ABERDEENCOLLEGE®

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[Introduction](#)

6th Edition 2011 351

Formula Handbook

including
Engineering
Formulae,
Mathematics,
Statistics
and
Computer Algebra

Name________________________________

Course______________________________

http://ubuntuone.com/p/ZOF/ - pdf
http://ubuntuone.com/p/dAn - print
http://ubuntuone.com/p/ZOE/ - OOo (edit)

Introduction

This handbook was designed to provide engineering students at Aberdeen College with the formulae required for their courses up to Higher National level (2nd year university equivalent).

In order to use the interactive graphs you will need to have access to Geogebra (see 25). If you are using a MS Windows operating system and you already have Java Runtime Environment loaded then no changes will be required to the registry. This should mean that no security issues should be encountered. For Mac and Linux (and for MS Windows if you have problems)
see **http://www.geogebra.org/cms/en/portable**

It is typed in Open Office.org. Future developments will include more hyperlinks within the handbook and to other maths sites, with all the illustrations in it produced with Geogebra (see 25) or OOo.

Any contributions will be gratefully accepted and acknowledged in the handbook. If you prefer, you can make changes or add to the handbook within the terms of the Creative Commons licence . Please send me a copy of your work and be prepared to have it incorporated or adapted for inclusion in my version.
My overriding concern is for the handbook to live on and be continuously improved. I hope that you find the handbook useful and that you will enjoy using it and that that you will feel inspired to contribute material and suggest hyperlinks that could be added.

Many thanks to my colleagues at Aberdeen College for their contributions and help in editing the handbook. Special thanks are due to Mark Perkins at Bedford College who adopted the handbook for his students, helped to format the contents and contributed to the contents. Without Mark's encouragement this project would have never taken off.

If you find any errors or have suggestions for changes please contact the editor:
Peter K Nicol. (p.nicol@abcol.ac.uk) (peterknicol@gmail.com) **Contents**

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1 Recommended Books

referred to by author name in this handbook

1.1 Maths

General pre-NC and NC :	**Countdown to Mathematics**; Graham and Sargent Vol. 1 ISBN 0-201-13730-5, Vol. 2 ISBN 0-201-13731-3
NC	**Foundation Maths**, Croft and Davison ISBN 0-131-97921-3
NC and HN and Degree**:** **K Singh**	**Engineering Mathematics through Applications**; Kuldeep Singh, ISBN 0-333-92224-7. **www.palgrave.com/science/engineering/singh**
	Engineering Mathematics, 6th Edition, J Bird ISBN 1-8561-7767-X
HN and degree: **J Bird**	**Higher Engineering Mathematics**, 4th Edition, J Bird, ISBN 0-7506-6266-2
Degree	**Engineering Mathematics** 6th Edition , K A Stroud ISBN 978-1- 4039-4246-3

1.2 Mechanical and Electrical Engineering

NC	**Advanced Physics for You**, K Johnson, S Hewett et al. ISBN 0 7487 5296 X

Mechanical Engineering

NC and HN	**Mechanical Engineering Principles**, C Ross, J Bird ISBN 0750652284

Electrical Engineering

NC and HN	**Basic Electrical Engineering Science** Ian McKenzie Smith, ISBN 0-582-42429-1

2 Useful Web Sites

If you use any of the sites below please read the instructions first. When entering mathematical expressions **the syntax MUST be correct**. See section 26 of this book.
Most sites have examples as well as instructions. It is well worth trying the examples first.
If you find anything really useful in the sites below or any other site please tell us so that we can pass on the information to other students.

Efunda	A US service providing a wealth of engineering information on materials, processes, **Maths**, unit conversion and more. Excellent calculators (like quickmath). **http://www.efunda.com**
Freestudy	Mechanical engineering notes and exercises and Maths notes and exercises. **http://www.freestudy.co.uk**
matek.hu	An online calculator which also does calculus and produces graphs. (Based on Maxima). **http://www.matek.hu**
Mathcentre **MC**	Try the **Video Tutorials**. **http://www.mathcentre.ac.uk** The other stuff is excellent too. Also see **http://www.mathtutor.ac.uk**
QuickMath	Links you to a computer running MATHEMATICA - the most powerful mathematical software. **http://www.quickmath.com**
Mathway	Try the problem solver for algebra, trig and calculus and it draws graphs too. See 26 for input syntax. **http://www.mathway.com**
Mathsnet	Look under Curriculum for Algebra for some excellent online exercises. **http://www.mathsnet.net**
BetterExplained **BE**	It is true – maths and some other topics explained better. **http://BetterExplained.com/** how to learn maths **how to learn maths**
Just the Maths	A complete text book – all in pdf format **http://nestor.coventry.ac.uk/jtm/contents.htm**
WolframAlpha	Almost any maths problem solved! **http://www.wolframalpha.com/**
Khan Academy	The "free classroom of the World" Many video lectures using a blackboard **http://www.khanacademy.org**

