Flux $\downarrow$ Flux Error $\downarrow$

HW #3

Brayan Hernandez

#1 $F + 0.03F$

$$m_{F+F_e} - m_F = -2.5\log_{10}\left(\frac{F+0.03F}{F}\right)$$

$$= -2.5\log_{10}\left(\frac{\cancel{F}(1+0.03)}{\cancel{F}}\right)$$

$$= -2.5\log_{10}(1.03) \boxed{\approx -0.032 \text{ mag}}$$

is about 0.03 mag

#2 $I_\nu = 1 \frac{MJy}{sr}$ $D = 2.3$ m $1 \text{ pix} = 1$ arcsec

$$(1'')^2 = \left(\frac{1}{206265} \text{ radians}\right)^2 = 2.37\times10^{-11} \frac{sr}{pix}$$

$$f = I_\nu \cdot \left(2.37\times10^{-11} \frac{sr}{pix}\right) = 2.37\times10^{-11} \frac{MJy}{pix} = 2.37\times10^{-5} \frac{Jy}{pix}$$

$$f_\nu = 2.37\times10^{-5} \frac{\cancel{Jy}}{pix} \cdot \left(10^{-26} \frac{W\cdot m^{-2}\cdot Hz^{-1}}{\cancel{Jy}}\right) = 2.37\times10^{-31} \frac{W}{m^2\cdot Hz\cdot pix}$$

$$f_\nu = 2.37\times10^{-31} \frac{\cancel{W}}{\cancel{m^2}\cdot Hz\cdot pix} \cdot 10^7 \frac{erg/s}{\cancel{W}} \cdot 10^{-4} \frac{\cancel{m^2}}{cm^2} = 2.37\times10^{-28} \frac{erg}{s\cdot cm^2\cdot Hz\cdot pix}$$

$$\nu = \frac{c}{\lambda}$$

$$\nu f_\nu = \lambda f_\lambda \Rightarrow f_\lambda = \frac{c}{\lambda^2} f_\nu = \frac{\left(3\times10^{18} \frac{\text{Å}}{s}\right)}{(5500^2 \text{Å}^2)} \left(2.37\times10^{-28} \frac{erg}{s\cdot cm^2\cdot \cancel{Hz}\cdot pix}\right)$$

$$f_\lambda = 2.35\times10^{-17} \frac{erg}{s\cdot cm^2\cdot \text{Å}\cdot pix}$$

Integrated flux $\Rightarrow$ Assume $\Delta\lambda \approx 1000$ Å

$$F = f_\lambda \cdot \Delta\lambda = 2.35\times10^{-14} \frac{erg}{s\cdot cm^2\cdot pix}$$

$\Downarrow$ continues

$D = 23m = 230cm$

#2 (cont.) $F = 2.35 \times 10^{-14} \frac{erg}{s \cdot cm^2 \cdot pix}$

$$L = F \cdot (Area) = 2.35 \times 10^{-14} \frac{erg}{s \cdot cm^2 \cdot pix} \cdot \left(\pi \left(\frac{230\ cm}{2} \right)^2 \right)$$

$$L = \cancel{}\ 9.78 \times 10^{-10} \frac{erg}{s \cdot pix}$$

Energy of photon (@ 5500Å): $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-27})(3 \times 10^{10})}{(5 \times 10^{-5} cm)}$

$$E_\gamma \approx 3.98 \times 10^{-12}\ erg$$

photon rate

$$n = \frac{L}{E_\gamma} = \frac{9.78 \times 10^{-10} \frac{erg}{s \cdot pix}}{3.98 \times 10^{-12}\ erg} \Rightarrow \boxed{\therefore\ n \approx 245.84 \frac{\cancel{photons}}{s \cdot pix}}$$

It says 1038 in the HW PDF but I believe it's suppose to be 10^{38} since it's a galaxy

#3 $d = 10\,\text{Mpc}$ $\quad L_\lambda = 10^{38}\,\frac{\text{erg}}{\text{s}\cdot\text{Å}}$ $\quad D = 2.3\,\text{m}$

$$L_\lambda = 4\pi d^2 f_\lambda \Rightarrow f_\lambda = \frac{L_\lambda}{4\pi d^2}\left[\frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{Å}}\right]$$

