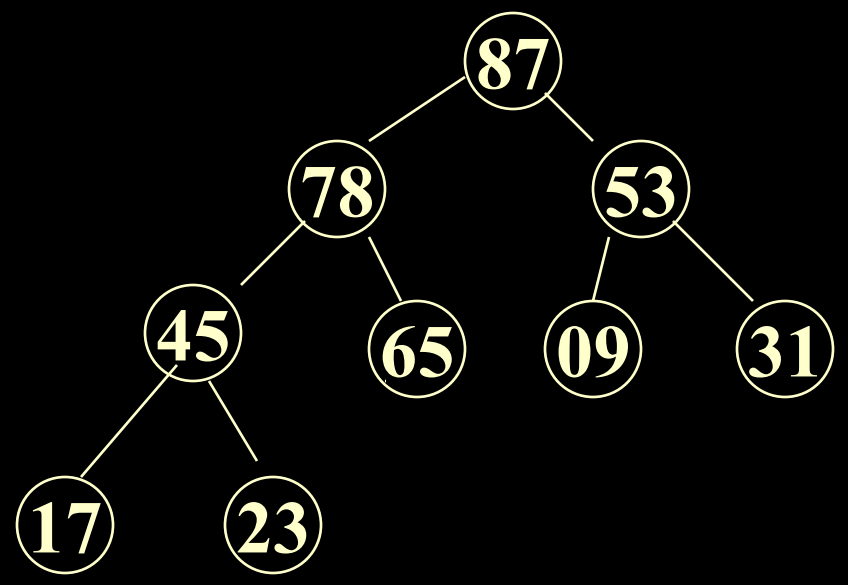
1.definition: <u>A max heap(min Heap)</u>

- is A complete binary tree
- The value in each node is greater(less) than or equal to those in its children(if any).

6.3 Heaps

Example of a max heap

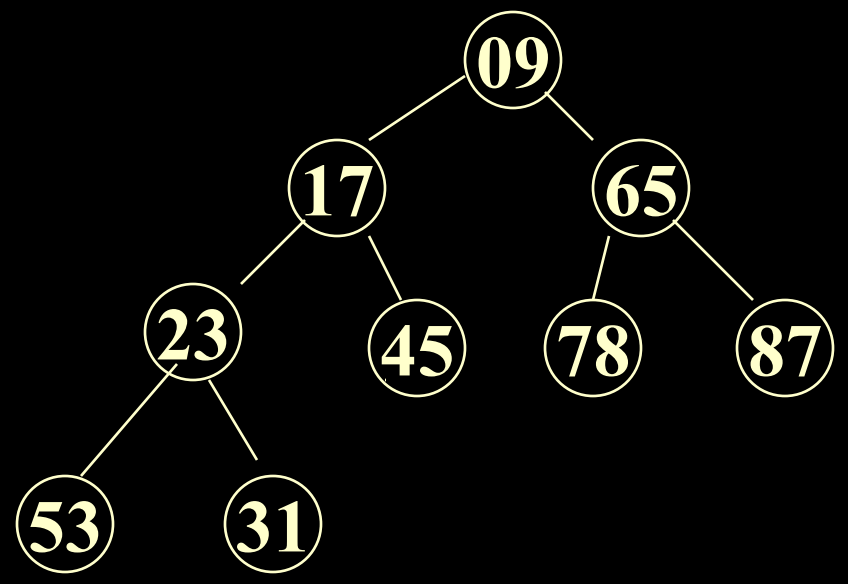
k={87,78,53,45,65,09,31,17,23}



6.3 Heaps

Example of a min heap

k={09,17,65,23,45,78,87,53,31}



6.3 Heaps

2. class MaxHeap

Data member of heap: <u>T * heap, int MaxSize, CurrentSize</u>

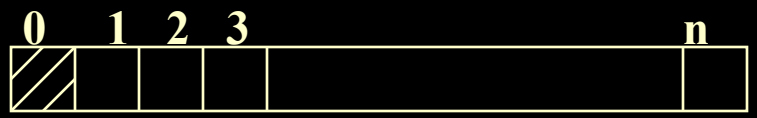
```
template<class T>class MaxHeap
{ public:
        MaxHeap(int MaxHeapSize=10);
        ~MaxHeap(){delete[]heap;}
        int size()const{return CurrentSize;}
        T Max(){ if (CurrentSize= =0)throw OutOfBounds();
                    return heap[1];}
       MaxHeap<T>&insert(const T&x);
       MaxHeap<T>& DeleteMax(T& x);
       void initialize(T a[], int size,  int ArraySize);
  private:
       int CurrentSize, MaxSize;
      T * heap;
}
```

