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ASTR 8060
Homework 3

Learning goals: practice the S/N equation to compute S/N and exposure times for common astronomical situations.

1. Show that an error of 3% in flux units is very nearly the same as an error of 0.03 magnitudes.

2. For source A from HW2 (surface brightness of 1 MJy per steradian at 5500 Angstroms), suppose that you are observing with a V filter with a CCD camera on a 2.3 m diameter telescope like WIRO. How many photons will you collect per second in a single ccd pixel assuming that the pixel equals 1 square arcsecond on the sky and assuming you have a typical V filter bandpass width. Assume that the efficiency of the camera, CCD, and all the optics is 100%.

3. If a galaxy at a distance of 10 Mpc has a monochromatic luminosity of 1038 erg s^{-1} Angstrom^{-1} over the optical part of the spectrum, how many photons per second do you expect to detect with a 2.3 m telescope in the V filter, assuming your optics and CCD have a total efficiency of 50%? Assume you observe your source at 2 airmasses.

4. If a point source has a total count rate of 0.2 ph s^{-1}, the sky background has a count rate of 0.5 ph s^{-1} pixel^{-1}, the dark current is 10 electrons hour^{-1} pixel^{-1}, and the read noise is 5 electrons, how many 1-minute exposures does it take to reach a S/N of 100? Assume the stellar PSF is distributed over 4 pixels.

5. At WIRO with the prime focus imager (a 2.3m f/2.1 telescope with 13.5 μm pixels), assuming observations are taken at 1 airmass, detector $QE = 0.90$, and other telescope efficiencies are 0.70, how long would be required to achieve a S/N of 100 on a $V = 22$ magnitude star in the V filter if the moon phase is full ($\mu_V = 20$ mag arcsec^{-2})? Assume 1.1″ seeing. The WIRO Prime read noise is 4.5 electrons pixel^{-1} and the dark current is effectively zero. Compare your first answer to when the moon phase is new ($\mu_V = 22$ mag arcsec^{-2}, e.g. from Kitt Peak Imaging Manual). Are you detector, source, or background limited?

6. At Keck, an imager with a QE of 80% is used to image a stellar object with a S/N of 50 in 10 minutes through a narrowband filter of width 50 Å. Compute how long would be required to obtain the same S/N on WIRO using our imager with 95% QE and a broadband V filter. Assume the source noise limited case and state any other assumptions you needed to make to solve the problem.

1. Show that an error of 3% in flux units is very nearly the same as an error of 0.03 magnitudes.

Say $f_\nu = 3310$ (Jy)

$\sigma_\nu = 0.03(3310) = 99.3$ Jy

$\Delta^+ f_\nu = 3409.3$ Jy

$m_{AB} = -2.5\log(f_\nu [\text{Jy}]) + 8.9$

$m_{AB} = -2.5\log(3310) + 8.9 \rightarrow m_{AB} = 0.10$

$m_{AB} = -2.5\log(3409.30) + 8.9 \rightarrow m_{AB} = 0.068$

$\hookrightarrow \boxed{\Delta m_{AB} = 0.032 \sim 0.03}$

2. For source A from HW2 (surface brightness of 1 MJy per steradian at 5500 Angstroms), suppose that you are observing with a V filter with a CCD camera on a 2.3 m diameter telescope like WIRO. How many photons will you collect per second in a single ccd pixel assuming that the pixel equals 1 square arcsecond on the sky and assuming you have a typical V filter bandpass width. Assume that the efficiency of the camera, CCD, and all the optics is 100%.