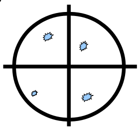
The Open University	There are a lot of excellent courses to study and if you want to improve your maths I suggest that you start here http://mathschoices.open.ac.uk/ Read the text very carefully on all the pages and then go to **http://mathschoices.open.ac.uk/routes/p6/index.html** and try the quizzes.
Plus Magazine	Plus magazine opens a door to the world of maths, with all its beauty and applications, by providing articles from the top mathematicians and science writers on topics as diverse as art, medicine, cosmology and sport. You can read the latest mathematical news on the site every week, browse our blog, listen to our podcasts and keep up-to-date by subscribing to Plus (on email, RSS, Facebook, iTunes or Twitter). **http://plus.maths.org/content/**
Paul's Online Math Notes	Recommended by June Cardno, Banff and Buchan College **http://tutorial.math.lamar.edu/**
Waldomaths	Some excellent interactive tools - Equations 1 and 2 in particular for transposition practice. **http://www.waldomaths.com/**
HND Engineer	As Alasdair Clapperton says “The aim of this website to assist, enlighten and inspire Scottish NC/HNC/HND engineering students within the current Scottish Government drive towards renewable energy targets”. **http://www.hndengineer.co.uk/**

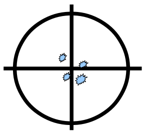
If you come across any Engineering or Mathematics sites that might be useful to students on your course please tell me (Peter Nicol) - p.nicol@abcol.ac.uk

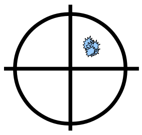
3 Evaluation

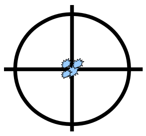
3.1.1 Accuracy and Precision

Example: Target = 1.234 - 4 possible student answers

Not Accurate, not Precise  1.270, 2.130, 0.835, 1.425

Accurate but not Precise  1.231, 1.235, 1.232, 1.236

Precise but not Accurate  1.276, 1.276, 1.276. 1,276

Precise and Accurate  1.234, 1.234, 1.234, 1.234

3.1.2 Units

Treat units as algebra -

for example $KE=\frac{1}{2}m\,v^2$ where $m=5\,kg$ and $v=12\frac{m}{s}$.

$$KE=\frac{1}{2}\times 5\times kg\times\left(\frac{12\times m}{s}\right)^2$$

$$KE=\frac{1}{2}\times 5\times kg\times\frac{12^2\times m^2}{s^2}$$

$$KE=\frac{1}{2}\times 5\times 12^2\times\frac{kg\times m^2}{s^2}$$

$$KE=360\frac{kg\,m^2}{s^2} \qquad KE=360\,J$$

Standard workshop tolerance ± 0.2 mm

3.1.3 Rounding

Do not round calculations until the last line.
Round to significant figures preferably in engineering form

Example: $A=\frac{\pi\,d^2}{4}$ where $d=40$

$A=1256.637061$ $\qquad A=1.256637061\times 10^3$

$A=1.257\times 10^3$ rounded to 4 sig fig ($A=1257$)

There should be **at least** 2 more significant figures in the calculation than in the answer.

4 Electrical Formulae and Constants

4.1 Basic

		Unit symbol
Series Resistors	$R_T = R_1 + R_2 + R_3 \ldots .$	Ω
Parallel Resistors	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots .$	Ω 8
Potential Difference	$V = I\,R$	V
Power	$P = I\,V$ or $P = I^2 R$ or $P = \frac{V^2}{R}$	W
Energy (work done)	$W = P\,t$	J or kWh
Frequency	$f = \frac{1}{T}$	Hz

4.2 Electrostatics

Series Capacitors	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots .$	F
Parallel Capacitors	$C_T = C_1 + C_2 + C_3 \ldots .$	F
Charge	$Q = I\,t$ or $Q = C\,V$	C
Capacitance	$C = \frac{A\,\varepsilon}{d} = \frac{A\,\varepsilon_0 \varepsilon_r}{d}$	F
Absolute Permittivity	$\varepsilon_0 \approx 8.854 \times 10^{-12}$	F/m