6.3 Heaps

3.member function of MaxHeap

1)Constructor for MaxHeap

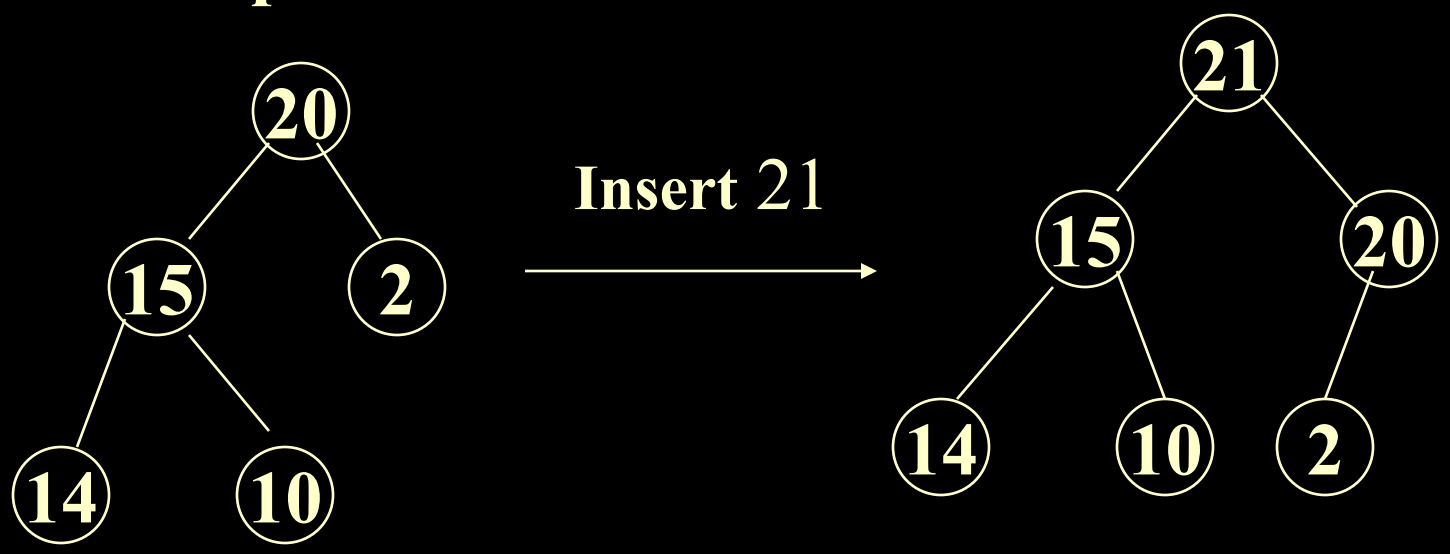
```
 template<class T>
 MaxHeap<T>::MaxHeap(int MaxHeapSize)
{  MaxSize=MaxHeapSize;
   Heap=new T[MaxSize+1];
   CurrentSize=0;
}
```

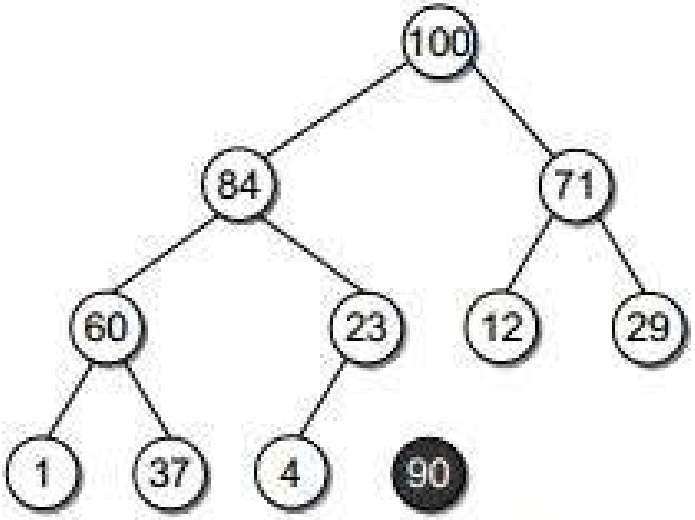


6.3 Heaps

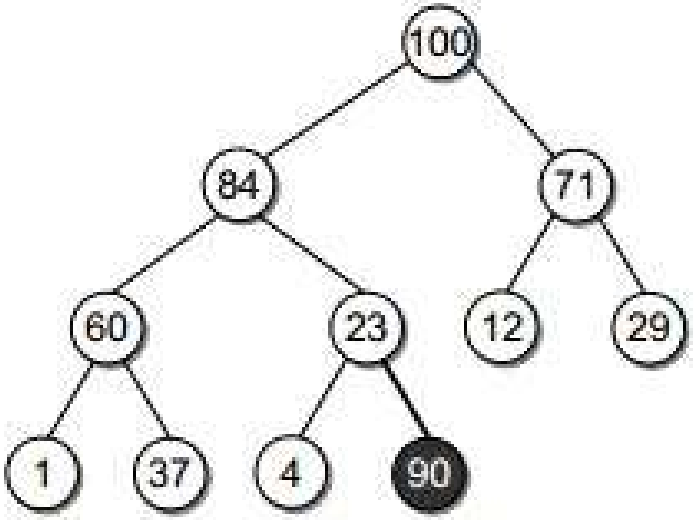
2)Insertion

Example:

