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ME EN 575 Optimization
Homework 2
Limestone Mill
Jan 26, 2018

Report

Objective

Minimize the total cost of starting and running a Limestone mill operation over the course of seven years by determining optimal pipe diameter, limestone slurry velocity, and average lump size of limestone particles after grinding (D, V and d respectively)

I. Main Optimization Results

```
D = 0.1819 ft
V = 7.2357 ft/s
d = 0.0005 ft

total_cost = $399,193.86
P_g = 174.3648 HP
P_f = 223.2327 HP
P_total = 397.5975 HP
c_slurry = 0.3999
Q_w = 0.1127
rho = 1.0484e+02
```

Constraints (`c <= 0`)

```
c(1) = D - 0.5
c(2) = -V + 1.1*V_c                    - binding
c(3) = c_slurry - 0.4                  - binding
```

<u>II. Procedure</u>

Part a and b:

Equation Sequence

Get values of design vars. from Optimization

D, V, d

Compute Slurry flow rate

$$Q = \frac{1}{4}\pi D^2 V$$

Compute Flow Rate of Limestone

$$Q_l = W/\gamma$$

Compute Flow Rate of water

$$Q_w = Q - Q_l$$

Compute Slurry Consentration

$$c = Q_l/Q$$

Compute Slurry density

$$\rho = \rho_w + c(\gamma - \rho_w)$$

Compute Power for Grinding

$$P_g = 218W\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{a}}\right)$$

Compute R_w

$$R_w = \frac{\rho_w V D}{\mu}$$

Compute f_w

$$f_w = 0.3164/R_w^{0.25} \quad \text{if } R_w \leq 10e^5$$

$$f_w = 0.0032 + 0.221 R_w^{-0.237} \quad \text{if } R_w \geq 10e^5$$

Compute $C_d R_p^2$

$$C_d R_p^2 = \frac{4g\rho_w d^3(\gamma - \rho_w)}{3\mu^2}$$

Compute f

$$f = f_w\left(\frac{\rho_w}{\rho} + 150c\frac{\rho_w}{\rho}\left(\frac{gD(s-1)}{V^2\sqrt{C_d}}\right)^{1.5}\right)$$

Compute ΔP

$$\Delta P = \frac{f\rho L V^2}{2Dg_c}$$

Compute Power for Pumping

$$P_f = \Delta P Q$$

Compute Initial Cost

$$\text{cost\_initial} = 300\left(P_g/550\right) + 200\left(P_f/550\right)$$

Compute Yearly Operating Cost

$$\text{cost\_yearly} = 0.07\left(\frac{P_g}{550}\right)8*300 + 0.05\left(\frac{P_f}{550}\right)8*300$$

Compute Net Present Value for Operating Costs

$$P_{cost} = \text{cost-yearly}\,\frac{(1+i)^n - 1}{i(1+i)^n}$$

Compute Total Cost

$$\text{cost\_total} = \text{cost\_initial} + P_{cost}$$

Compute V_c (for constraint)

