CONTENT

Problem 1

Hash table

Key Value	Probe Sequence
43	0
23	6
1	3
0	1
15	7
31	2
4	9
7	5
11	5, 6, 7, 8
3	7, 8, 9, 10
5	0, 1, 2, 3, 4
9	10, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

	Final Hash Table Contents
0	43
1	0
2	31
3	1
4	5
5	7
6	23
7	15
8	11
9	4
10	3

Problem 2

e

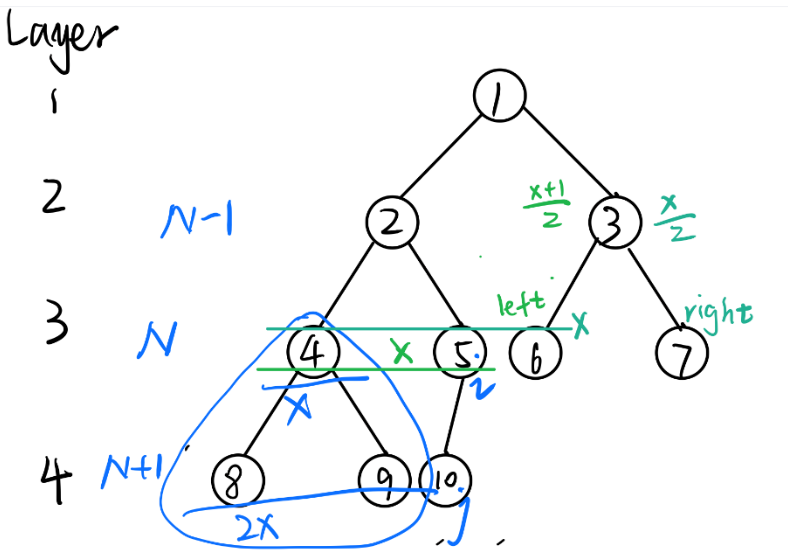
A collision occurs when two keys go through the hash function and arrive at the same slot. The hash function if h(k) = k mod m. If the table is not chosen well enough such like m is 2 or exact power of 2, a collision can easily occur. The hash table should have as many hash values as possible, so a prime number should be chosen for the table as size

Sample run.

```
For hash table with size m1:
min: 0
max: 23
average: 9.07216
variance: 34.9928

For hash table with size m2:
min: 0
max: 417
average: 9.22449
variance: 56.3122

For hash table with size m3:
min: 0
max: 209
average: 7.86
variance: 385.196

For hash table with size m4:
min: 0
max: 55
average: 8.9703
variance: 135.893
```

Problem 3

proof.



Consider a complete tree is a full tree except the most deep layer(last layer), and if the last layer is not full, the nodes is filled from left to right, i is the global index of a node and i start from 1.

Assume a node i in layer N with index i, and there are x nodes before this node in this layer.

We have the total number of nodes in layers N is 2^n-1, total number of nodes in layers N-1 is 2^{n-1}-1, and then i=2^{n-1}-1+x

The child nodes of node i are in layer N+1.

For left child node j, because every node before node x in layer N will have two child nodes which also before node j, we have j=$(2^n$-1)-1 +2x=2($2^{n-1} + x$-1)=2i

So the right child node j+1=2i+1.

Similarly, the parent node k of node i is in the layer N-1. Assume node i is the left child of node k, the parent node is (x+1)/2th in this layer. The total number of nodes in layer N-2 is 2^{n-2}-1

So the parent node k=(2^{n-2}-1) +(x+1/2) =((2^{n-1}+x-1)/2)=i/2

If node i is the right child of node k, the parent node k is the m/2th node.

k=(2^{n-2}-1) +(x)/2=((2^{n-1}+x-2)/2)=(i-1)/2

Therefore, the parent node of node i is $\lfloor i/2 \rfloor$