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ME EN 575 Optimization
Homework 1
Spring Design
Jan 19, 2018

Report

Objective
Maximize the force of a spring at its preloaded height, h0 by varying design parameters wire diameter, coil diameter, number of coils, and free height (d, D, n, hf respectively).

I. Main Optimization Results

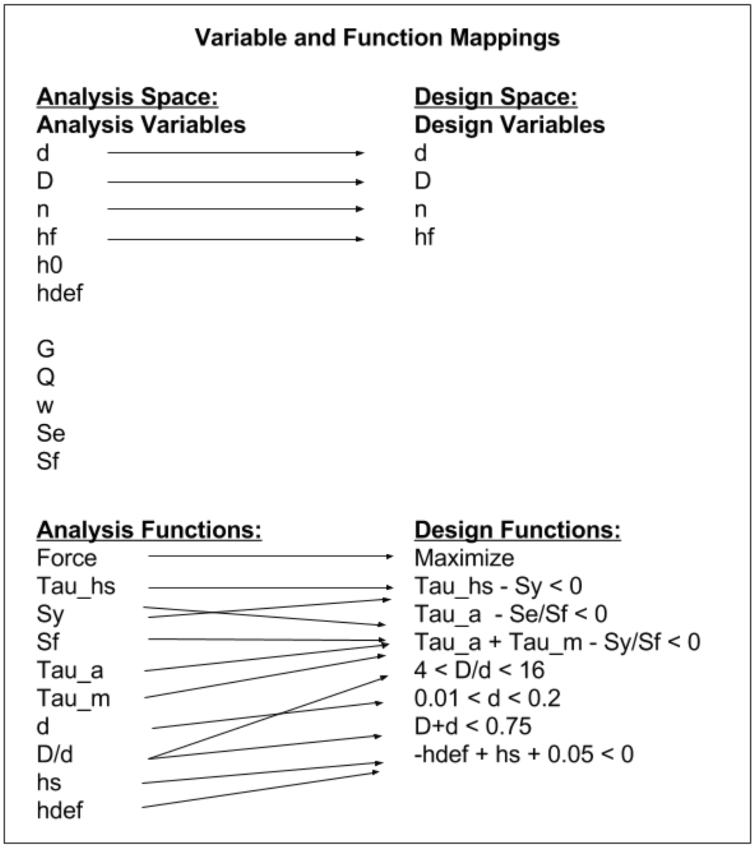
```
d = 0.0724
D = 0.6776
n = 7.5928
hf = 1.3691

F = 6.4541
k = 17.4854
K = 1.1561
hs = 0.55
Tau_m = 5.2224e+04
Tau_a = 1.8353e+04
Tau_hs = 7.5165e+04
Sy = 1.0586e+05
```

Constraints (c <= 0)

```
c(1) = Tau_hs - Sy                  - binding
c(2) = Tau_a - Se/Sf
c(3) = Tau_a + Tau_m - Sy/Sf        - binding
c(4) = (D/d) - 16                   - binding
c(5) = -(D/d) + 4
c(6) = d - 0.2
c(7) = -d + 0.01
c(8) = D + d - 0.75                 - binding
c(9) = -hdef + hs + 0.05            - binding
```

