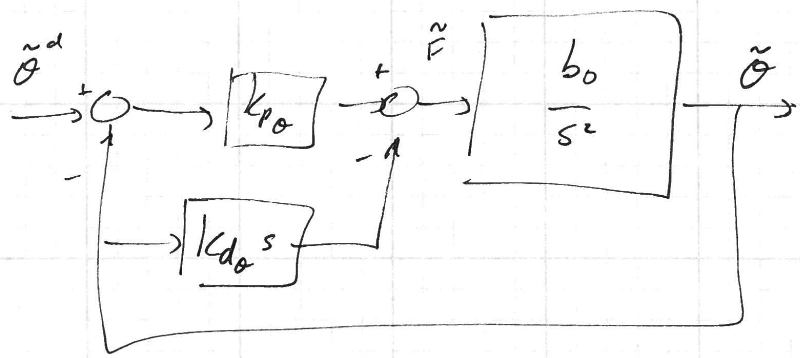
Homework E.11 - Solution

E.11

The block diagram for the inner loop is



where $b_0 = \dfrac{l}{\left(\frac{m_2 l^2}{3} + m_1 z_e^2\right)}$ and $z_e = \frac{1}{2} l$ is the equilibrium position

The closed loop transfer function is

$$\tilde{\theta}(s) = \frac{k_p b_0}{s^2 + k_D b_0 s + k_p b_0} \tilde{\theta}^d$$

The closed loop char eqn is

$$\Delta_{cl} = s^2 + k_D b_0 s + k_p b_0$$

The DC gain is

$$k_{DC_\theta} = \frac{k_p b_0}{k_p b_0} = 1$$

The desired char eq is $\Delta_{cl}^d = s^2 + 2\zeta_\theta \omega_{n_\theta} s + \omega_{n_\theta}^2$

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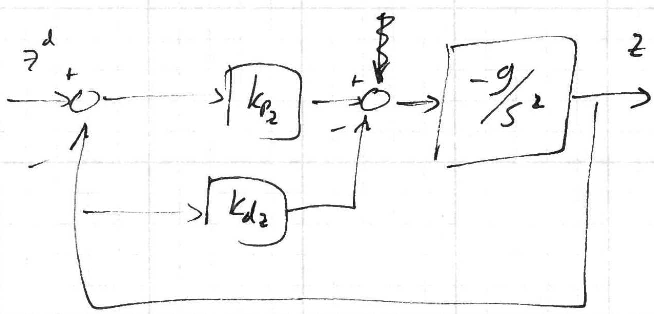
$$\therefore \quad k_P = \frac{\omega_{n_\theta}^2}{b_0}$$

$$k_D = \frac{2\zeta_\theta \omega_{n_\theta}}{b_0}$$

The block diagram for the outer loop (since $k_{DC_\theta} = 1$) is



The closed loop transfer function is

$$Z(s) = \frac{-g k_{P_z}}{s^2 - g k_{D_z} s + -g k_{P_z}} Z^d(s)$$

The closed loop char eq is

$$\Delta_{cl} = s^2 - g k_{D_z} s - g k_{P_z}$$

The desired closed loop char eq is

$$\Delta_{cl}^d = s^2 + 2\zeta_z \omega_{n_z} s + \omega_{n_z}^2$$

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$\therefore$

$$k_{D_z} = -\frac{2\zeta_z \omega_{n_z}}{g}$$

$$k_{P_z} = -\frac{\omega_{n_z}^2}{g}$$

The DC gain of the outer loop is

$$k_{DC_z} = 1$$

$b_0 = 2.6519$

$t_{r_\theta} = 1$

$\omega_{n_\theta} = 2.2$

$k_{P_\theta} = 1.8251$

$k_{D_\theta} = 1.173$

$k_{DC_\theta} = 1$

$t_{r_z} = 10$

$\omega_{n_z} = 0.22$

$k_{P_z} = -0.0049$

$k_{D_z} = -0.0317$