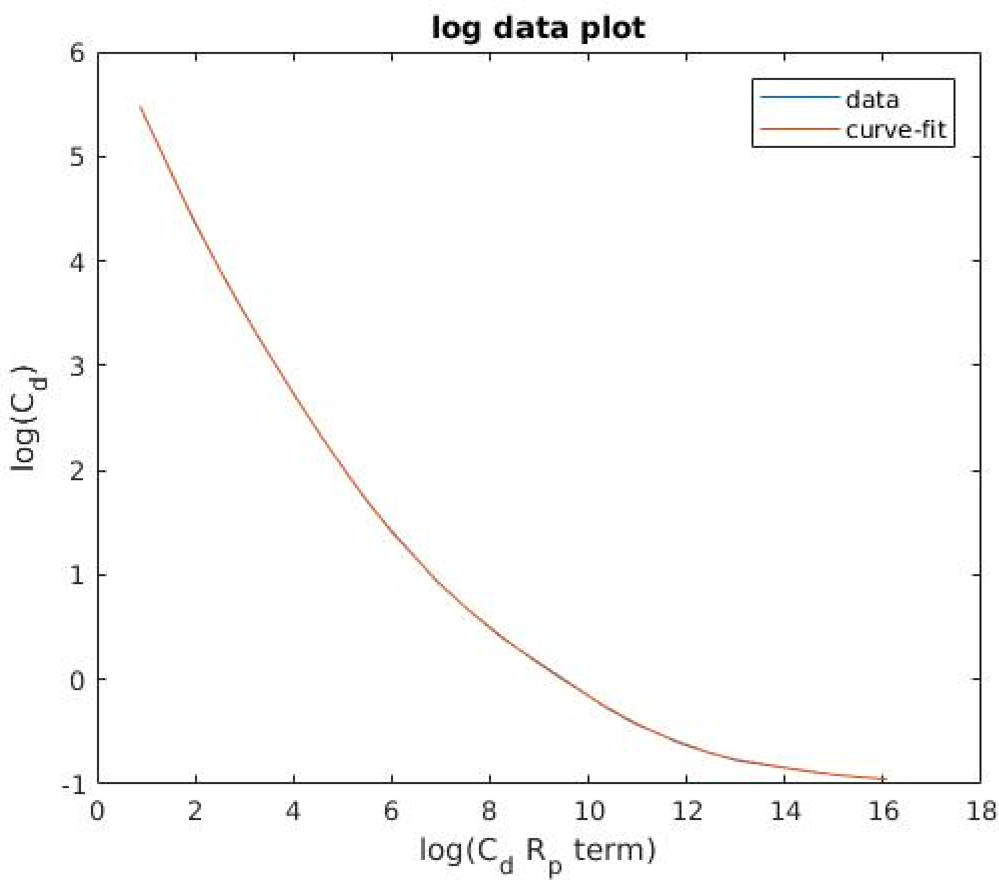
$$V_c = \left(\frac{40gc(S-1)D}{\sqrt{C_d}}\right)^{1/2}$$

Loop until optimum found

Part c:

Model validation was done by first carefully checking units in the supplied equations, and then by checking the results of each equation for feasibility. This was done by carefully stepping through the code using breakpoints. Ultimately the model was verified when it produced a correct result. In the real world where there is not a correct result to compare against, further verification would be necessary in the form of verification of the validity of the equations used, and a thorough model and design review by a team of engineers.

For determining the drag coefficient C_d, I fit a curve to the log of the data supplied. Curve fitting was accomplished by fitting a 10^{th} order polynomial to the log data using a least-squares approach I implemented in MatLab. Once I had the polynomial coefficients, I wrote a simple look-up function that used the coefficients to find the log value of C_d. This value was then exponentiated and the result passed out of the function as C_d. The accuracy of my curve fit was assessed first by visually inspecting the result, and then by computing the Mean Square Error (MSE). I found the fit to be exceptionally good with an MSE of `2.07e-05`.



III. Results and Discussion

Part a.

Optimum Values of Variables and Functions:

```
D = 0.1819 ft
V = 7.2357 ft/s
d = 0.0005 ft
c_slurry = 0.3999
Q_w = 0.1127
rho = 1.0484e+02

total_cost = $399,193.86
P_g = 174.3648 HP
P_f = 223.2327 HP
P_total = 397.5975 HP

**Highlight Indicates variable at constraints or bounds
```

Constraints (c <= 0)