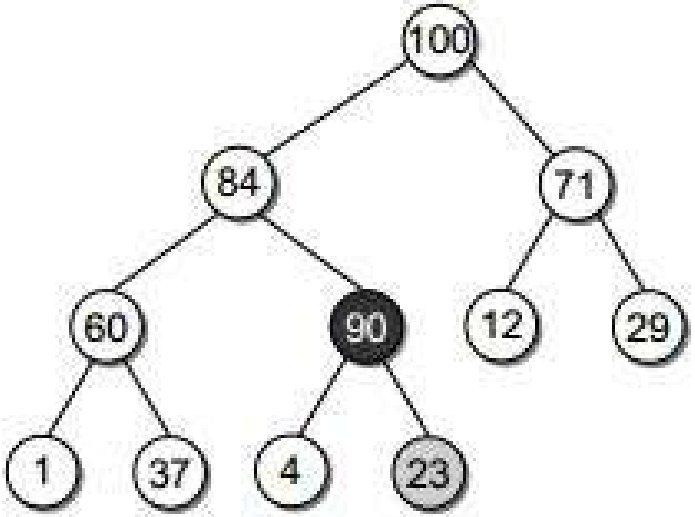




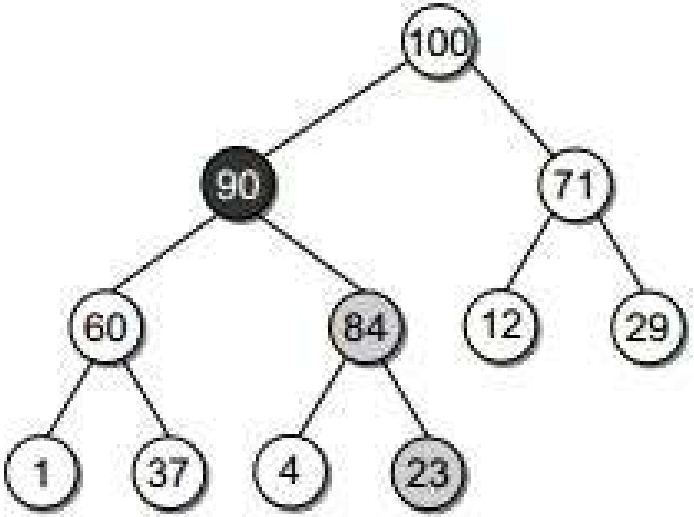
(a) create a new node for 90.



(b) link the node as the last child.



(c) sift-up: swap 23 and 90.



(d) sift-up: swap 84 and 90.

6.3 Heaps

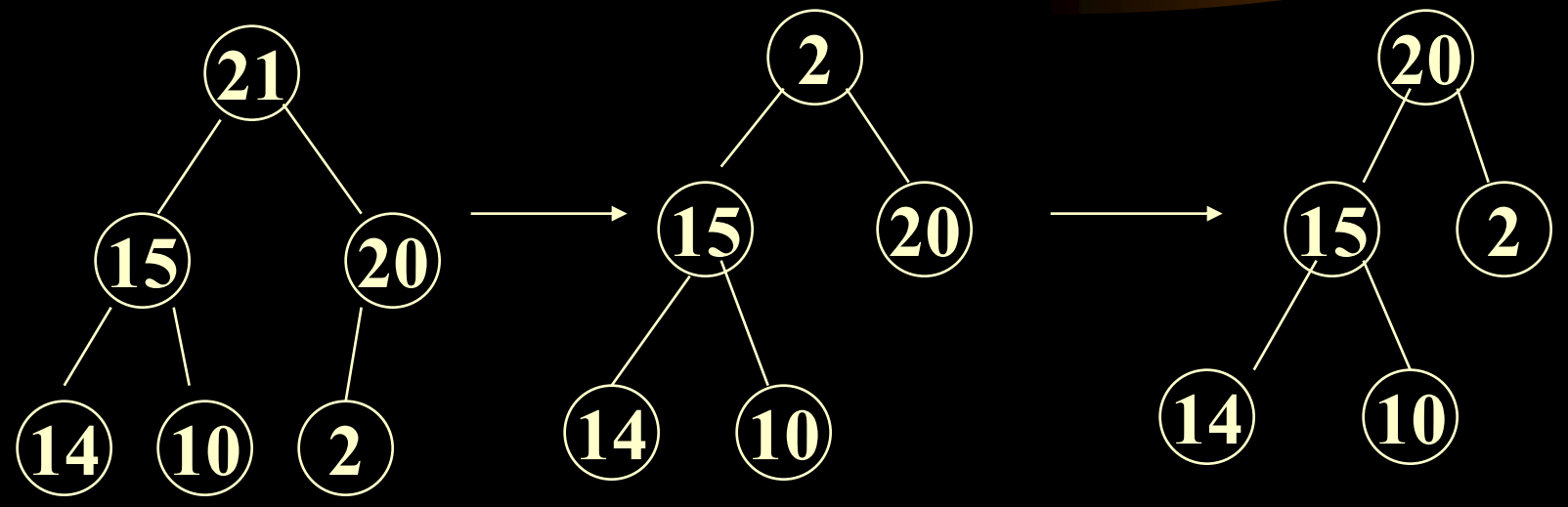
Insertion

```
template<class T>MaxHeap<T>& MaxHeap<T>::
  Insert(const T& x)
{  if(CurrentSize= =MaxSize)throw NoMem();
    int i=++CurrentSize;
    while(i!=1&&x>heap[i/2])
      { heap[i]=heap[i/2];  i/=2; }
     heap[i]=x;
     return *this;
}
```

time complexity is $O(\log_2 n)$

6.3 Heaps

3)deletion



6.3 Heaps

deletion from a max heap

```
template<class T>MaxHeap<T>&  MaxHeap<T>:: DeleteMax(T& x)
  {  if (CurrentSize= =0)throw OutOfBounds();
     x=heap[1];
     T y=heap[CurrentSize--];
     int i=1;  ci=2;
     while(ci<=CurrentSize)
       {  if(ci<CurrentSize&&heap[ci]<heap[ci+1]) ci++;
          if(y>=heap[ci]) break;
          heap[i]=heap[ci];
          i=ci; ci*=2;
       }
     heap[i]=y;    return *this;
  }
```