4.3 Electromagnetism

Magnetomotive Force	$F = I\,N$	At or A
Magnetisation	$H = \frac{I\,N}{\ell}$	At/m or A/m
Reluctance	$S = \frac{l}{\mu\,A} = \frac{l}{\mu_o \mu_r\,A}$	At/Wb or A/Wb
Absolute Permeability	$\mu_0 = 4\,\pi \times 10^{-7}$	H/m

4.4 AC Circuits

			Unit Symbol
Force on a conductor	$F = B\,I\,\ell$		N
Electromotive Force	$E = B\,\ell\,v$		V
Instantaneous emf	$e = E\sin\theta$		V
Induced emf	$e = N\dfrac{d\,\phi}{dt}$	$e = L\dfrac{di}{dt}$	V
RMS Voltage	$V_{rms} = \dfrac{1}{\sqrt{2}} \times V_{peak}$	$V_{rms} \approx 0.707\,V_{peak}$	V
Average Voltage	$V_{AV} = \dfrac{2}{\pi} \times V_{peak}$	$V_{AV} \approx 0.637\,V_{peak}$	V
Angular Velocity	$\omega = 2\,\pi\,f$		rad/s 17.7
Transformation Ratios	$\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} = \dfrac{I_p}{I_s}$		
Potential Difference	$V = I\,Z$		V
Power Factor	$\text{pf} = \cos\phi$		
Capacitive Reactance	$X_C = \dfrac{1}{2\pi\,f\,C}$		Ω
Inductive Reactance	$X_L = 2\pi\,f\,L$		Ω
Admittance	$Y = \dfrac{1}{Z}$		S
True Power	$P = V\,I\cos\phi$		W
Reactive Power	$Q = V\,I\sin\phi$		VAr
Apparent Power	$S = V\,I^*$	$= P + j\,Q$	VA

Note: I^* is the complex conjugate of the phasor current. See 17

Thanks to Iain Smith, Aberdeen College

5 Mechanical Engineering

[K Singh pp 2 – 98 especially 32 – 40 and 69 - 73]

5.1.1 Dynamics: Terms and Equations

Linear			**Angular**		
$s=$	displacement	(m)	$\theta =$	angular displacement	(rad)
$u=$	initial velocity	(m/s)	$\omega_1=$	initial velocity	(rad/s)
$v=$	final velocity	(m/s)	$\omega_2=$	final velocity	(rad/s)
$a=$	acceleration	(m/s^2)	$\alpha =$	acceleration	(rad/s^2)
$t=$	time	(s)	$t =$	time	(s)

5.1.2 Conversions

Displacement $s=r\theta$

Velocity $v=r\omega$ $\quad v=\frac{s}{t}$ $\quad \omega=\frac{\theta}{t}$

Acceleration $a=r\alpha$

2π radians = 1 revolution = 360^o, i.e. $1\,rad=\left(\frac{360}{2\pi}\right)^o\approx 57.3^o$ see 17.4.1

If N = rotational speed in revolutions per minute (rpm), then $\omega=\frac{2\pi N}{60}$ rad/s

5.2 Equations of Motion

Linear	**Angular**
$v=u+at$	$\omega_2=\omega_1+\alpha t$
$s=\frac{1}{2}(u+v)t$	$\theta=\frac{1}{2}(\omega_1+\omega_2)t$
$s=ut+\frac{1}{2}at^2$	$\theta=\omega_1 t+\frac{1}{2}\alpha t^2$
$v^2=u^2+2as$	$\omega_2^2=\omega_1^2+2\alpha\theta$
$a=\frac{v-u}{t}$	$\alpha=\frac{(\omega_2-\omega_1)}{t}$

5.3 Newton's Second Law

Linear

$\sum F = ma$

Angular

$\sum T = I\,\alpha$

where $T = F\,r$, $I = m\,k^2$
and k = radius of gyration

5.3.1 Centrifugal Force

$$\mathrm{CF} = \frac{m\,v^2}{r}$$

$$\mathrm{CF} = m\,\omega^2\,r$$

5.4 Work done and Power

	Linear	Angular
Work Done	$\mathrm{WD} = F\,s$	$\mathrm{WD} = T\,\theta$
Power	$P = \frac{\text{Work done}}{\text{Time taken}} = \frac{F\,s}{t} = F\,v$	$P = T\,\omega$

