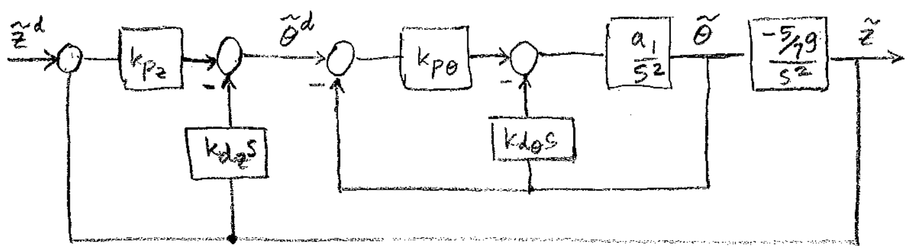
# Homework E.12 - Solution

V.8 $\quad F_{max} = 15\ \text{N} \qquad$ Select $z_e = \frac{L}{2}$

(a)




(b)

$$\frac{\tilde{\Theta}}{\tilde{\Theta}^d} = \frac{k_{p\theta} \frac{a_1}{s^2}}{1 + (k_{d_\theta} s + k_{p\theta}) \frac{a_1}{s^2}}$$

$$\boxed{\frac{\tilde{\Theta}}{\tilde{\Theta}^d} = \frac{a_1 k_{p\theta}}{s^2 + a_1 k_{d_\theta} s + a_1 k_{p\theta}}}$$

(c)

$$F_e = \frac{1}{2} m_2 g + m_1 \frac{z_e}{L} g \qquad z_e = \frac{L}{2}$$

$$= \frac{1}{2} g (m_1 + m_2)$$

$$= \frac{1}{2} (9.81\ \text{m/s}^2)(2.35\ \text{kg})$$

$$F_e = 11.5\ \text{N}$$

$$|F| \leq F_{max} \Rightarrow -F_{max} \leq F \leq F_{max} \qquad F = \tilde{F} + F_e$$

$$\Rightarrow -F_{max} \leq \tilde{F} + F_e \leq F_{max}$$

$$\Rightarrow -F_{max} - F_e \leq \tilde{F} \leq F_{max} - F_e$$

This will be satisfied if

$$\underline{|\tilde{F}| \leq F_{max} - F_e}$$

Substituting,

$$|\tilde{F}| \leq 15\ \text{N} - 11.5\ \text{N}$$

$$\boxed{|\tilde{F}| \leq 3.5\ \text{N}}$$

(d) For a step in $\theta^d$ of $A_\theta$,

$$\tilde{F}_{max} \approx k_{p\theta} A_\theta$$

$$k_{p\theta} = \frac{\tilde{F}_{max}}{A_\theta} = \frac{3.5\ N}{(1\ deg)\left(\frac{\pi\ rad}{180\ deg}\right)}$$

$$\boxed{k_{p\theta} = 200\ \frac{N}{rad}}$$

(e) CLCE desired: $s^2 + 2\zeta_\theta \omega_{n_\theta} s + \omega_{n_\theta}^2 = 0$

CLCE actual: $s^2 + k_{d\theta} a_1 s + k_{p\theta} a_1 = 0$

$$\omega_{n_\theta} = \sqrt{k_{p\theta} a_1} \qquad a_1 = \frac{L}{\frac{1}{3} m_2 L^2 + m_1 z_e^2}$$

$$k_{d\theta} = \frac{2\zeta_\theta \omega_{n\theta}}{a_1} = 2\zeta_\theta \sqrt{\frac{k_{p\theta}}{a_1}}$$

let $\zeta_\theta = 0.7$

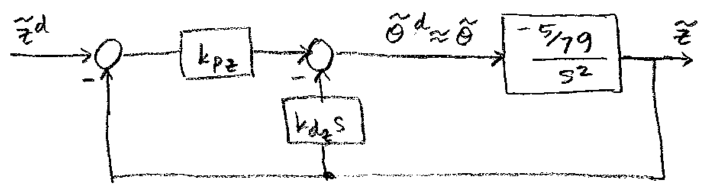
$$\Rightarrow k_{d\theta} = 2(0.7)\sqrt{\frac{200}{a_1}}$$

$$\boxed{k_{d\theta} = 10.4\ s^{-1}}$$

(f) CL poles are roots of CLCE.

From Matlab, $\boxed{s_{1,2} = -13.8 \pm 18.4\ rad/s}$

(g)




$$\frac{\tilde{z}}{\tilde{z}^d} = \frac{k_{pz}\left(\frac{-5/7g}{s^2}\right)}{1 + (k_{d_z}s + k_{pz})\left(\frac{-5/7g}{s^2}\right)}$$

$$\boxed{\frac{\tilde{z}}{\tilde{z}^d} = \frac{-\frac{5}{7} g k_{pz}}{s^2 - \frac{5}{7} g k_{d_z} s - \frac{5}{7} g k_{pz}}}$$

(h) From block diagram, $\tilde{\theta}^d \approx k_{p_z} \tilde{z}^d$

$\tilde{z}^d = A_z = 0.25 m$ , $\tilde{\theta}^d = -A_\theta = -1 \deg$

$$\Rightarrow \quad k_{p_z} = -\frac{A_\theta}{A_z}$$

$$= -\frac{1 \deg \cdot \frac{\pi}{180}}{0.25 m}$$

$$\boxed{k_{p_z} = -0.0698}$$

(i) desired CLCE: $s^2 + 2\zeta_z \omega_{n_z} s + \omega_{n_z}^2 = 0$

actual CLCE: $s^2 - \frac{5}{7} g k_{d_z} s - \frac{5}{7} g k_{p_z} = 0$

$$\omega_{n_z} = \sqrt{-\frac{5}{7} g k_{p_z}} = \sqrt{\frac{5}{7} \frac{A_\theta}{A_z} g}$$

$$\boxed{\omega_{n_z} = 0.699 \text{ rad/s}}$$

$$-\frac{5}{7} g k_{d_z} = 2\zeta_z \omega_{n_z}$$

$$k_{d_z} = -2\zeta_z \sqrt{\frac{7 A_\theta}{5 g A_z}}$$

For $M_p < 5\% \Rightarrow \zeta > 0.7$

Let $\zeta = 0.75$

$$k_{d_z} = -2(0.75)\sqrt{\frac{7(1 \deg)(\frac{\pi}{180})}{5(9.81 \, m/s^2)(0.25 m)}}$$

$$\boxed{k_{d_z} = -0.150}$$

(j) poles are roots of CLCE. From Matlab,

$$\boxed{s_{1,2} = -0.524 \pm j\, 0.462 \text{ rad/s}}$$