Integrated Flux: (assume $\Delta\lambda \approx 1000\,\text{Å}$)

$$F = f_\lambda \cdot \Delta\lambda = \frac{L_\lambda}{4\pi d^2}\cdot\Delta\lambda\left[\frac{\text{erg}}{\text{s}\cdot\text{cm}^2}\right]$$

$$L = F\cdot \underset{\text{area}}{A} = \frac{L_\lambda}{4\pi d^2}\cdot\Delta\lambda\cdot\pi\left(\frac{D}{2}\right)^2\left[\frac{\text{erg}}{\text{s}}\right]$$

Energy of photon (@ 5000 Å): $E_\gamma = \frac{hc}{\lambda}$

+

efficiency of 50% $\Rightarrow \therefore\ \epsilon = 0.5$

$$n = \epsilon\cdot\frac{L}{E_\gamma} = \epsilon\cdot\frac{L_\lambda}{4\pi d^2}\cdot\Delta\lambda\cdot\pi\left(\frac{D}{2}\right)^2\cdot\frac{\lambda}{hc}\left[\frac{\text{photons}}{\text{s}}\right]$$

$$n = (0.5)\frac{10^{38}\,\frac{\text{erg}}{\text{s}\cdot\text{Å}}}{\left(4\pi\cdot 10\,\text{Mpc}\cdot\frac{3.09\times10^{24}\,\text{cm}}{1\,\text{Mpc}}\right)^2}(1000\,\text{Å})\cdot\pi\left(\frac{230^2\,\text{cm}^2}{4}\right)\cdot\frac{\lambda}{hc}$$

$n \approx 4.35\times10^4$ Photons/s

Extinction coefficient for V mag = 0.2 $\frac{\text{mag}}{\text{airmass}}$

2 airmasses $\Rightarrow A_V = 0.4$ mag

↳ Reduction factor $= 10^{-0.4A_V} = 10^{-0.4(0.4)}$

≈ 0.69

$$n_{\text{final}} = 0.69\,(4.35\times10^4)\,\frac{\text{photons}}{\text{s}}\ \boxed{\approx 3.01\times10^4\,\frac{\text{photons}}{\text{s}}}$$

#4 for 1, 1 min exposure:

$$\frac{S}{N} = \frac{R_s \cdot t}{\sqrt{R_s t \cdot n_{pix}\left(R_B t + R_D t + R_N^2\right)}}$$

$$\frac{S}{N} = \frac{0.2(60s)}{\sqrt{0.2(60s) + 4\left(0.5(60s) + \frac{10}{3600}(60) + 5\right.}}$$

$$\frac{S}{N} \approx 0.97$$

For N, 1 min exposures

$$\left.\frac{S}{N}\right|_N = \frac{R_s \cdot t \cdot N}{\sqrt{N\left[R_s t \cdot n_{pix}\left(R_B t + R_D t + R_N^2\right)\right.}} = \sqrt{N}\,\frac{S}{N}$$

$$\left.\frac{S}{N}\right|_N = \sqrt{N}\left(\frac{S}{N}\right) \Rightarrow N = \left(\frac{\left.\frac{S}{N}\right|_N}{\frac{S}{N}}\right)^2 = \left(\frac{100}{0.97}\right)^2$$

$$\boxed{\therefore N \approx 1.06 \times 10^4 \text{ exposures}}$$

#5 $D = 2.3\,m$ $f/2.1$ Pixel size $= 13.5\,\mu m$

$QE = 0.9$ other efficiency $= 0.7$

total efficiency $\epsilon = 0.9 \cdot 0.7$

$f = 2.1 \times 2.3 = 4.83\,m$

plate scale: $s = \frac{206265''}{4.83\,m} = 4.27 \times 10^{4} \frac{''}{m}$

$\theta_{pix} = s \cdot (\text{pixel size}) = 4.27 \times 10^{4} \frac{''}{m} \cdot 13.5\,\mu m \cdot \frac{1\,m}{10^{6}\,\mu m}$