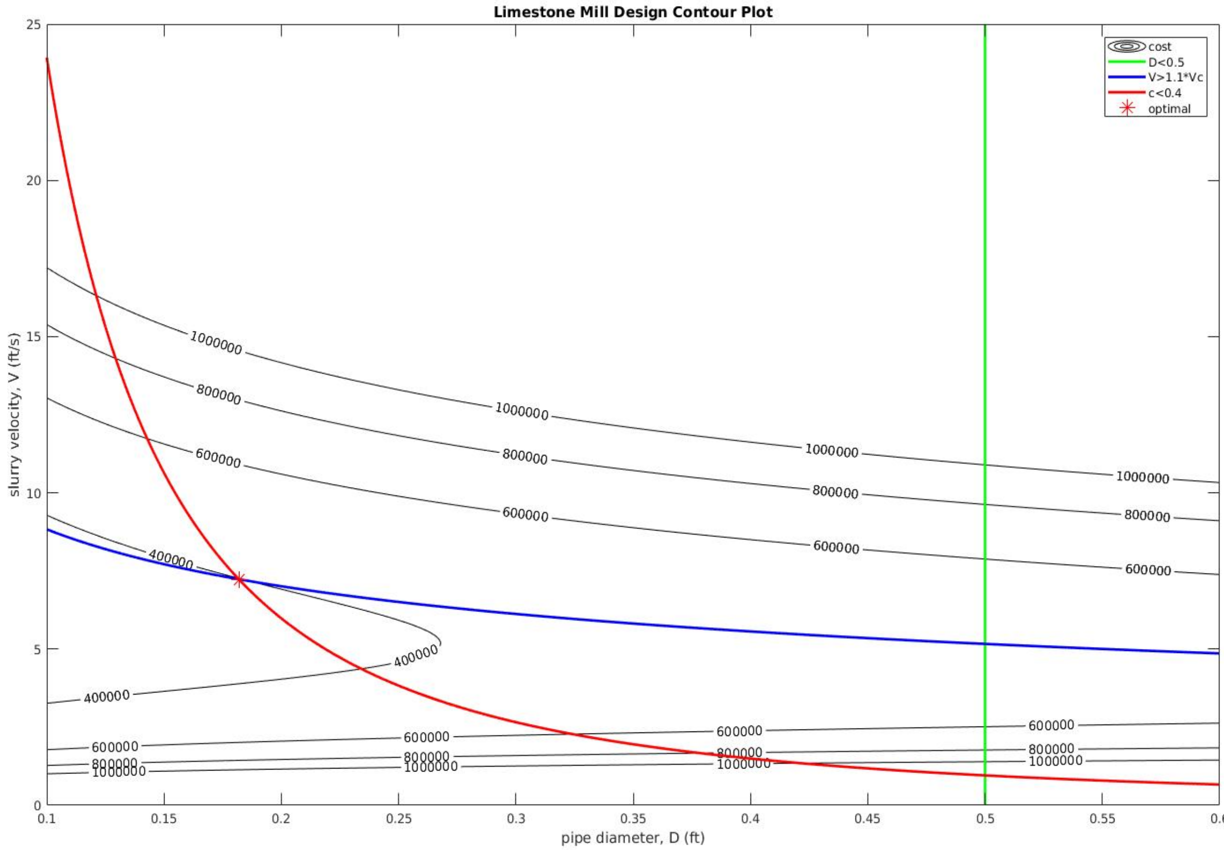
```
c(1) = D - 0.5
c(2) = -V + 1.1*V_c                    - binding
c(3) = c_slurry - 0.4                  - binding
```

Part b.

The optimum lies at the intersection of the slurry concentration constraint and the slurry velocity constraint. This makes these constraints binding constraints and the diameter constraint a non-binding constraint. As can be seen below, this location in the contour plot also corresponds to the lowest cost contour that is inside the feasible design space. This optimum is a local min and there is good evidence that it is also a global minimum because the result converges to this location even when the optimization is started outside the design space.

Part c.

Contour Plot of Design Space



Part d.

The primary observation I made was that this problem is very sensitive to small changes of the design variables as far as how much the total cost ends up being. This high sensitivity further highlights the importance of carefully validating the model.