- $\mu_{5500\,Å} = 1$ MJy ; Bandpass ~1000 Å on UVBRI filters
- from HW 2, flux for this source $6.46 \times 10^{-6} \frac{\text{phot}}{\text{s cm}^2\,Å\ \text{arcsec}^2}$

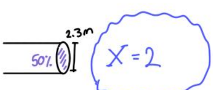
→ photons collected over 1□″, @ 5500Å, over 2.3m aperture, assuming 100% efficiency

$A_{WIRO} = \pi r^2 = \pi\left(\frac{2.3}{2}\right)^2 \rightarrow A_{WIRO} = \frac{4.15\text{m}^2 \mid 100^2\text{cm}^2}{\mid 1\text{m}^2} \rightarrow 41547.6\text{ cm}^2$

100%

$6.46\times10^{-6}\,\frac{\text{phot}}{\text{s}} \cdot \frac{1000\,Å}{Å} \cdot \frac{41547.6\text{cm}^2}{\text{cm}^2} \cdot \frac{1\,□''}{□''} \cdot 1 \longrightarrow \boxed{\text{Expect}: 268.4 \text{ phot/s/pix}}$

3. If a galaxy at a distance of 10 Mpc has a monochromatic luminosity of 1038 erg s^{-1} Angstrom^{-1} over the optical part of the spectrum, how many photons per second do you expect to detect with a 2.3 m telescope in the V filter, assuming your optics and CCD have a total efficiency of 50%? Assume you observe your source at 2 airmasses.



$L = 1038 \frac{\text{erg}}{\text{s}Å}$

$d = 10\times10^6$ pc

$f_\lambda = \frac{L_\lambda}{4\pi d^2} \rightarrow \frac{10^{38}\frac{\text{erg}}{\text{s}Å}}{4\pi(3\times10^{25}\text{cm})^2} \rightarrow f_\lambda \sim 9\times10^{-15}\frac{\text{erg}}{\text{s cm}^2 Å}$

$\hookrightarrow \frac{9\times10^{-15}\text{ erg}}{\text{s}Å\text{cm}^2} \Big| \frac{1\text{ phot}}{3.61\times10^{-12}\text{erg}} \rightarrow f_\lambda \sim 2.5\times10^{-3}\frac{\text{phot}}{\text{s cm}^2 Å}$

$d = \frac{10\times10^6\text{pc} \mid 3.09\times10^{18}\text{cm}}{\mid 1\text{pc}} \rightarrow d \sim 3\times10^{25}\text{cm}$

$V_{extinct} = 0.2$ from Notes

$\eta_{total\ efficiency} = 0.5 * (2.5^{(\text{airmass}\,\cdot\,\text{extinction})})^{-1}$

$0.5 * (2.5^{2\cdot0.2})^{-1} \rightarrow \eta_{tot} \sim 0.34$

$\hookrightarrow 2.5\times10^{-3}\,\frac{\text{phot}}{\text{s}} \cdot \frac{1000\,Å}{Å} \cdot \frac{41547.6\text{cm}^2}{\text{cm}^2} \cdot 0.34$

$\hookrightarrow \boxed{\text{expect}: \sim 35{,}315 \text{ ph/s/pix}}$

4. If a point source has a total count rate of 0.2 ph s^{-1}, the sky background has a count rate of 0.5 ph s^{-1} $pixel^{-1}$, the dark current is 10 electrons $hour^{-1}$ $pixel^{-1}$, and the read noise is 5 electrons, how many 1-minute exposures does it take to reach a S/N of 100? Assume the stellar PSF is distributed over 4 pixels.

$$S/N = \frac{R_S \cdot t}{\sqrt{(R_S \cdot t) + n_{pix}(R_B \cdot t + R_D \cdot t + N_R^2)}}, \quad PSF = 4\,pix$$

$1e^- = 1$ photon

- $R_S = 0.2$ ph/s
- $R_B = 0.5$ ph/s/pix
- $R_D = 10$ ph/hr/pix $\rightarrow 10 \frac{ph}{hr\,pix} \cdot \frac{1\,hr}{60\,min} \cdot \frac{1\,min}{60\,sec} \rightarrow 0.003$ ph/s/pix
- $N_R^2 = 5$ ph/pix

$$100 = \frac{0.2t}{\sqrt{0.2t + 4(0.5t + 0.003t + 5)}}$$

Desmos

$t \sim 553010\,s/60 \rightarrow$ $\boxed{\text{approx } 9217 \text{ exposures!}}$

