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ASTR 8060
HW #3

1) Show that an error of 3% in flux units is very nearly the same as an error of 0.03 magnitudes.

We can use $m_v = -2.5\log\left(\frac{f}{F_v}\right)$ to show this relation, I'm borrowing numbers from HW #2 question 5...

$$m_v = -2.5\log\left(\frac{3.63\times10^{-5}\ Jy}{3540\ Jy}\right) \Rightarrow m_v = 19.97$$

We have magnitude $m_v = 19.97 \pm 0.03$, use $f = F_v \cdot 10^{-0.4(m_v)}$

- Add

$$f = 3540 \cdot 10^{-0.4(20.00)} = 3.54\times10^{-5}\ Jy$$

- Subtract

$$f = 3540 \cdot 10^{-0.4(19.94)} = 3.74\times10^{-5}\ Jy$$

Consider that 3% of 3.63×10^{-5} Jy is 0.1089×10^{-5}

- Add

$$m_v = -2.5\log\left(\frac{3.74\times10^{-5}\ Jy}{3540\ Jy}\right) \Rightarrow m_v = 19.94$$

- Subtract

$$m_v = -2.5\log\left(\frac{3.52\times10^{-5}\ Jy}{3540\ Jy}\right) \Rightarrow m_v = 20.00$$

So for m_v 19.97 ± 0.03
- $m_v = 20.00 \Leftrightarrow 3.54\times10^{-5}$ Jy
- $m_v = 19.94 \Leftrightarrow 3.74\times10^{-5}$ Jy

And for $f = 3.63\times10^{-5}\ Jy \pm 0.1089\times10^{-5}$
- $m_v = 20.00 \Leftrightarrow 3.52\times10^{-5}$ Jy
- $m_v = 19.94 \Leftrightarrow 3.74\times10^{-5}$ Jy

All very nearly the same!

2) Source A has surface brightness of 1 MJy per str @ 5500 Å

1 pixel = 1 arcsec^2

QE = 100%

2.3m diameter

V filter

ph s^{-1} for a single pixel

$R_S = \underset{1}{\eta_{total}} \cdot \underset{880Å}{\Delta\lambda} \cdot A \cdot f_\lambda$

Find Area

$A = \pi \left(\frac{230\ cm}{2}\right)^2 = 41547.56\ cm^2$

Use f_λ from HW #2 Q6

$f_\lambda = 6.46\times10^{-6}\ ph\ s^{-1}\ cm^{-2}\ Å^{-1}\ arcsec^{-2}$

$R_s = 1(880Å)(41547.56\ cm^2)(6.46\times10^{-6}\ ph\ s^{-1}cm^{-2}Å^{-1}arcsec^{-2})(1\ arcsec^2)$

$\boxed{R_s = 236\ ph/s}$ 2

3.) $R_s = Q \cdot \Delta\lambda \cdot Area \cdot f_\lambda$

- $QE = 50\%$
- $\Delta\lambda = 880\,Å$
- $Area = \pi\left(\frac{D}{2}\right)^2 = \pi\left(\frac{230\,cm}{2}\right)^2 = 41547.56\,cm^2$
- $f_\lambda = 2.40\times10^{-38}\,ph\,s^{-1}Å^{-1}cm^{-2}$
- $X = 2$ airmass

Find f_λ

$$f_\lambda = \frac{L}{4\pi d^2} = \frac{1038\,erg\,s^{-1}Å^{-1}}{4\pi(10\,Mpc)} \Rightarrow \frac{1038\,erg\,s^{-1}Å^{-1}}{4\pi(3.086\times10^{25}\,cm)^2} \Rightarrow f_\lambda = 8.67\times10^{-50}\,erg\,s^{-1}cm^{-2}Å^{-1}$$

$$1\,ph = 6.624\times10^{-27}\,erg\,s\left(\frac{3\times10^{16}\,Å/s}{5500\,Å}\right) = 3.61\times10^{-12}\,erg/ph$$

$$f_\lambda = \frac{8.67\times10^{-50}\,erg\,s^{-1}cm^{-2}Å^{-1}}{3.61\times10^{-12}\,erg/ph} = 2.40\times10^{-38}\,ph\,s^{-1}Å^{-1}cm^{-2}$$

$\eta_{atmo} = (2.5^{X\cdot k})^{-1}$ I've seen a couple different $\Rightarrow$ values for k_v from 0.15–0.20 $\Rightarrow \eta_{atmo} = (2.5^{2\cdot0.15})^{-1} \approx 0.75966$

$$R_s = 0.5(.76)(880\,Å)(41547.56\,cm^2)(2.40\times10^{-38}\,ph\,s^{-1}Å^{-1}cm^{-2})$$