IV. Appendix

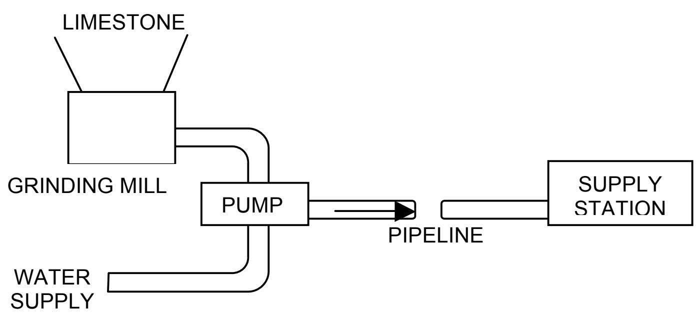
Matlab Script:
```
function [xopt, fopt, exitflag, output] = mill_optimization()

    % ------------Starting point and bounds------------
    % design variables: x0 = [D, V, d]
    x0 = [0.3, 10.0, 0.001];   % starting point
    ub = [1.0, 1000, 0.01];  % upper bound
    lb = [0.1, 0.5, 0.0005];  % lower bound

    % ------------Linear constraints------------
    A = [];
    b = [];
    Aeq = [];
    beq = [];

    % ------------Objective and Non-linear Constraints------------
    function [f, c, ceq] = objcon(x)

        % set objective/constraints here

        % design variables (things we'll adjust to find optimum)
        D = x(1);  % internal pipe dia. (ft)
        V = x(2);  % avg flow velocity (ft/sec)
        d = x(3);  % avg limestone particle size after grinding (ft)

        % other analysis variables (constants that the optimization won't touch)
        % L = 15;  % pipe length (miles)
        L = 15*5280;  % pipe length (ft)
        W = 12.67;  % flowrate of limestone (lbm/sec)
        als = 0.01;  % avg lump size of limestone before grinding (ft)

        gamma = 168.5;  % limestone density (lb_m/ft^3)
        rho_w = 62.4;  % water density (lb_m/ft^3)
        g = 32.17;  % gravity (ft/s^2)
        g_c = 32.17;  % conversion factor
        mu = 7.392e-4;  % water viscosity (lb_m/ft-sec)
        S = gamma/rho_w;  % specific gravity of the limestone

        % analysis functions
        Q = 0.25*pi*(D^2)*V;  % flow rate of the slurry (volumetric)
        Q_l = W/gamma;  % flow rate of limestone (volumetric)
        Q_w = Q - Q_l
        c_slur = Q_l/Q
        % c_w = (c_slur*gamma)/((1-c_slur)*rho_w + c_slur*gamma);  % consentration of solid by
weight in the slurry
        % rho = 1/(c_w/gamma + (1 - c_w)/rho_w);  % density of the slurry (lb_m/ft^3)
        rho = rho_w + c_slur*(gamma - rho_w)

        P_g = 218*W*((1/sqrt(d)) - (1/sqrt(als)));  % power for grinding (ft-lbf/sec)

        R_w = rho_w*V*D/mu;
        if R_w <= 10e5
            f_w = 0.3164/(R_w^0.25);
        else
            f_w = 0.0032 + 0.221*(R_w^-0.237);
```

```matlab
        end

        Cd_Rp_term = 4*g*rho_w*(d^3)*((gamma - rho_w)/(3*(mu^2)));
        C_d = cd_lookup(Cd_Rp_term);

        fric = f_w*((rho_w/rho)+150*c_slur*(rho_w/rho)*(g*D*(S-1)/((V^2)*sqrt(C_d)))^1.5)

        delta_p = (fric*rho*L*(V^2))/(D*2*g_c);

        P_f = delta_p*Q;

        total_power = (P_g + P_f)/550

        grinder_hp = P_g/550
        pump_hp = P_f/550

        V_c = ((40*g*c_slur*(S-1)*D)/(sqrt(C_d)))^0.5

        % COST STUFF
        initial_cost = 300*(P_g/550) + 200*(P_f/550);
        hrs_per_year = 8*300;  % 8 hrs per day, 300 days per year
        yearly_operating_cost = 0.07*(P_g/550)*hrs_per_year + 0.05*(P_f/550)*hrs_per_year;
        ir = 0.07;
        n = 7;  % number of years

        P = yearly_operating_cost*(((1 + ir)^n - 1)/(ir*(1 + ir)^n));

        total_cost = initial_cost + P

        % objective function (what we're trying to optimize)
        f = total_cost;  % minimize total cost

        % inequality constraints (c<=0)
        c = zeros(3,1);
        c(1) = D - 0.5;       % D <= 0.5
        c(2) = -V + 1.1*V_c;  % V >= 1.1*V_c
        c(3) = c_slur - 0.4;  % c_slur <= 0.4

        % equality constraints (ceq=0)
        ceq = [];  % empty when we have none

    end

    % ------------Call fmincon------------
    options = optimoptions(@fmincon, 'display', 'iter-detailed');
    [xopt, fopt, exitflag, output] = fmincon(@obj, x0, A, b, Aeq, beq, lb, ub, @con, options);

    % ------------Separate obj/con (do not change)------------
    function [f] = obj(x)
            [f, ~, ~] = objcon(x);
    end
    function [c, ceq] = con(x)
            [~, c, ceq] = objcon(x);
    end
end
```

ME 575
Homework #2 Pipeline Design
Due Jan 26 at 2:50 p.m.