II. Optimization Setup

Table 1 Variable and Function Mappings



III. Results

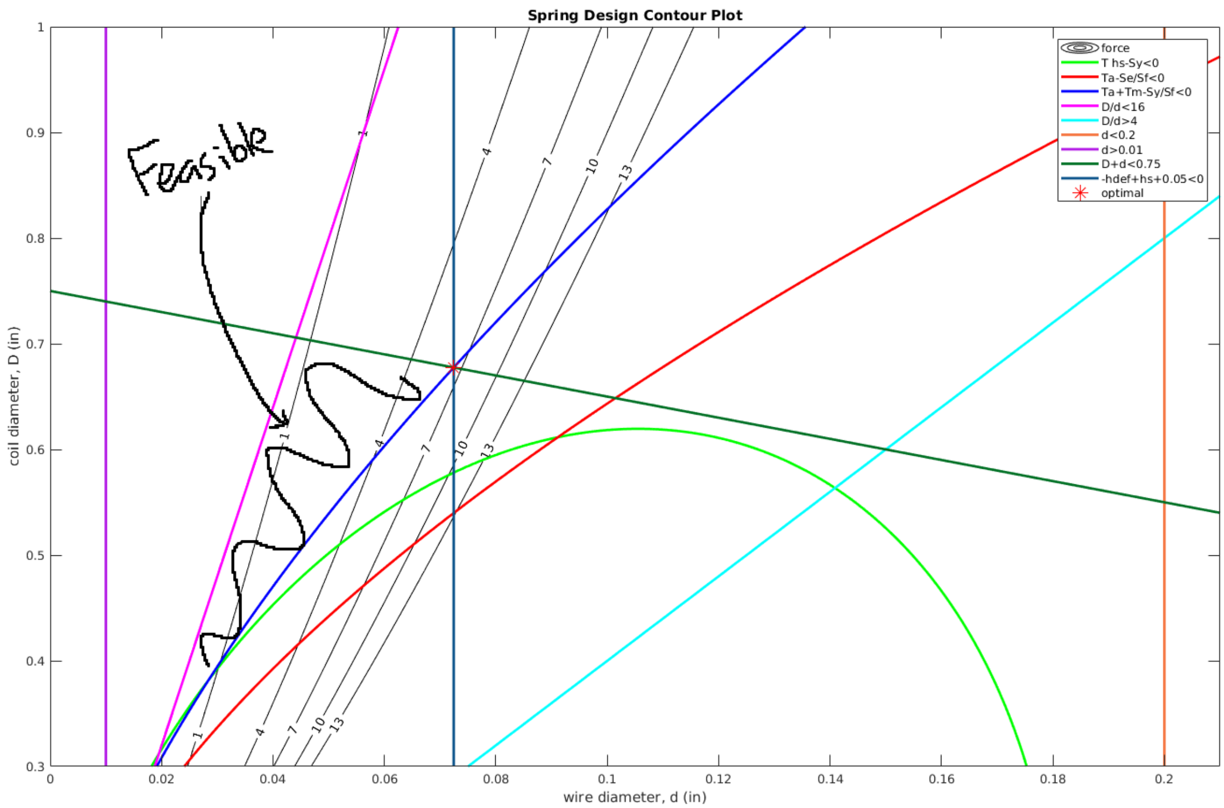
Table 2 Optimum Values (variables and functions)

Variables	Value
d	0.0724
D	0.6776
n	7.5928
hf	1.3691
Functions	Value
F	6.4541
k	17.4854
K	1.1561
hs	0.5500
Tau_m	5.2224e+04
Tau_a	1.8353e+04
Tau_hs	7.5165e+04
Sy	1.0586e+05

