Homework #2:

Complete the following problems from HK: 3.20, 4.3(3), 4.14, 4.16, 4.21, 4.27

Submit a short (2-5 sentence) explanation of what you would like to do for your final project.

Complete the following two problems:

1. For a scalar nonautonomous differential equation in the form $\dot{x} = -a(t)x$, define sufficient conditions on $a(t)$, so that the equilibrium of the scalar dynamics is (1) stable and (2) asymptotically stable.
2. Consider the Euler equations,

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + \mu_1$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3 + \mu_2$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + \mu_3$$

where $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T \in R^3$ is the angular velocity with respect to an inertial frame from a body-fixed frame. And $I_1 > I_2 > I_3 > 0$ are the principle moments of inertial. Let $\boldsymbol{h}_G = [I_1\omega_1, I_2\omega_2, I_3\omega_3]^T$ represent the angular momentum of the body about its center of mass, G. $\boldsymbol{\mu} = [\mu_1, \mu_2, \mu_3]^T$ are the torques that represent the control of the system. When $\boldsymbol{\mu} = \boldsymbol{0}$ these are the Euler's equations for a free rigid body.

a) Assume $\boldsymbol{\mu} = \boldsymbol{0}.$ Show that $\|\boldsymbol{h}_G\|$ is conserved, i.e. $\frac{d}{dt}\|\boldsymbol{h}_G\| = 0.$
b) Find the equilibrium points of the system when $\|\boldsymbol{h}_G\| = \lambda \neq 0$, still assume $\boldsymbol{\mu} = \boldsymbol{0}$.
c) Linearize the system about each equilibrium point. What do the non-zero eigenvalues of the Jacobian matrix suggest about the local behavior of the solutions around each equilibrium point?
d) Use MATLAB to plot the phase portrait of the system on the surface of the angular momentum sphere. Let $I_1 = 3, I_2 = 2, I_3 = 1, and\ \lambda = 1$. Include MATLAB figure and code. (Hint: use ode45 to compute $\omega(t)$ for a variety of initial conditions that satisfy $\|\boldsymbol{h}_G\| = \lambda$; for each solution use plot3 to plot $\boldsymbol{h}_G(t)$.
e) Briefly describe what happens to a free rigid body spun around an axis very nearly aligned with the $\boldsymbol{b}_1$axis (first body axis). The $\boldsymbol{b}_2$ axis? The $\boldsymbol{b}_3$ axis?