$\theta_{pix} = 0.577''$

pixel area $= \theta_{pix}^{2} = 0.332\ \text{arcsec}^{2}$

V-Band zero point: 3631 Jy

$f_\nu = (3631\ Jy) \cdot 10^{-0.4(22)} \approx 5.75 \times 10^{-6}\ Jy$

$f_\nu = 5.75 \times 10^{-6}\ \cancel{Jy} \cdot \frac{10^{-23}\ erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}}{\cancel{Jy}}$

$f_\nu = 5.75 \times 10^{-29}\ erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$

$\nu f_\nu = \lambda f_\lambda \Rightarrow f_\lambda = \frac{c}{\lambda^2} f_\nu = \frac{3 \times 10^{18}\ \cancel{Å} \cdot \cancel{Hz}}{(5500\,Å)^2} \cdot 5.7 \times 10^{-29} \frac{erg}{s \cdot cm^2 \cdot \cancel{Hz}}$

$f_\lambda = 5.71 \times 10^{-18}\ erg \cdot s^{-1} \cdot cm^{-2} \cdot Å^{-1}$

$F = f_\lambda \cdot \Delta\lambda \xRightarrow{\Delta\lambda = 1000\,Å} F = 5.71 \times 10^{-15} \frac{erg}{s \cdot cm^2}$

Area

$L = F \cdot A = 5.71 \times 10^{-15} \frac{erg}{s \cdot cm^2} \cdot \pi \left(\frac{230\ cm}{2}\right)^2$

$L \approx 2.37 \times 10^{-10} \frac{erg}{s}$

continues

#5 (cont.) $L = 2.37\times10^{-10}\ \frac{erg}{s}$

$E_\gamma = \frac{hc}{\lambda} \approx 4\times10^{-12}\ erg$

$$n = \frac{L}{E_\gamma} = \frac{2.37\times10^{-10}\ \frac{erg}{s}}{4\times10^{-12}\ erg} \approx 59.61\ \frac{photons}{s}$$

$$R_S = \epsilon n = 0.63\left(59.61\ \frac{photons}{s}\right) \sim 37.55\ \frac{pho}{s}$$

Now for the background

seeing $= 1.1''$ $\Rightarrow$ $p = \frac{1.1''}{2} = 0.55''$

$\wedge$ Nyquist sampling

Aperature Area $= \pi p^2$

$= \pi(0.55'')^2 = 0.95\ arcsec^2$

$$n_{pixels} = \frac{\text{Aperature Area}}{\text{Pixel Area}} = \frac{0.95\ \cancel{arcsec^2}}{0.332\ \cancel{arcsec^2}} = 2.86\ pix$$

~ 3 pixels

Sky Background

Full Moon: $\mu_V = 20\ \frac{mag}{arcsec^2}$

$$f_{\nu,FM} = (3631\ Jy)\cdot 10^{-0.4(20)} \approx 3.63\times10^{-5}\ Jy$$

$$\approx 3.63\times10^{-28}\ \frac{erg}{s\cdot cm^2\cdot Hz}$$

$$f_\lambda = \frac{c}{\lambda^2} f_\nu \approx 3.60\times10^{-17}\ \frac{erg}{s\cdot cm^2\cdot Å}$$

$$F_{FM} = 3.60\times10^{-14}\ \frac{erg}{s\cdot cm^2} \Rightarrow L_{FM} = F_{FM}\cdot A \approx 1.50\times10^{-9}\ \frac{erg}{s}$$

$$n_{FM} = \frac{\epsilon L}{E_\gamma} \approx 236.94\ \frac{photons}{s} \xRightarrow[\text{3 pix}]{\text{across}} R_{B,FM} = 78.98\ \frac{photons}{s\cdot pix}$$

$\Downarrow$ continues

#5 (cont.)