$\boxed{R_s = 3.33\times10^{-31}\,ph\,s^{-1}}$ 3

This value seems to be very small and confusing. It's entirely possible that my calculations are wrong, but I was wondering if the luminosity is smaller than normal. It seems that monochromatic luminosities can range from $10^{35}-10^{46}$ which is much larger than 1038. If L = 1e38 or 10e38 then $R_s = 32117.142\,ph\,s^{-1}$ and $R_s = 321171.42\,ph\,s^{-1}$ respectively

4.) $R_S = 0.2 \text{ ph s}^{-1}$

$R_B = 0.5 \text{ ph s}^{-1} \text{pix}^{-1}$

$R_D = 10 \text{ e}^- \text{hr}^{-1} \text{pix}^{-1} \Rightarrow 0.00278 \text{ e}^- \text{s}^{-1} \text{pix}^{-1}$

$N_R = 5 \text{ e}^-$

Exposure time = 1 min = 60 sec

$n_{pix} = 4$

$$\frac{S}{N} = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_D t + R_D t + N_R^2)}}$$

$$\frac{S}{N} = \frac{(0.2 \text{ ph s}^{-1}) t}{\sqrt{(0.2 \text{ ph/s}) t + 4((0.5 \text{ ph s}^{-1} \text{pix}^{-1}) t + (0.00278 \text{ e}^- \text{s}^{-1} \text{pix}^{-1}) t + (5 \text{e}^-)^2}}$$

Plug in different values for t

- $t = 60 \Rightarrow \frac{S}{N} = 0.78$
- $t = 120 \Rightarrow \frac{S}{N} = 1.26$
- $t = 120\,000 \Rightarrow \frac{S}{N} = 46.58$
- $t = 420\,000 \Rightarrow \frac{S}{N} = 87.16$
- $t = 500\,000 \Rightarrow \frac{S}{N} = 95.10$
- $t = 552\,840 \Rightarrow \frac{S}{N} = 100.00 \Rightarrow$ $\boxed{9214 \text{ 1-minute exposures}}$

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5.) WIRO

- prime focus imager: 2.3m f/2.1 telescope w 13.5 μm pixels
- $X = 1$ airmass
- $QE = 0.40, 0.70 \Rightarrow 0.63$
- $S/N = 100$
- $V = 22$ mag star
- 1.1" seeing
- $N_R = 4.5\ e^- \text{pix}^{-1}$
- $R_D = 0$
- $t = ?$
- $R_B = \mu_V = 20$ mag arcsec^{-2} , $\mu_V = 22$ mag arcsec^{-2}
- V filter zeropoint = 3540 Jy

Find f_V

$$f_V = 3540\ \text{Jy} \cdot 10^{-0.4(22)} = 5.61\times10^{-6}\ \text{Jy} \cdot 10^{-23}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}$$

$$f_V = 5.61\times10^{-29}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}$$

$$f_\lambda = f_V \cdot \frac{c}{\lambda^2} = \frac{3\times10^{18}\ \text{Å/s}}{(5500\ \text{Å})^2} \cdot 5.61\times10^{-29}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1} \Rightarrow f_\lambda = 5.56\times10^{-18}\ \text{erg s}^{-1}\text{cm}^{-2}\ \text{Å}^{-1}$$

$$1\text{ph} = 6.624\times10^{-27}\ \text{erg s} \left(\frac{3\times10^{18}\ \text{Å/s}}{5500\ \text{Å}}\right) = 3.61\times10^{-12}\ \text{erg/ph}$$

$$f_\lambda = \frac{5.56\times10^{-18}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Å}^{-1}}{3.61\times10^{-12}\ \text{erg/ph}} = 1.54\times10^{-6}\ \text{ph s}^{-1}\ \text{cm}^{-2}\ \text{Å}^{-1}$$

$$\eta_{atmo} = (2.5^{X\cdot K})^{-1} \Rightarrow (2.5^{1\cdot0.15})^{-1} = 0.872$$

$$R_S = \eta_{total} \cdot \Delta\lambda \cdot \text{Area} \cdot f_\lambda$$

$$= (0.63)\cdot(0.87)(880)\left(\pi\left(\frac{230\text{ cm}}{2}\right)^2\right)(1.54\times10^{-6}\ \text{ph s}^{-1}\ \text{cm}^{-2}\ \text{Å}^{-1})$$

$$R_S = 30.92\ \text{ph s}^{-1}$$

HW #2
Q6

Find R_B

• For $\mu_V = 20$ mag arcsec^{-2}

$\mu_V = -2.5\log\left(\frac{f_V}{3540}\right)^{\leftarrow Jy/arcsec^2} \Rightarrow f_V = 3540 \cdot 10^{-0.4(20\ mag/arcsec^2)}$