For this assignment, you will gain additional experience in developing an engineering model and optimizing it. This problem includes experimental data, some conditional statements in the model, and an economic objective function. This problem is a modified version taken from James Siddall, *Optimal Engineering Design*, pp. 281-285, Dekker.

Minimize the total cost (capital and operating) for the source station (grinder and pump) for a pipeline which transports crushed limestone from a quarry to a terminal located some distance away, using water as a transporting medium. This will require that you determine the diameter of the pipeline, even though it will not be included in the cost.



SOURCE STATION

The limestone is crushed at the quarry, mixed with water to form a slurry, and pumped through the pipe.

The following specifications are given,

L = length of pipeline = 15 miles
W = flowrate of limestone = 12.67 lb_m/sec
a = average lump size of limestone before grinding = 0.01 ft.

The designer wishes to determine:

V = average flow velocity, ft/sec
c = volumetric concentration of slurry
D = internal diameter of pipe, ft.
d = average limestone particle size after grinding, ft.
Q_w = water flow rate, ft^3/sec
ρ = density of slurry lb_m/ft^3

but only three of the above values can be changed; the others are then determined. For example, the volumetric concentration of slurry can be expressed as,

$$c = \frac{Q_l}{Q_w + Q_l} = \frac{Q_l}{Q}$$

where,

Q = slurry flow rate (ft^3/sec)
Q_l = limestone flow rate (ft^3/sec) (fixed in problem statement)
Q_w = water flow rate (ft^3/sec)

Recall also that mass flow rate is $\dot{m} = \rho A V$ and volumetric flow rate is $Q = AV$.

Other Considerations:
The velocity, V, must exceed that at which sedimentation and clogging would occur. The formula for grinding power is not valid for a particle sizes below 0.0005 ft (particle size after grinding). The concentration of limestone in the pipe must be less than that at which pipe blockage would occur. The pipe diameter should not exceed 6 inches, above which the initial cost for the pipeline would be excessive.

The following expressions will be used to build a model:

Power for Grinding
The power for grinding is given by,

$$P_g = 218\ W\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{a}}\right) \tag{1}$$

where P_g has units of ft-lb_f/sec; W is in lb_m/sec and d, a are in ft. The constant 218 is a conversion factor that also has units; we will assume the units are such as to give P_g in ft-lb_f/sec.

Power for Pumping
The friction factor for the slurry is estimated by

$$f = f_w\left[\frac{\rho_w}{\rho} + 150c\frac{\rho_w}{\rho}\left(\frac{gD(S-1)}{V^2\sqrt{C_d}}\right)^{1.5}\right] \tag{2}$$

where

f_w = friction factor of water
g = acceleration due to gravity = 32.17 ft/sec^2
ρ_w = density of water = 62.4 lb_m/ft^3
C_d = average drag coefficient of the particles
S = specific gravity of the limestone (density of limestone divided by the density of water)

The friction factor of water is given by,

$$f_w = \frac{0.3164}{R_w^{0.25}} \quad \text{if} \quad R_w \le 10^5$$

$$f_w = 0.0032 + 0.221 R_w^{-0.237} \quad R_w \ge 10^5$$

where

$$R_w = \frac{\rho_w V D}{\mu}$$

where

ρ_w = density of water
V = Velocity
D = diameter of pipe
μ = viscosity of water = 7.392×10^{-4} $lb_m/(ft\text{-}sec)$

Equation (2) above contains C_d, the drag coefficient of the particles. The drag coefficient combines with Reynold's number for the particle as a dimensionless quantity which depends on particle diameter:

$$C_d R_p^2 = 4 g \rho_w d^3 \left(\frac{\gamma - \rho_w}{3\mu^2} \right) \tag{3}$$

where

γ = limestone density = 168.5 lb_m/ft^3
μ = viscosity of water ($lb_m/(ft\text{-}sec)$)
R_p = Reynolds number for the particle at terminal settling velocity (not calculated)

There is an empirical relationship between C_d and $C_dR_p^2$ defined by the following table:

C_d	240	120	80	49.5	36.5	26.5	14.6	10.4
$C_d R_p^2$	2.4	4.8	7.2	12.4	17.9	26.5	58.4	93.7