Time complexity is $O(\log_2 n)$

6.3 Heaps

```
 java program(MinHeap)
public class BinaryHeap
 {  public BinaryHeap( )
    public BinaryHeap( int capacity )
    public void insert( Comparable x ) throws Overflow
    public Comparable findMin( )
    public Comparable deleteMin( )

    public boolean isEmpty( )
    public boolean isFull( )
    public void makeEmpty( )

    private static final int DEFAULT_CAPACITY = 100;
    private int currentSize;
    private Comparable [ ] array;
    private void percolateDown( int hole )
    private void buildHeap( )
 }
```

6.3 Heaps

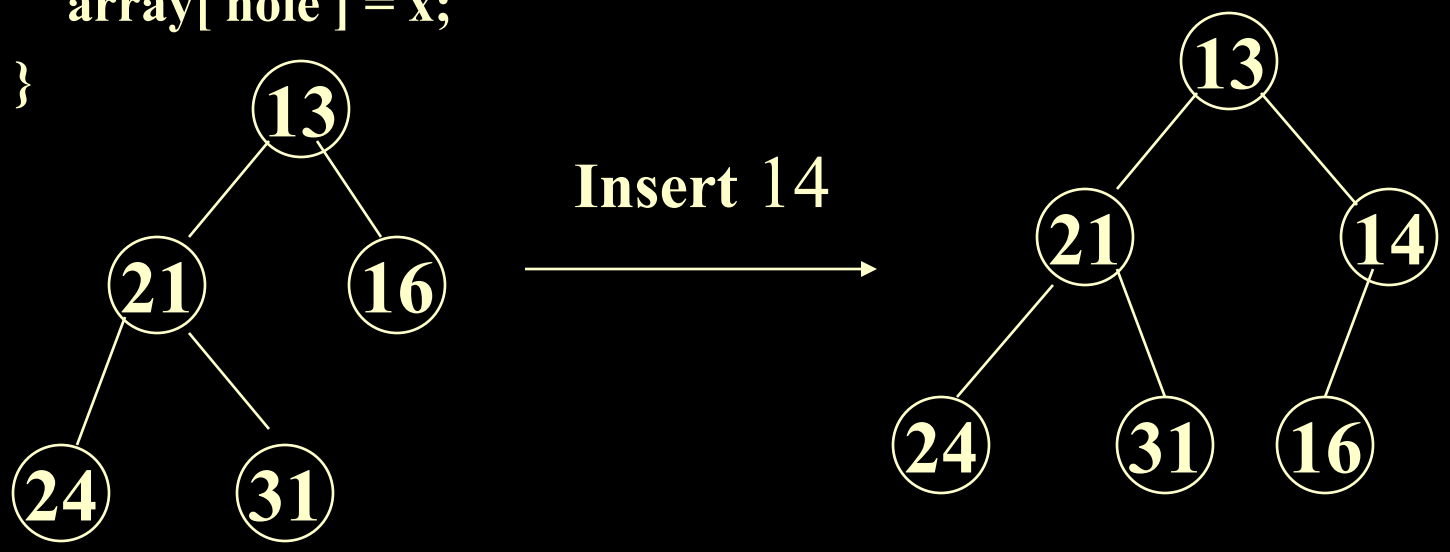
```
public BinaryHeap( )
{   this( DEFAULT_CAPACITY );
}

public BinaryHeap( int capacity )
{   currentSize = 0;
    array = new Comparable[ capacity + 1 ];
}

 public void makeEmpty( )
{   currentSize = 0;
}
```

6.3 Heaps

```
public void insert( Comparable x ) throws Overflow
{  if( isFull( ) )
      throw new Overflow( );
   int hole = ++currentSize;
   for( ; hole > 1 && x.comparebleTo( array[ hole / 2 ] ) < 0;
          hole /= 2 )
        array[ hole ] = array[ hole / 2 ];
   array[ hole ] = x;
}
```