5. At WIRO with the prime focus imager (a 2.3m f/2.1 telescope with 13.5 μm pixels), assuming observations are taken at 1 airmass, detector $QE = 0.90$, and other telescope efficiencies are 0.70, how long would be required to achieve a S/N of 100 on a $V = 22$ magnitude star in the V filter if the moon phase is full ($\mu_V = 20$ mag arcsec^{-2})? Assume 1.1″ seeing. The WIRO Prime read noise is 4.5 electrons pixel^{-1} and the dark current is effectively zero. Compare your first answer to when the moon phase is new ($\mu_V = 22$ mag arcsec^{-2}, e.g. from Kitt Peak Imaging Manual). Are you detector, source, or background limited?

R

V filter 1000 Å

$$S/N = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + \cancel{R_D t} + N_R^2)}}$$

? ?

→ detector stuff

- $A_{tele} = \pi\left(\frac{2.3}{2}\right)^2 \rightarrow A_{WIRO} = 4.15\,m^2$

Not 100% Efficient

$\eta_{tot} = QE \cdot OE \cdot (2.5^{X \cdot k})^{-1} \rightarrow 0.9 \cdot 0.7 \cdot (2.5^{(1 \cdot 0.2)})^{-1} \rightarrow \eta_{tot} = 0.52$

effective area: $A_{Eff} = A_{WIRO} \cdot 0.63 \rightarrow \underline{A_{Eff} = 2.61\,m^2} \rightarrow 26100\,cm^2$

- Plate Scale: $S = \frac{206265}{f}$, $f = RD \rightarrow 2.1 \cdot 2.3 \rightarrow f = 4.83\,m$ or $4.83 \times 10^3\,mm$, $\underline{S = 42.7\,[''/mm]}$

→ Get R_s!

get out of mags!

$V = 22_{mag} \rightarrow f_\nu = 3631 \cdot 10^{-0.4(22)} \rightarrow f_\nu = 5.8 \times 10^{-6}\,Jy \left| \frac{10^{-23}\,erg\,s^{-1}cm^{-2}Hz^{-1}}{Jy} \right. \rightarrow f_\nu = 5.8 \times 10^{-29}\,erg\,s^{-1}cm^{-2}Hz^{-1}$

$\frac{\partial \nu}{\partial \lambda} = \frac{c}{\lambda^2} = \frac{3 \times 10^{18}\,Å/s}{(5300Å)^2} \rightarrow 1.1 \times 10^{11}\,\frac{Hz}{Å} \implies f_\lambda = 5.8 \times 10^{-29}\,erg\,s^{-1}cm^{-2}\cancel{Hz^{-1}} \cdot 1.1 \times 10^{11}\,\cancel{Hz}\,Å^{-1} \rightarrow f_\lambda = 6.4 \times 10^{-18}\,erg\,s^{-1}cm^{-2}Å^{-1}$

- get to Phot/s → $E = \frac{hc}{\lambda} \rightarrow \frac{6.626 \times 10^{-27}\,ergs \cdot 3 \times 10^{18}\,\frac{Å}{s}}{5300\,Å} \rightarrow 3.8 \times 10^{-12}\,erg/phot$

↳ $f_\lambda = \frac{6.4 \times 10^{-18}\,erg\,s^{-1}cm^{-2}Å^{-1}}{3.8 \times 10^{-12}\,erg\,phot^{-1}} \rightarrow \underline{f_\lambda = 1.7 \times 10^{-6}\,phot\,s^{-1}cm^{-2}Å^{-1}}$

↳ for WIRO: $1.7 \times 10^{-6}\,\frac{phot}{s} \cdot \frac{26100cm^2}{cm^2} \cdot \frac{1000\,\cancel{Å}}{Å} \cdot 0.52 \rightarrow$ $\underline{\text{Source rate}: 23.1\,\frac{phot}{s}}$ $= R_s$!