5.5 Energy

	Linear	Angular
Kinetic Energy	$\mathrm{KE} = \frac{1}{2}\,m\,v^2$	$\mathrm{KE} = \frac{1}{2}\,I\,\omega^2$ $\mathrm{KE} = \frac{1}{2}\,m\,k^2\,\omega^2$
Potential Energy	$\mathrm{PE} = m\,g\,h$	

KE of a rolling wheel = KE (linear) + KE (angular)

5.6 Momentum / Angular Impulse

Impulse = Change in momentum

Linear	**Angular**
$Ft = m_2 v - m_1 u$	$Tt = I_2 \omega_2 - I_1 \omega_1$

If the mass does not change: $Ft = m v - mu$

5.7 Specific force / torque values

Force to move a load: $F = \mu\, m\, g \cos\theta + m\, g \sin\theta + m\, a$

Force to hoist a load vertically $(\theta = 90^o)$ $F = m\, g + m\, a = m(g + a)$

Force to move a load
along a horizontal surface $(\theta = 0^o)$ $F = \mu\, m\, g + m\, a$

Winch drum torque $T_{app} = T_F + F_r + I\alpha$

5.8 Stress and Strain

Stress (σ) = load / area $\sigma = \frac{F}{A}$

Strain = change in length / original length $\varepsilon = \frac{\delta l}{l}$ or $\varepsilon = \frac{x}{l}$

$E =$ Stress / Strain $E = \frac{\sigma}{\varepsilon}$

Bending of Beams $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

2nd Moment of Area (rectangle) $I = \frac{b\, d^3}{12} + A\, h^2$

Torsion Equation $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

2nd Moment of Area (cylinder) $J = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$

Thanks to Frank McClean and Scott Smith, Aberdeen College

5.9 Fluid Mechanics

Mass continuity $\dot{m}=\rho A V$, or $\dot{m}=\rho A C$

Bernoulli's Equation $\frac{p}{\rho g}+\frac{C^2}{2g}+z$ = constant

or $\frac{p_1}{\rho g}+\frac{C_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{C_2^2}{2g}+z_2+z_F$

Volumetric flow rate $Q=A v$

Actual flow for a venturi-meter $Q_{actual}=A_1 c_d \sqrt{\dfrac{2gh\left(\dfrac{\rho_m}{\rho_f}-1\right)}{\left(\dfrac{A_1}{A_2}\right)-1}}$

Efunda Calculator

Actual flow for an orifice plate $Q=A_0 c_d \sqrt{\dfrac{2gh\left(\dfrac{\rho_m}{\rho_f}-1\right)}{1-\left(\dfrac{D_0}{D_1}\right)^4}}$

Reynold's number $\mathrm{Re}=\frac{\rho V D}{v}$ $\mathrm{Re}=\frac{V D}{\gamma}$ Efunda calculator

Darcy formula for head loss $h=\frac{4 f l v^2}{2 g d}$, $h=\frac{4 f l v^2}{2 d}$ energy loss

Efunda Calculator

5.10 Heat Transfer

Through a slab $\dot{Q}=\frac{k A(T_1-T_2)}{x}$

Through a composite $\dot{Q}=\frac{\Delta T}{\Sigma R}$ where $\Sigma R=\frac{x_1}{k_1}+\frac{x_2}{k_2}+\frac{1}{h_1}+\frac{1}{h_2}+\ldots$

Through a cylindrical pipe $\dot{Q}=\frac{\Delta T}{\Sigma R}$

where $$\Sigma R=\frac{1}{2\pi R_1 h_1}+\frac{\ln\left(\dfrac{R_2}{R_1}\right)}{2\pi k_1}+\frac{\ln\left(\dfrac{R_3}{R_2}\right)}{2\pi k_2}+\frac{1}{2\pi R_3 h_3}$$

5.11 Thermodynamics

Boyle's Law	$p_1 V_1 = p_2 V_2$
Charles's Law	$\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$
Combined Gas Law	$\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$
Perfect Gas	$pV = mRT$
Mass flow rate	$\dot{m} = \rho A C$
Polytropic Process	pV^n = constant
Isentropic Process (reversible adiabatic)	pV^γ = constant where $\gamma = \dfrac{c_P}{c_V}$
Gas constant	$R = c_p - c_v$
Enthalpy (specific)	$h = u + pv$
Steady flow energy equation	$\dot{Q} = \dot{m}\left(h_2 - h_1 + \dfrac{C_2^2}{2} - \dfrac{C_1^2}{2} + g(z_2 - z_1)\right) + \dot{W}$
Vapours	$v_x = x v_g$
	$u_x = u_f + x(u_g - u_f)$
	$h_x = h_f + x(h_g - h_f)$ or $h_x = h_f + x h_{fg}$

Thanks to Richard Kaczkowski and Scott Smith, Aberdeen College.