C_d	6.9	5.3	4.1	2.55	2.0	1.5	1.27	1.07
$C_d R_p^2$	173	260	410	1020	1800	3750	6230	10,700

C_d	0.77	0.65	0.55	0.5	0.46	0.42	0.40	0.385
$C_d R_p^2$	30,800	58,500	138,000	245,000	460,000	1,680,000	3,600,000	9,600,000

The slurry density can be expressed as,

$$\rho = \rho_w + c(\gamma - \rho_w) \quad \text{(units are } lb_m/ft^3\text{)} \tag{4}$$

The pressure drop in the pipe due to friction is given by,

$$\Delta p = \frac{f \rho L V^2}{D 2 g_c} \quad \text{(units are } lb_f/ft^2\text{)} \tag{5}$$

where

- f = friction factor for the slurry, given by (2) above
- ρ = density of slurry, given by (4) above
- L = length of pipeline
- V = velocity of slurry
- D = diameter of pipe
- g_c = conversion between lb_f and lb_m = $32.17 \dfrac{lb_m\text{-}ft}{lb_f\text{-}sec^2}$

Finally, the friction power loss is given by

$$P_f = \Delta p\, Q \quad \text{(units in ft-lb}_f\text{/sec)} \tag{6}$$

where

- Δp = is pressure drop from Eq. (5)
- Q = slurry flow rate (ft^3/sec)

Sedimentation

Sedimentation and clogging may occur if the velocity V is less than a critical velocity V_c. This velocity is estimated by the equation,

$$V_c = \left(\frac{40gc(S-1)D}{\sqrt{C_d}} \right)^{0.5}$$

As a factor of safety, we would like the slurry velocity to be 10% higher than V_c.

Pipe Blockage

Blockage can occur due to simply too high a fraction of solids in the slurry. If the particles were idealized into spheres of equal size and jammed together, the percent of unoccupied space, or voidage would be 26% or a concentration of 0.74. For irregular particles it is estimated that a safe concentration should be less than 0.4.

Cost

Cost should include the capital cost of the grinder and pump and the energy cost to operate the grinder and pump. As a first estimate, we will assume the cost of the grinder is \$300 per horsepower and the pump is \$200 per horsepower. Assume the plant will operate 8 hours per day, 300 days per year, with a plant life of seven years. The interest rate is 7%. For reasons we won't elaborate on, cost of energy is \$0.07 per hp-hr for the grinder and \$0.05 per hp-hour for the pump. Estimate the total cost using a net present value method.

Limits of Model

The model for grinding power is not valid for an average particle diameter below 0.0005 feet.

Comments

Note that this model requires a curve fit of some data to relate $C_d \,\mathrm{Re}^2$ to C_d. Make sure the curve fit is good enough that you do not introduce significant error in the problem (this implies the goodness-of-fit is high). There are several ways this might be approached, including curve fitting the log of the data.

As a "ballpark" value, the total power should be approximately 400 hp (±100).
Check your units!

Turn in a report with the following sections:

1) Title Page with Summary. The Summary should be short (less than 50 words), and give the values of the variables and objective at the optimum.

2) Procedure:
    a. Show the equation sequence in manner similar to that given for the Heat Pump example of Section 2.8.5 of the notes. This can be done by hand.
    b. Provide a table showing the mapping between analysis space and design space.
    c. Explain anything you did to validate the model. Discuss briefly how you handled the curve fit and how accurate your fit was.

3) Results and Discussion of Results:
    a. Provide a table showing the optimum values of variables and functions, with binding constraints and/or variables at bounds highlighted. Note, in addition to cost, include the power required by the pump and grinder in hp.
    b. Briefly discuss the optimum and the design space around the optimum. Do you feel this is a global optimum? Provide support for your conclusion.
    c. Include at least one contour plot with the feasible region and optimum marked. What did you learn from this plot?
    d. Include any other observations you feel are pertinent. These may relate to the model, the results, the optimization process, the nature of the optimum, etc. This section should not be longer than a paragraph.

4) Appendix:
    a. Listing of `MATLAB or other programs`
    b. Copy of the assignment

Please turn in as a pdf on Learning Suite. Note: Include requested items (such as graphs or tables) in their respective sections as given above, and not in the Appendix. Any output from MATLAB should be integrated into the report with captions, explanatory comments, etc.