6.3 Heaps

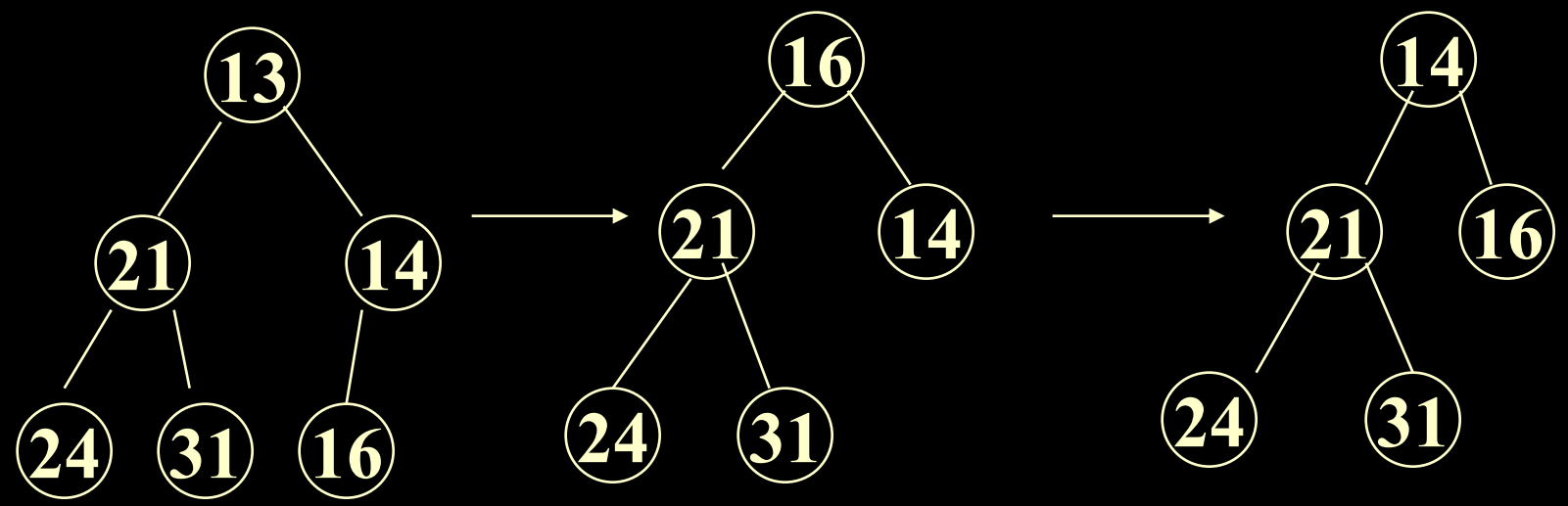
```
public Comparable deleteMin( )
{  if( isEmpty( ) )
      return null;

   Comparable minItem = findMin( );
    array[ 1 ] = array[ currentSize-- ];
    percolateDown( 1 );

    return minItem;
}
```

6.3 Heaps

```
private void percolateDown( int hole )
{   int child;
    Comparable tmp = array[ hole ];
    for( ; hole *2 <= currentSize; hole = child )
    {   child = hole * 2;
        if ( child != currentSize &&  array[ child + 1 ].compareTo( array[ child ] ) < 0 )
                child++;
        if( array[child ].compareTo( tmp ) < 0 )
            array[ hole ] = array[ child ];
        else
           break;
    }
    array[ hole ] = tmp;
}
```

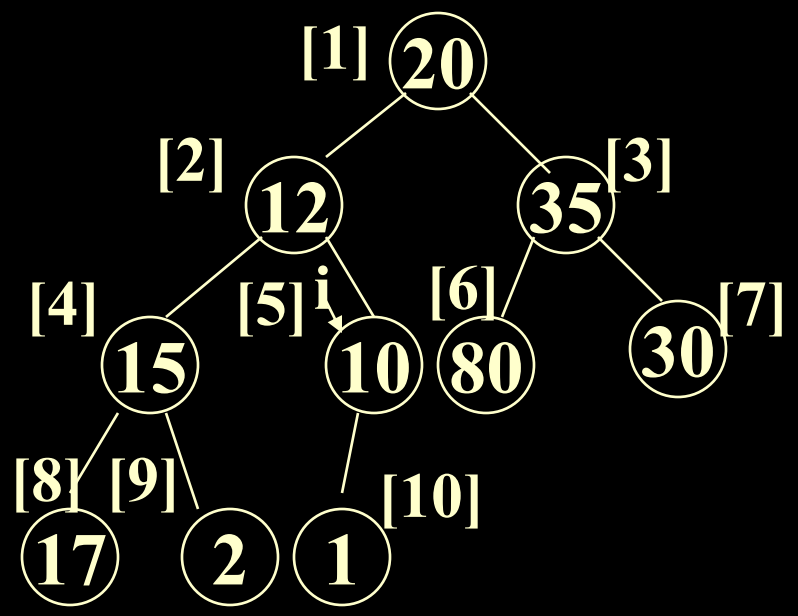


6.3 Heaps

4)Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

书中称为由底向上:



$i=[n/2], [n/2]-1, \ldots, 1$

Turn into max heap from these subtree roots

6.3 Heaps

initialize (C++ program)

```
Template<class T> void MaxHeap<T>::
                    Initialize (T a[],int size,int ArraySize)
 {  delete[] heap;
    heap=a;   CurrentSize=Size;    MaxSize=ArraySize;
    for( int i=CurrentSize/2; i>=1; i--)
        {  T y=heap[i];    int c=2*i;
           while(c<=CurrentSize)
             { if(c<CurrentSize && heap[c]<heap[c+1])  c++;
               if(y>=heap[c]) break;
               heap[c/2] = heap[c];
               c*=2;
             }
           heap[c/2]=y;
        }
 }
```

6.3 Heaps

java initialize

```
private void buildHeap( )
{  for( int i = currentSize / 2; i > 0 ; i-- )
        percolateDown( i );
}
```

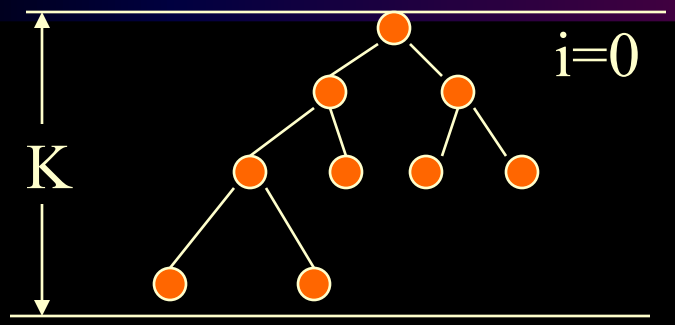
6.3 Heaps

Complexity of Initialize:

