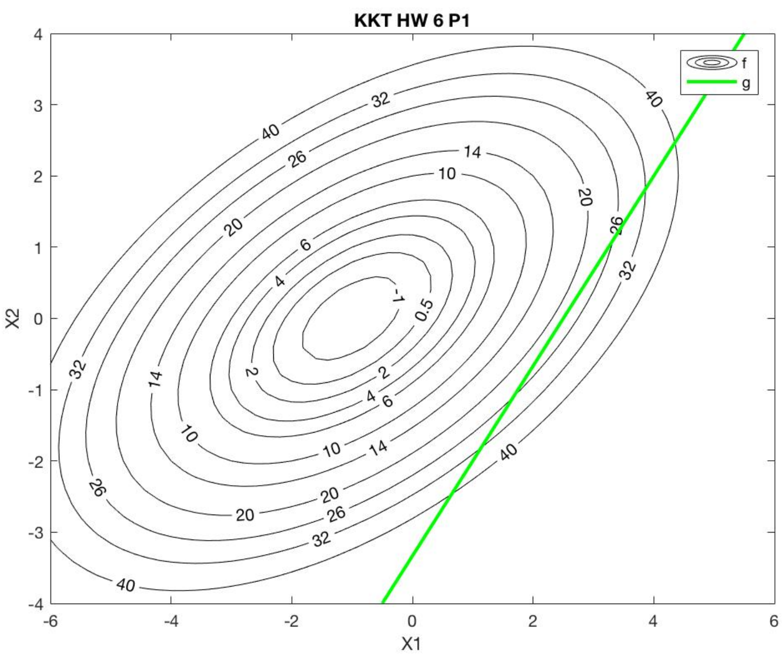
ME 575
Homework #6 Due: March 28 11:50 p.m.
KKT conditions, LaGrange Multipliers

Turn in through Learning Suite. Show your work.

1. (a) (4 points) Solve the following problem using KKT conditions (a contour plot is given):

$$Min\ f = 4x_1 - 3x_2 + 2x_1^2 - 3x_1x_2 + 4x_2^2$$
$$g_1(\mathbf{x}):\quad 2x_1 - 1.5x_2 = 5$$

Does the optimum from the KKT conditions agree with the graphical optimum?



(b) (3) Change the constraint to be,

$$g_1(\mathbf{x}):\quad 2x_1 - 1.5x_2 = 5.1$$

Solve again for the optimum. Does the Lagrange multiplier from (a) accurately predict the change in the objective? Compare the actual change to the predicted change.

(c) (3) Are the KKT equations for a problem with a quadratic objective and a linear equality constraint always linear? Is this true for a problem with a quadratic objective and a linear inequality constraint?

3. (6) Solve the following problem using the KKT conditions:

$$Min\ f(x) = x_1^2 + 2x_2^2 + 3x_3^2$$
$$g_1(x):\ x_1 + 5x_2 = 12$$
$$g_2(x):\ -2x_1 + x_2 - 4x_3 \le -18$$

4. For the problem:

$$Min\ f(x) = x_1^2 + x_2$$

$$g_1(x) = x_1^2 + x_2^2 - 9 = 0$$
$$g_2(x) = x_1 + x_2^2 - 1 \le 0$$
$$g_3(x) = x_1 + x_2 - 1 \le 0$$

A contour plot of this problem looks like:

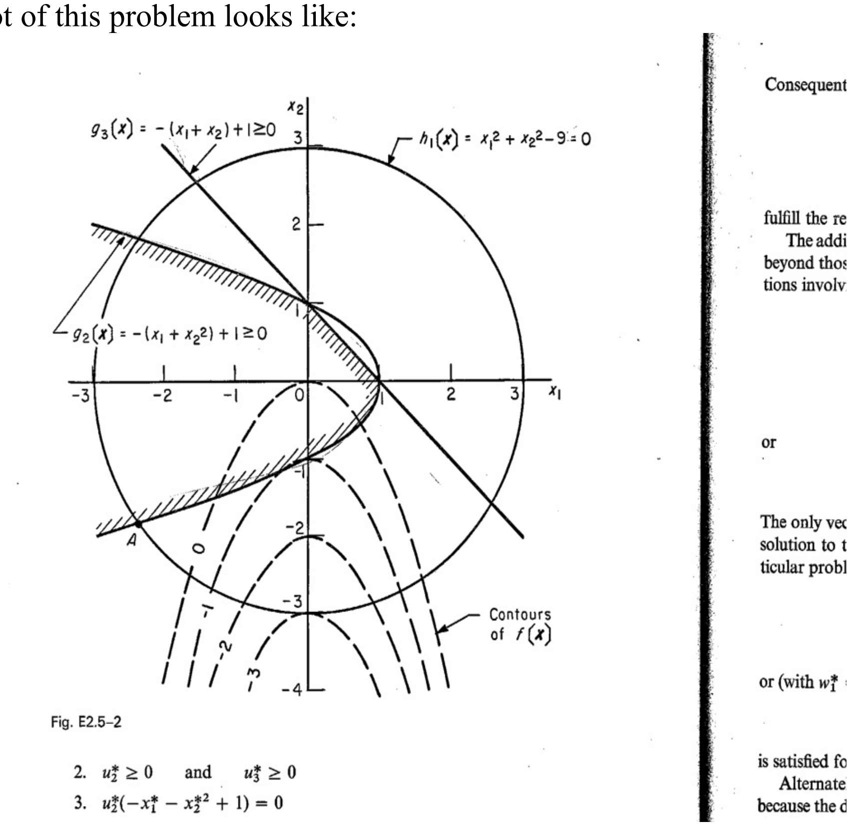


Figure taken from Himmelblau, David, *Applied Nonlinear Programming*. McGraw-Hill.

Using the KKT equations (constraints should be considered satisfied within acceptable round-off):

a. (4) Verify that the point $\lfloor -2.3723, -1.8364 \rfloor$ is a local optimum (point A)

b. (4) Verify that the point $\lfloor -2.5000, -1.6583 \rfloor$ is <u>not</u> a local optimum

c. (6) Drop the equality constraint from the problem. Using the contour plot above to see where the optimum lies (and thereby determine which constraints are binding), solve for the optimum using the KKT conditions. Note the equations will not be linear.