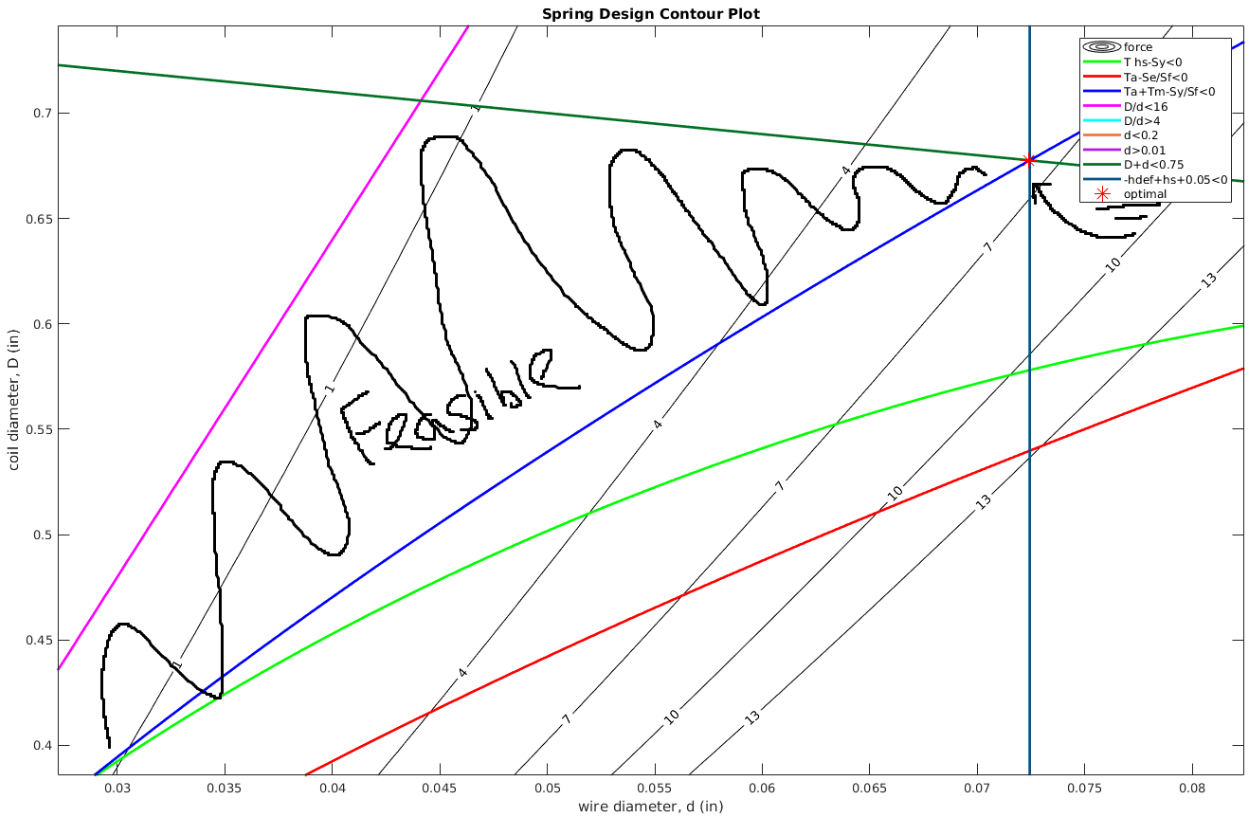
Table 3 Starting Points & Results

Starting Points				Optimized Results			
d	D	n	hf	d	D	n	hf
0.015	0.5	10.0	1.5	0.0724	0.6776	7.5928	1.3691
0.15	1.0	1.0	7.0	0.0724	0.6776	7.5928	1.3691
0.08	0.75	3.0	0.9	0.0724	0.6776	7.5928	1.3691
0.01	0.2	9.0	5.0	0.0724	0.6776	7.5928	1.3691
0.2	0.9	4.0	1.0	0.0724	0.6776	7.5928	1.3691

Zoomed Out Contour Plot of Design Space



Zoomed In Contour Plot of Design Space



<u>IV. Discussion</u>

Using MatLab's 'fmincon' funtion, an optimal design that meets all requirements and constraints was found. By starting the optimization at several different points in the design space, and arriving at the same result each time (see Table 3), there is high confidence that the optimum found represents the true optimum. By the same argument there is also evidence that this is both a local and a global optimum. As can be seen clearly by the plots, there are five bounding constraints. Since we are trying to maximize force, and force increases as we move to the right within the design space, we see that the optimum lies near the right-most corner of the feasible design space.