- **Create Heap time complexity:**

算法分析

初始建堆：n个结点，$K=\lfloor \log_2 n \rfloor$，从0层开始



第i层交换的最大次数为k-i

第i层有2^i个结点

总交换次数：$\sum_{i=0}^{k-1} 2^i \cdot (k-i) = \sum_{j=1}^{k} j \cdot 2^{k-j} = \sum_{j=1}^{k} j(2^k \cdot 2^{-j})$

令k-i=j

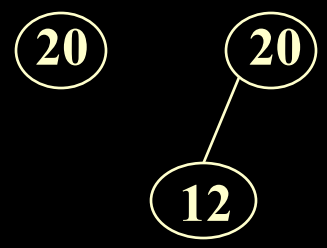
$$=2^k \cdot \sum_{j=1}^{k} j \cdot 2^{-j} \le 2^k \cdot 2 \le 2^{\log n} \cdot 2 = 2n = O(n)$$

6.3 Heaps

4)Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

还可以这样做: 依次插入一个元素到堆中.书中称为由顶向下(也可见书中例子).



Complexity of Initialize:

$$\sum_{K=1}^{n} \lfloor \lg k \rfloor <= \sum_{K=1}^{n} \lg k = \lg 1 + \ldots\ldots + \lg n = \lg(1 \cdot 2 \ldots. n) =$$

$$\lg(n!) = O(n \lg n)$$

(提示: 因 $n! = n*(n-1) * (n-2) * \ldots.. *2 * 1$

显然 该n 项都小于n, 所以$\lg(n!) <= \lg(n*n*n*\ldots.*n)$)

6.4.Applications of Priority Queues

1.heap sort

Method:

1)initialize a max heap with the n elements to be sorted O(n)

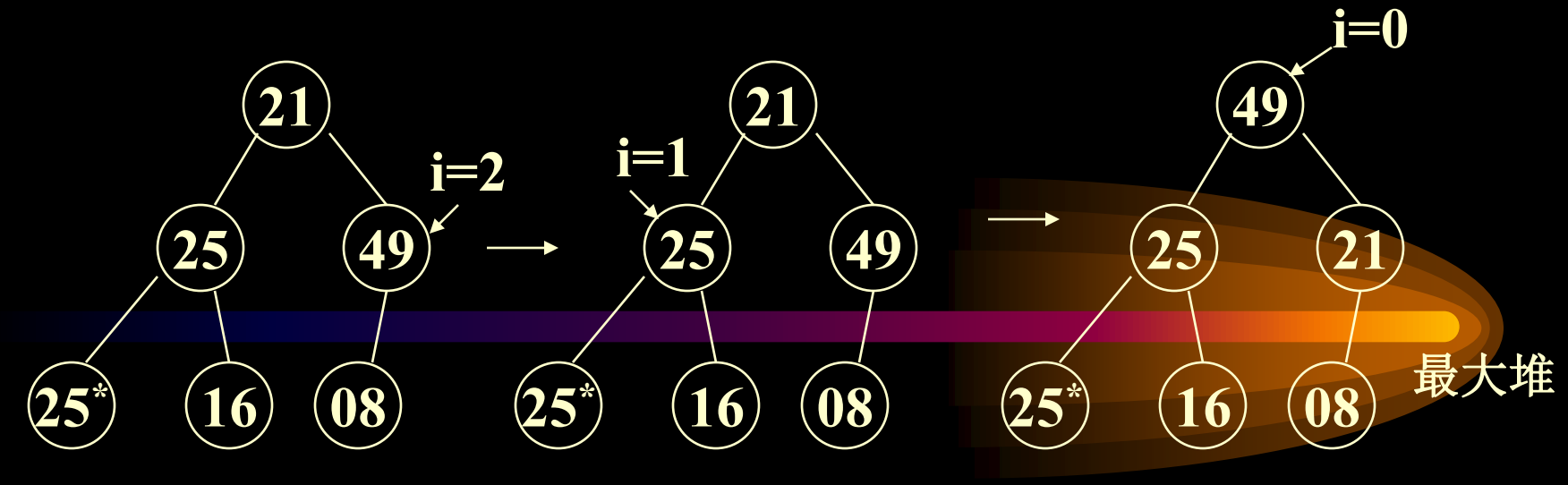
2)each time we delete one element, then adjust the heap $O(\log_2 n)$

