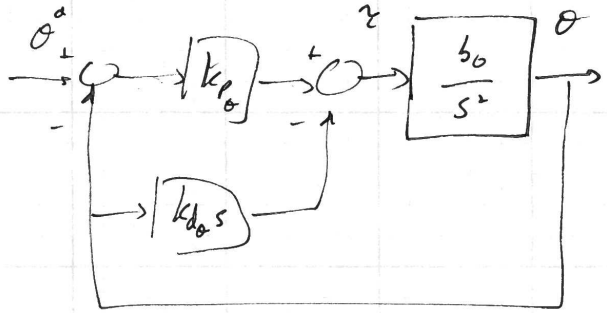
Homework G.11 - Solution

G.11 The block diagram for the inner loop is



where $b_0 = \frac{1}{J_t + 2m_r d^2}$

The closed loop transfer function is

$$\theta(s) = \frac{b_0 k_{p_\theta}}{s^2 + b_0 k_{d_\theta} s + b_0 k_{p_\theta}} \theta^d(s)$$

The closed loop char eq is

$$\Delta_{cl} = s^2 + k_D b_0 s + k_p b_0$$

The DC gain is

$$k_{DC_\theta} = 1$$

The desired char eq is

$$\Delta_{cl}^d = s^2 + 2\zeta\omega_{n_\theta} s + \omega_{n_\theta}^2$$

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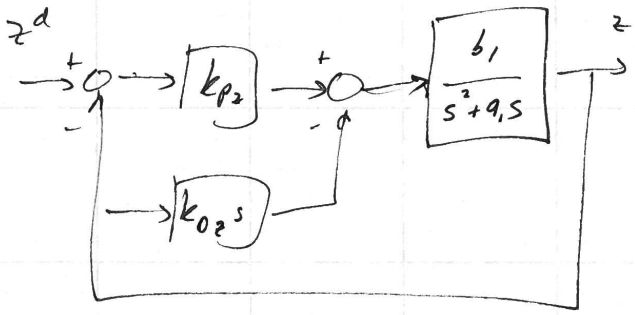
$$k_{p_\theta} = \frac{\omega_{n_\theta}^2}{b_0}$$

$$k_{D_\theta} = \frac{2\xi_\theta \omega_{n_\theta}}{b_0}$$

The block diagram for the outer loop is



~~The closed loop transfer~~

where $$b_1 = \frac{-F_0}{m_c + 2m_r}$$

$$a_1 = \frac{\mu}{m_c + 2m_r}$$

The closed loop transfer function is

$$Z(s) = \frac{b_1 k_{p_2}}{s^2 + (a_1 + b_1 k_{D_2}) s + b_1 k_{p_2}} z^d(s)$$

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Note that the DC-gain is

$$k_{DC_z} = 1$$

The closed loop char eq is

$$\Delta_{cl} = s^2 + (a_1 + b_1 k_{D_z}) s + b_1 k_{P_z}$$

The desired closed loop char eq is

$$\Delta_{cl}^d = s^2 + 2\zeta_z \omega_{n_z} s + \omega_{n_z}^2$$

$\therefore$

$$k_{P_z} = \frac{\omega_{n_z}^2}{b_1}$$

$$k_{D_z} = \frac{2\zeta_z \omega_{n_z} - a_1}{b_1}$$

$b_0 = 20.3253$

$t_{r_\theta} = 0.8$

$\omega_{n_\theta} = 2.75$

$k_{P_\theta} = 0.3721$

$k_{D_\theta} = 0.1913$

$b_1 = -9.81$

$a_1 = 0.0667$

$t_{r_z} = 8$

$\omega_{n_z} = 0.2750$

$k_{P_z} = -0.0077$

$k_{D_z} = -0.0328$