MeEn 537 Homework #3

1. Work the following problems from your textbook:
   (a) 7-2
   (b) 7-4
   (c) 7-6
   (d) 8-2
   (e) 8-3
2. Do the following:
   (a) Verify the following equation by direct calculation:

$$[\omega]_\times p = \omega \times p$$

   (b) Given the following Euler angle rotation matrix:

$$R = R_{z,\psi} R_{y,\theta} R_{z,\phi}$$

   show that $\frac{d}{dt}R = [\omega]_\times R$ for the specific case where

$$\omega = \{c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}\}\hat{x} + \{s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}\}\hat{y} + \{\dot{\psi} + c_\theta \dot{\phi}\}\hat{z}$$

   HINTS: Start by using the product rule with the first equation and then using chain rule ($\dot{R}_{z,\psi} = \frac{dR_{z,\psi}}{d\psi}\dot{\psi}$) along with what we know about the derivative of a rotation matrix. You can definitely use symbolic tools in MATLAB or continue to manipulate equations by hand after that point.
   (c) Calculate the Jacobian from Problem 2d) from HW 2 (also pictured below) by hand. Find this symbolically first, then let each link length be 30 cm and compare the resulting Jacobian at a few different joint configurations to the Jacobian that is calculated by the Robotics Toolbox using the same DH parameters. How are the columns affected by the change in position and what does this mean physically (use at least one concrete example to discuss)?

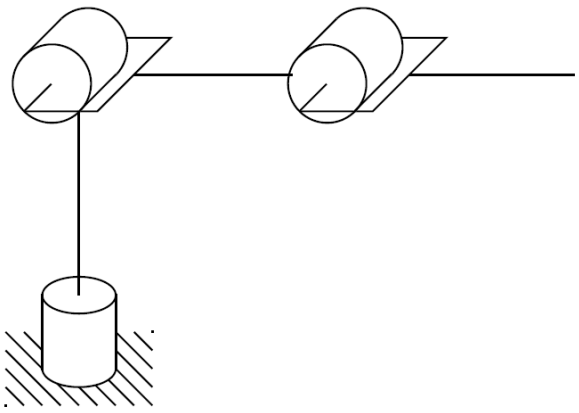
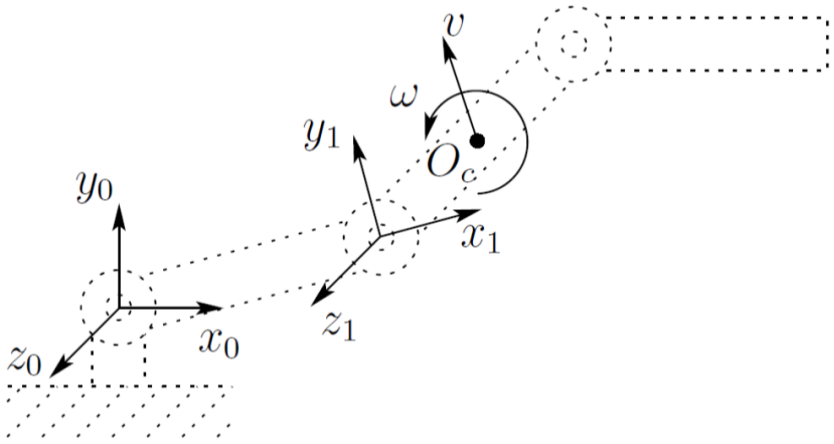


Fig. 3.27 Three-link articulated robot

(d) For the three-link planar manipulator shown below, compute the forward kinematics for the vector o_c (in terms of joint angles and DH parameters) and derive the manipulator Jacobian matrix in symbolic form. Let the first link length be a_1 and the distance from the second joint to point c be a_c.



(e) Write the Jacobian for the 2 link manipulator that we looked at in class in the end effector frame using the following two methods (which should agree in terms of the result):

i. by direct calculation

ii. by transforming the original Geometric Jacobian that we found in class

(f) Using the Jacobian for the 2 link manipulator from class, find the required torques to apply a force in the following directions at the following joint configurations (make note of anything that you think is interesting or increases your understanding). How do your answers change if the force given is a reaction force instead of a force applied by the end-effector?:

i. $q = [0, \pi/4]$, $F = [-1, 0, 0]$ Newtons

ii. $q = [0, \pi/2]$, $F = [-1, 0, 0]$ Newtons

iii. $q = [\pi/4, \pi/4]$, $F = [-1, -1, 0]$ Newtons
iv. $q = [0, 0]$, $F = [0, 0, 1]$ Newtons
v. $q = [0, 0]$, $F = [1, 0, 0]$ Newtons