Time complexity is $O(n)+O(n*\log_2 n)=O(n*\log_2 n)$

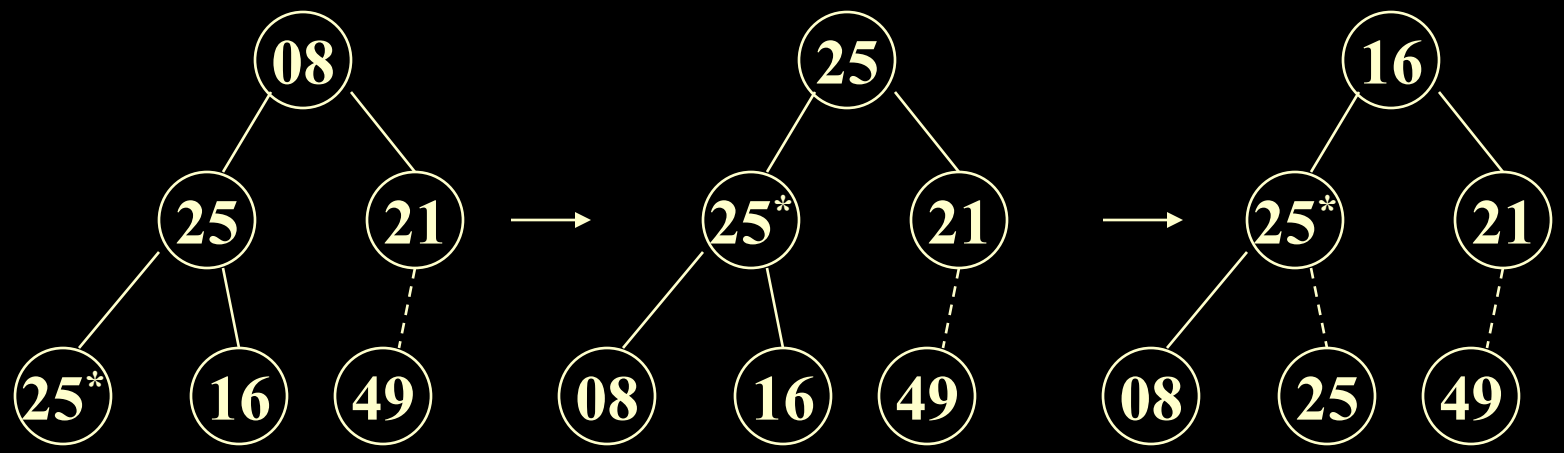
6.4.Applications of Priority Queues

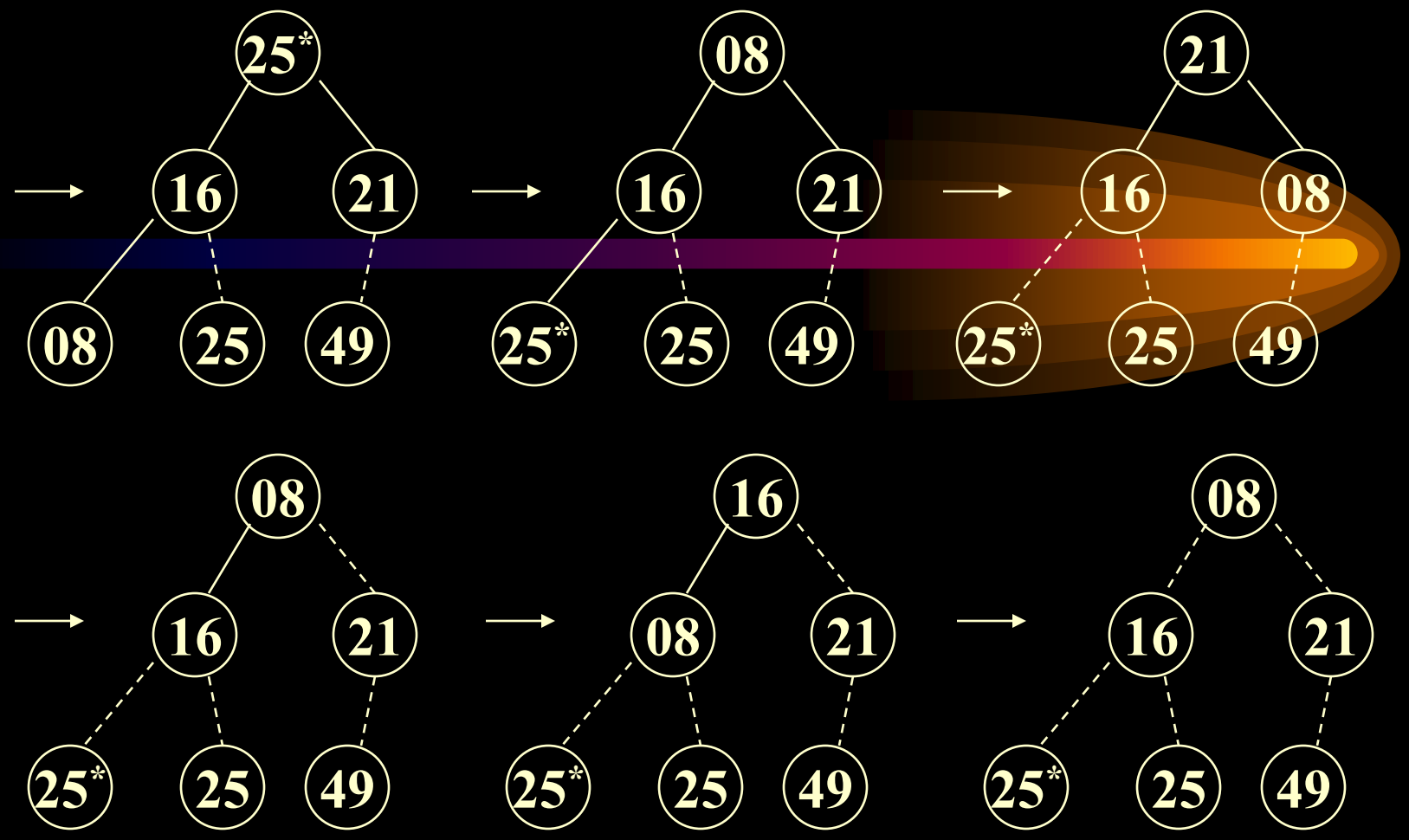
heap sort

Example :{21,25,49,25*,16,08}


i=2
i=1
i=0
21
25
49
25*
16
08
21
25
49
25*
16
08
49
25
21
25*
16
08
最大堆

调整


08
25
21
25*
16
49
25
25*
21
08
16
49
16
25*
21
08
25
49



从以上例子可以看出堆排序是不稳定的

heap sort
算法：c++

```
Template<class  Type>void HeapSort(datalist<Type>&list)
{for(int i=(list.currentsize)/2;i>=1;i--)
        FilterDown(i,list.currentsize);
 for(i=list.currentsize;i>1;i--)
        {Swap(list.Vector[1],list.vector[i]);
         FilterDown(1,i-1);
        }
}
```

heap sort

java program

```
public static void heapsort( Comparable [ ] a )
{  for( int i = a.length / 2; i >= 1; i-- )
       percDown( a, i, a.length );
   for( int i = a.length ; i > 1; i-- )
   {  swapReferences( a, 1, i );
       percDown( a, 1, i-1);
    }
}
```

heap sort

```
private static void percDown( Comparable [ ] a, int i, int n )
{   int child;
    Comparable tmp;

    for( tmp = a[ i ]; leftChild( i ) < n; i = child )
    {    child = leftChild( i );
         if( child != n – 1 && a[child ].compareTo( a[ child + 1 ]) < 0 )
              child++;
         if( tmp.compareTo( a[ child ] ) < 0 )
              a[ i ] = a[ child ];
         else  break;
    }
    a[ i ] = tmp;
}
private static int leftChild( int i )
{   return 2 * i + 1;
}
```

6.4.Applications of Priority Queues

2. The Selection Problem

6.4.Applications of Priority Queues

2. The Selection Problem

在N个元素中找出第K个最大元素。

1A算法：读入N个元素放入数组， 并将其选择排序，返回适当的元素。

运行时间：$O(N^2)$

1B算法:

1) 将K个元素读入数组， 并对其排序（按递减次序）。
最小者在第K个位置上。

2) 一个一个地处理其余元素：
每读入一个元素与数组中第K个元素(在K个元素中为最小)比较，