→ Get n_{pix}!

need areas for n_{pix}

- seeing = 1.1″ → $\pi(1.1^2) \rightarrow A_{seeing} = 3.8\,\square''$
- $S = 42.7\,''/mm$
- WIRO size = 13.5 μm → $13.5 \times 10^{-3}\,mm$

Size $= 42.7\,''/mm \cdot 13.5 \times 10^{-3}\,mm \rightarrow P_{size} = 0.58''$

$P_{area} = 0.336\,\square''$

↳ $n_{pix} = \frac{3.8\,\square''}{0.336\,\square''/pix} \rightarrow \underline{n_{pix} \sim 11\,pix}$

→ Get R_B!

$\mu_V = 20\,\frac{mag}{\square''}$; $f_\nu = 3631 \times 10^{-0.4(20)} \rightarrow f_\nu = 3.6 \times 10^{-5}\,Jy/\square'' \longrightarrow f_\lambda = 1. \times 10^{-5}\,phot\,s^{-1}cm^{-2}Å^{-1}\,arcsec^{-2}$

↳ BKG rate: $135.7\,\frac{phot}{s\,\square''} \cdot \underline{0.336\,\square''} \longrightarrow \underline{R_B = 45.6\,\frac{phot}{s}}$

P_{area}

$$S/N = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + N_R^2)}}$$

- $R_S = 23.1\ \frac{phot}{s}$
- $n_{pix} = 11\,pix$
- $R_B = 45.6\ \frac{phot}{s}$
- $N_R^2 = 4.5\ phot/s$
- $S/N = 100$

Background limited

$$100 = \frac{23.1t}{\sqrt{23.1t + 11(45.6t + 4.5)}}$$

solve on desmos

$t = 9833\,s \,/\, 360 \rightarrow 27.314 \rightsquigarrow \boxed{t \sim 27\ \text{hours}}$

repeat for New moon, $M_V = 22\ mag/\square''$ comparable flux to obj $f_{\lambda_{moon}} \sim 1.7\times10^{-6}\ phot\,s^{-1}cm^{-2}\text{Å}^{-1}arcsec^{-2}$

$\hookrightarrow 23.1\ \frac{phot}{s\,\square''} \cdot 0.336\,\square'' \longrightarrow R_B = 7.8\ phot/s$

$$S/N = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + N_R^2)}}$$

- $R_S = 23.1\ \frac{phot}{s}$
- $n_{pix} = 11\,pix$
- $R_B = 7.8\ phot/s$
- $N_R^2 = 4.5\ phot/s$
- $S/N = 100$

$$100 = \frac{23.1t}{\sqrt{23.1t + 11(7.8t + 4.5)}}$$

Desmos

$t = 2041\,s \,/\, 360 \rightarrow 5.669 \rightsquigarrow \boxed{t \sim 6\ \text{hours}}$

6. At Keck, an imager with a QE of 80% is used to image a stellar object with a S/N of 50 in 10 minutes through a narrowband filter of width 50 Å. Compute how long would be required to obtain the same S/N on WIRO using our imager with 95% QE and a broadband V filter. Assume the source noise limited case and state any other assumptions you needed to make to solve the problem.

$\longrightarrow$ Source limited case: $S/N \sim \sqrt{R_s t}$

Find R_s! $\rightarrow 50 \sim \sqrt{0.8 R_s \cdot (10 \cdot 60 s)} \rightarrow 50 \sim \sqrt{0.8(600)}\sqrt{R_s} \rightarrow R_s \sim 5.21\ \frac{phot}{s}$

Source rate does not depend on detector, plug into WIRO?

$\rightarrow 50 \sim \sqrt{0.95 R_s t} \rightarrow 50 \sim \sqrt{0.95 \cdot 5.21\, t} \rightarrow \rightarrow 505.1 \sim t_{(s)} \rightarrow \boxed{t_{exp} \sim 8.4\ \text{minutes}}$

Is this legal?