NEW MOON: $\mu_V = 22 \ \frac{mag}{arcsec^2}$

Same process as full moon $\Rightarrow$ yields $R_{B,NM} = 12.52 \ \frac{\text{Photons}}{s \cdot pix}$

$$\frac{S}{N} = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_B t + \underbrace{\cancel{R_D t}}_{\approx 0} + R_N^2)}}$$

$$\frac{S}{N} = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_B t + R_N^2)}}$$

FULL MOON:

$$100 = \frac{(37.55 \ \frac{ph}{s})t}{\sqrt{(37.55 \ \frac{ph}{s})t + 3(78.98 \ \frac{ph}{s \cdot pix} + 4.5)}}$$

plotted on desmos as a function of t. Looked at what t-value gave S/N = 100

$t = 1,946.52 s \ \boxed{\approx 32.44 \text{ min}}$

NEW MOON

Same eqn but w/ $R_B = 12.52 \ \frac{ph}{s \cdot pix}$

$t = 532.76 s \ \boxed{\approx 8.87 \text{ min}}$

$\#6 \quad R_s \propto \epsilon \, \Delta\lambda \, \overset{\text{Area}}{A}$

Source Limited $\Rightarrow \frac{S}{N} = \frac{R_s t}{\sqrt{R_s t}} = \sqrt{R_s t}$

$\therefore \frac{S}{N} \propto \left[\epsilon \Delta\lambda A t\right]^{1/2}$

$$\left.\frac{S}{N}\right|_{keck} \propto \left[(0.8)(50\text{Å})\left(\pi\left(\frac{10\,m}{2}\right)^2\right)(\overset{10\,min}{t_{keck}})\right]^{1/2}$$

$$\left.\frac{S}{N}\right|_{WIRO} \propto \left[(0.95)(1000\text{Å})\left(\pi\left(\frac{2.3}{2}\right)^2\right)(t_{WIRO})\right]^{1/2}$$

$$\frac{\left.\frac{S}{N}\right|_{keck}}{\left.\frac{S}{N}\right|_{WIRO}} = 1 = \frac{(0.8)(50\text{Å})\left(\pi \frac{100}{4} m^2\right) t_{keck}}{(0.95)(1000\text{Å})\left(\pi \frac{4.9}{4} m^2\right) t_{WIRO}}$$

$$\Rightarrow \therefore t_{WIRO} = \frac{(0.8)(50\cancel{\text{Å}})(\cancel{\pi}\, 25\, \cancel{m^2})}{(0.95)(1000\cancel{\text{Å}})(\cancel{\pi}\, \frac{4.9}{4}\cancel{m^2})} t_{keck}$$

$t_{WIRO} \approx 0.796\, t_{keck}$

$t_{keck} = 10\,min$

$$\boxed{t_{WIRO} \approx 7.96\ min}$$

#6 $R_s \propto \epsilon \, \Delta\lambda \, \overset{\text{Area}}{A}$

Source Limited $\Rightarrow$ $\frac{S}{N} \approx \frac{R_s t}{\sqrt{R_s t}} = \sqrt{R_s t}$

$$\therefore \frac{S}{N} \propto \left[\epsilon \Delta\lambda A t\right]^{1/2}$$

$$\left.\frac{S}{N}\right|_{\text{keck}} \propto \left[(0.8)(50\text{Å})\left(\pi\left(\frac{10\,\text{m}}{2}\right)^2\right)(\underset{}{\overset{10\,\text{min}}{t_{\text{keck}}}})\right]^{1/2}$$

$$\left.\frac{S}{N}\right|_{\text{WIRO}} \propto \left[(0.95)(1000\text{Å})\left(\pi\left(\frac{2.3}{2}\right)^2\right)(t_{\text{WIRO}})\right]^{1/2}$$

$$\frac{\left.\frac{S}{N}\right|_{\text{keck}}}{\left.\frac{S}{N}\right|_{\text{WIRO}}} = 1 = \frac{(0.8)(50\text{Å})\left(\pi \frac{100}{4} m^2\right) t_{\text{keck}}}{(0.95)(1000\text{Å})\left(\pi \frac{4.9}{4} m^2\right) t_{\text{WIRO}}}$$

$$\Rightarrow \therefore t_{\text{WIRO}} = \frac{(0.8)(50\cancel{\text{Å}})(\cancel{\pi}\, 25\, \cancel{m^2})}{(0.95)(1000\cancel{\text{Å}})(\cancel{\pi}\, \frac{4.9}{4} \cancel{m^2})} t_{\text{keck}}$$

$$t_{\text{WIRO}} \approx 0.796\, t_{\text{keck}}$$

$t_{\text{keck}} = 10\,\text{min}$

$$\boxed{t_{\text{WIRO}} \approx 7.96 \text{ min}}$$