如果 > , 则删除第K个元素，再将该元素放在合适的位置上。
如果 < , 则舍弃。
最后在数组K位置上的就是第K个最大元素。

例如：3, 5, 8, 9, 1, 10　　找第3个最大元素。

6.4.Applications of Priority Queues

运行时间(1B 算法)：

$O(K^2 + (N - K)*K)$

$= O(N*K)$

当 $K = \lceil N / 2 \rceil$，$O(N^2)$

试验： 在 N = 100 万个元素中， 找第 500,000 个最大元素。

以上两个算法在合理时间内均不能结束, 都要处理若干天才算完.

<u>用堆来实现：</u>

6A算法：假设求第K个最小元素

1）将N个元素建堆（最小） $O(N)$

2) 执行K次delete $O(K*\log N)$

$\Big\}$ $O(N + K * \log N)$

如果 $K = \lceil N / 2 \rceil$， $\theta(N * \log N)$

如果 $K = N$， $O(N * \log N)$ —— 堆排序

6.4.Applications of Priority Queues

6B算法：假设求第K个最大元素

1）读入前K个元素， 建立最小堆　　O(K)

2) 其余元素一一读入：

每读入一个元素与堆中第K个最大元素比（实际上是堆中最 小元素）

O(1)

大于，则将小元素去掉（堆顶），该元素进入，进行一次调整。

O(log K)

小于，则舍弃。

O(K + (N-K) * log K) = O(N*log K)

当 K =⌈N / 2⌉，θ(N * log N)

对6A, 6B,用同样的数据进行测试， 只需几秒钟左右给出问题解。

Chapter 6

2009年统考题:

8. 已知关键字序列 5,8,12,19,28,20,15,22 是最小根堆(最小堆), 插入关键字3,调整后得到的小根堆是

A. 3, 5, 12, 8, 28, 20, 15, 22, 19　　B. 3, 5, 12, 19, 20, 15, 22, 8, 28

C. 3, 8, 12, 5, 20, 15, 22, 28, 19　　D. 3, 12, 5, 8, 28, 20, 15, 22, 19

exercises:

1. a. Show the result of inserting 10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, and 2,one at a time, into an initially empty binary heap.

   b. Show the result of using the linear-time algorithm to build a binary heap using the same input.

2. Show the result of performing three deleteMin operations in the heap of the previous exercise.

Chapter 6

3.判别以下序列是否是堆？如果不是，将它调整为堆。

1) { 100, 86, 48, 73, 35, 39, 42, 57, 66, 21 }

2) { 12, 70, 33, 65, 24, 56, 48, 92, 86, 33 }

3) { 103, 97, 56, 38, 66, 23, 42, 12, 30, 52, 06, 20 }

4) { 05, 56, 20, 23, 40, 38, 29, 61, 35, 76, 28, 100 }

4.设待排序的关键码序列为{ 12, 2, 16, 30, 28, 10, 16^*, 20, 6, 18 }, 使用堆排序方法进行排序。写出建立的初始堆，以及调整的每一步。