V. Appendix

Matlab Script:

```
function [xopt, fopt, exitflag, output] = spring_design()

    % ------------Starting point and bounds------------
    % design variables: d D n hf
    x0 = [0.015, 0.5, 10.0, 1.5];   % starting point
%     x0 = [0.15, 1.0, 1, 7.0];   % starting point
%     x0 = [0.08, 0.75, 3, 0.9];   % starting point
%     x0 = [0.01, 0.2, 9, 5.0];   % starting point
%     x0 = [0.2, 0.9, 4, 1.0];   % starting point
    ub = [0.2, 1.0, 50.0, 10.0];  % upper bound
    lb = [0.01, 0.1, 1.0, 1.0];  % lower bound

    % ------------Linear constraints------------
    A = [];
    b = [];
    Aeq = [];
    beq = [];

    % ------------Objective and Non-linear Constraints------------
    function [f, c, ceq] = objcon(x)

        % set objective/constraints here

        % design variables (things we'll adjust to find optimum)
        d = x(1);  % wire dia (in)
        D = x(2);  % coil dia (in)
        n = x(3);  % num coils
        hf = x(4); % free height (no load) (in)

        % other analysis variables (constants that the optimization won't touch)
        h0 = 1.0;           % preloaded height (in)
        delta0 = 0.4;       % deflection (in)
        hdef = h0 - delta0; % deflected spring height (in)
        G = 12e6;
        Q = 150e3;
        w = 0.18;
        Se = 45e3;
        Sf = 1.5;

        % delta_x = 0.4;  % not sure if this is right??
        delta_x = (hf - h0);  % maybe this instead???

        % analysis functions
        k = G*d^4/(8*D^3*n)
        F = k*delta_x
        K = ((4*D-d)/(4*(D-d)))+0.62*(d/D)
        % Tau = (8*F*D/pi*d^3)*K;
        hs = n*d
        F_min = k*(hf - h0);
        % F_max = F_min + delta0*k;
        F_max = k*(hf - (h0 - delta0));
        F_hs = k*(hf - hs);
        Tau_min = 8*F_min*D*K/(pi*(d^3));
```

```matlab
        Tau_max = 8*F_max*D*K/(pi*(d^3));
        Tau_m = (Tau_max + Tau_min)/2
        Tau_a = (Tau_max - Tau_min)/2
        Tau_hs = 8*F_hs*D*K/(pi*(d^3))
        Sy = 0.44*(Q/d^w)

        % objective function (what we're trying to optimize)
        f = -F;  % maximize Force

        % inequality constraints (c<=0)
        c = zeros(5,1);
        c(1) = Tau_hs - Sy;
        c(2) = Tau_a - Se/Sf;
        c(3) = Tau_a + Tau_m - Sy/Sf;
        c(4) = (D/d) - 16;
        c(5) = -(D/d) + 4;
        c(6) = d - 0.2;
        c(7) = -d + 0.01;
        c(8) = D + d - 0.75;
        c(9) = -hdef + hs + 0.05;

        % equality constraints (ceq=0)
        ceq = [];  % empty when we have none

    end

    % ------------Call fmincon------------
    options = optimoptions(@fmincon, 'display', 'iter-detailed');
    [xopt, fopt, exitflag, output] = fmincon(@obj, x0, A, b, Aeq, beq, lb, ub, @con, options);


    % ------------Separate obj/con (do not change)------------
    function [f] = obj(x)
            [f, ~, ~] = objcon(x);
    end
    function [c, ceq] = con(x)
            [~, c, ceq] = objcon(x);
    end
end
```

ME 575
Homework #1 Spring Design
Due Friday, January 19, 2:50 p.m.

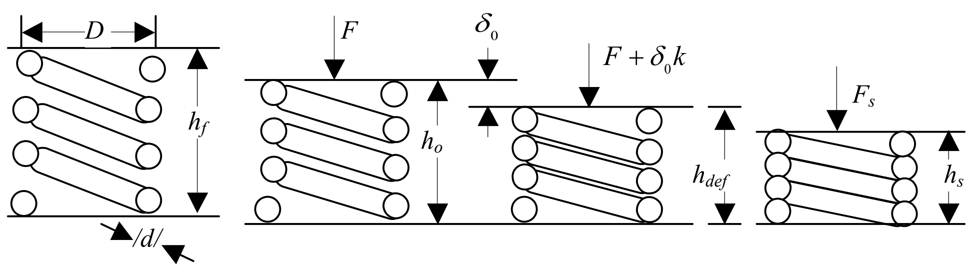
The specifications and modeling equations for compression spring design are given below. We wish to determine the spring design that maximizes the force of a spring at its preload height, h_o, of 1.0 inches. The spring is to operate an indefinite number of times through a deflection δ_o, of 0.4 inches, which is an additional deflection from h_o. The stress at the solid height, h_s, must be less than S_y to protect the spring from inadvertent damage.

The variables defining the design of a spring are d, D, n, h_o and h_f,

where	d	=	wire diameter
	D	=	coil diameter
	n	=	number of coils in the spring
	h_o	=	preload height
	h_f	=	free height (spring exerting no force)

and other variables/functions, as shown below, are,

δ_o	=	deflection from preload height
h_{def}	=	deflected height
h_s	=	solid height



The force in a linear spring is given by,

$$F = k\Delta x$$

where k is the spring stiffness and Δx is the deflection.