$f_V = 3.54\times10^{-5}\ Jy/arcsec^2 \Rightarrow f_V = 3.54\times10^{-28}\ erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,arcsec^{-2}$

$f_\lambda = \frac{c}{\lambda^2}\cdot f_V = \frac{(3\times10^{18}\ \text{Å}/s)}{(5500\text{Å})^2}\cdot 3.54\times10^{-28}\ erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,arcsec^{-2}$

$f_\lambda = 3.51\times10^{-17}\ erg\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}\,arcsec^{-2}$

$\frac{3.51\times10^{-17}\ erg\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}\,arcsec^{-2}}{3.61\times10^{-12}\ erg/ph} \Rightarrow f_\lambda = 9.72\times10^{-6}\ ph\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}\,arcsec^{-2}$

$R_B = \eta_{total}\,(880\text{Å})\left(\pi\left(\frac{230}{2}\right)^2\right)\cdot f_V$

$= (0.63)(0.87)(880\text{Å})\left(\pi\left(\frac{230}{2}\right)^2\right)(9.72\times10^{-6}\ ph\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}\,arcsec^{-2})$

$R_B = 195.07\ ph\,s^{-1}\,arcsec^{-2} \Rightarrow (0.5765''/pix)^2 \Rightarrow 64.83\ ph\,s^{-1}\,pix^{-1}$

• For $\mu_V = 22$ mag arcsec^{-2}

$R_B = 30.92 \cdot (0.5765)^2 \Rightarrow 10.28\ ph\,s^{-1}\,pix^{-1}$

Find pixels

$F = 2.3m(2.1) = 4.83m = 4830mm$

$s = \frac{206265}{F} = \frac{206265}{4830} = 42.70''/mm$

$13.5\ \mu m = 0.0135\ mm$

$P = 42.70''/mm\,(0.0135) \approx 0.5765''/pixel$

$N = \frac{seeing}{P} = \frac{1.1''}{0.5765''/pixel} = 1.908$ pixels ≈ 2 pixels, $N_{pix} = 4$

I'm not sure if this is the right way to find Npix but I believe a similar method was used in the notes where 1" seeing with a 2m f/10 w/ 13 μm pixels had ~7 pixels and $N_{pix} = 49$

$s = 10.31''/mm$, $P = 10.31''/mm\,(0.013\ mm/pix) \approx 0.1341''/pix$

$N = \frac{seeing}{P} = \frac{1''}{0.1341''/pixel} = 7.4 \approx 7\ pixel \xRightarrow{(7)^2} N_{pix} = 49$

Plug into $\frac{S}{N}$ eqn

- For Full moonphase

$$\frac{S}{N} = 100 = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

$$100 = \frac{(30.92\ ph\,s^{-1})\,t}{\sqrt{(30.92\ ph\,s^{-1})t + 4_{pix}((64.83\ ph\,s^{-1}pix^{-1})t + 0 + (4.5\ e^{-}pix^{-1})^2)}}$$

$t = 3037$ sec ≈ 50.61 min ≈ 51 mins

- For New moon phase

$$100 = \frac{(30.92\ ph\,s^{-1})\,t}{\sqrt{(30.92\ ph\,s^{-1})t + 4_{pix}((10.28\ ph\,s^{-1}pix^{-1})t + 0 + (4.5\ e^{-}pix^{-1})^2)}}$$

$t = 755$ sec $\approx 12.58 \approx 13$ mins

Limited case

	R_B		R_S
• For Full moonphase: background limited	64.83	>>	30.92
• For New moonphase: object limited case	10.28	<<	30.92

6) Keck

- $QE_K = 80\%$
- $S/N_K = 50$
- $t_K = 10$ min
- $\Delta\lambda_K = 50 Å$

WIRO

- $QE = 95\%$
- $S/N_W = 50$
- $t_w = ?$
- $\Delta\lambda_w = 880 Å$

$$S/N_K = S/N_W$$

$$(Area)_K \cdot QE_K \cdot \Delta\lambda_K \cdot t_K \approx (Area)_w \cdot QE_w \cdot \Delta\lambda_w \cdot t_w$$

$$t_w = t_K \left(\frac{(Area)_K}{(Area)_w}\right) \cdot \frac{QE_K}{QE_w} \cdot \frac{\Delta\lambda_K}{\Delta\lambda_w}$$

$$t_w = 600s \cdot \left(\frac{\pi \left(\frac{10}{2}\right)^2}{\pi \left(\frac{2.30}{2}\right)^2}\right) \cdot \left(\frac{.80}{.95}\right) \cdot \frac{50 Å}{880 Å} \Rightarrow t_w = 542.69\ s \Rightarrow \boxed{t_w = 9.044\ min}$$

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