The spring stiffness is,

$$k = \frac{Gd^4}{8D^3 n}$$

where G is the shear modulus of the material. The stress in a spring with an axial load of F is,

$$\tau = \frac{8FD}{\pi d^3} K$$

where K is the Wahl factor that accounts for stress concentration due to curvature of the spring as well as direct shear:

$$K = \frac{4D - d}{4(D - d)} + 0.62 \frac{d}{D}$$

Solid height, h_s, is the height at which the coils of the compressed spring close up. It is simply,

$$h_s = nd$$

If the spring is to operate indefinitely through a deflection δ_0, it must be designed so that it does not fail in fatigue. A fatigue criterion for compression spring design is

$$\tau_a \le S_e / S_f$$

$$\tau_a + \tau_m \le S_y / S_f$$

where τ_m is the mean shear stress and τ_a is the alternating shear stress, defined to be,

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} \qquad \tau_a = \frac{\tau_{\max} - \tau_{\min}}{2}$$

and where S_f is a factor of safety, S_e is the endurance limit, and S_y is the yield strength in shear. S_f and S_e are constants, but S_y is a function of material properties Q and w, according to the relation,

$$S_y = 0.44 \frac{Q}{d^w}$$

Also, to be reasonable, the ratio D/d should be $4 \le D/d \le 16$. The diameters of wire should be $0.01 \le d \le 0.2$ inches. The maximum allowable width for the spring, i.e., $(D + d)$, is 0.75 inches. To insure that the spring does not reach solid height in service, a clash allowance of 0.05 inches should be provided. This means the solid height should be at least 0.05 inches below the lowest point of deflection the spring reaches in service.

For this problem, assume

G = 12 x 10^6 psi	S_f = 1.5
S_e = 45,000 psi	Q = 150,000 psi
w = 0.18	

Also assume the number of coils is continuous for optimization.

Sample Design:
(Note, no guarantee this is a feasible design)

```
wire diameter            0.050  (in)
coil diameter            0.500  (in)
number of coils          10.00
free height              1.500  (in)

Spring constant                 7.500 (lb/in)
Wahl Factor                     1.14533
Force at Preload height       3.75 (lb)
Alternating Stress        17499 (psi)
Mean Stress                     61248 (psi)
Yield Strength                  113170 (psi)
Clash Allowance                 0.1 (in)
Diameter Sum              0.55 (in)
```

Assignment: Using whatever programming language you wish, solve for the optimum for several starting points, as mentioned below. Also create the requested contour plots. Optional: Develop a table of derivatives.

Report:

1) Title page with main optimization results (include values for design variables and functions, and indicate which constraints are binding).
2) Setup:
    a. A table showing the mapping between analysis variables, design variables, analysis functions and design functions, similar to Figure 1.7 in the notes.
    b. Optional: Print out the gradients for the functions (derivatives of each design function with respect to the design variables). Comment on whether some scaling would be appropriate.
3) Results:
    a. A table showing the optimum values of all variables and functions (analysis and design). Indicate (with arrows, highlighter, etc.) binding constraints and/or variables at bounds. (Hint: use Courier font to keep values aligned.)
    b. A table giving several starting points which were tried along with the optimal objective value reached from each point.
    c. A "zoomed out" contour plot showing the design space (both feasible and infeasible space) for coil diameter vs. wire diameter, with all constraints shown and the feasible region shaded and optimum marked.
    d. A "zoomed in" contour plot of the design space (mostly feasible space) for coil diameter vs. wire diameter, with all constraints shown and the feasible region shaded and optimum marked.
4) Discussion:
    a. Include any observations or comments about the model, process of optimization or the design space. What did you learn about optimizing this problem? Do you feel this is a global optimum? Keep this a half page or less.
5) Appendix:
    a. Listing of `MATLAB or other files`
    b. Copy of the assignment

Turn in through Learning Suite as a pdf file.

Please note: any output from `MATLAB (or other language)` should be integrated into the report as given in the sections above. Tables and figures should all have explanatory captions. Please do **not** just staple pages of output to your assignment.