6 Maths for Computing

a_n a to the base n

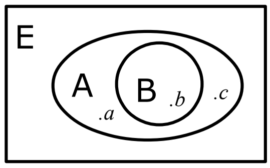
a_{10}	decimal; denary	(a d)	a_2	binary	(a b)
a_{16}	hexadecimal	(a h)	a_8	octal	(a o)

10^3	(1000)	kilo		2^{10}	(1024)	kilobyte
10^6		Mega		2^{20}	(1024^2)	megabyte
			but			
10^9		Giga		2^{30}	(1024^3)	gigabyte
10^{12}		Tera		2^{40}	(1024^4)	terabyte
10^{15}		Peta		2^{50}	(1024^5)	petabyte

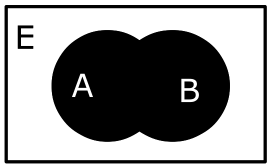
6.1.1 Notation for Set Theory and Boolean Laws

[J Bird pp 377 - 396]

E	universal set
$A=\{a, b, c\, ...\}$	a set A with elements a, b, c etc
$a\in A$	a is a member of A
$\{\ \}$	the empty set ($\emptyset$ is also used)
$B\subset A$	B is a subset of A

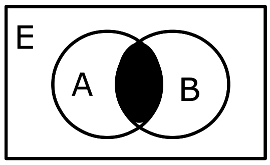


$B\subset A$

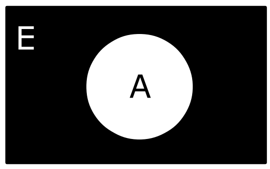


$A\cup B$ $A+B$

Set theory			**Boolean**	
$\cup$ union		$\vee$	$+$	**OR**
$\cap$ intersection		$\wedge$	$\cdot$	**AND**
A' complement of A			$\overline{A}$	**NOT**



$A\cap B$ $A\cdot B$



A' $\overline{A}$

7 Combinational Logic

$A+0=A$ $\quad$ $A\cdot 0=0$

$A+1=1$ $\quad$ $A\cdot 1=A$

$A\cdot A=A$ $\quad$ $A+A=A$

$A\overline{A}=0$ $\quad$ $A+\overline{A}=1$

$\overline{\overline{A}}=A$

$A\cdot B=B\cdot A$ $\quad$ $A+B=B+A$

$A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

$A+(B\cdot C)=(A+B)\cdot(A+C)$

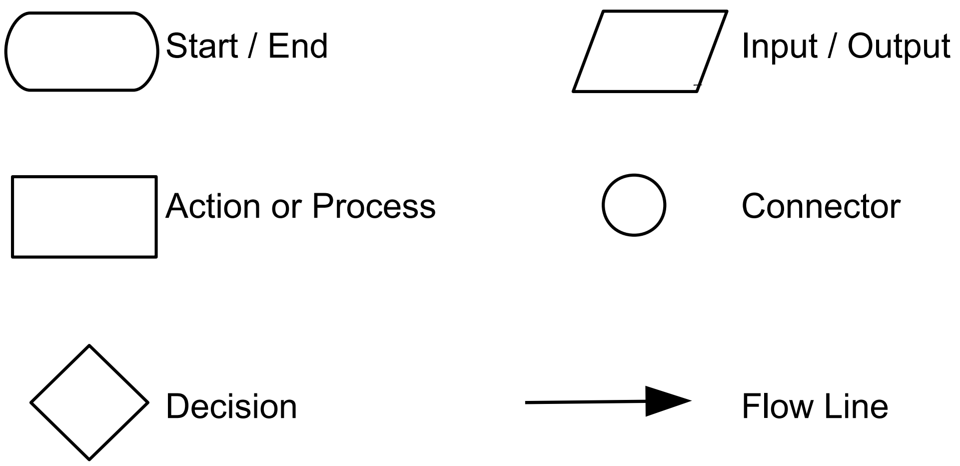
$A\cdot(B\cdot C)=C\cdot(A\cdot B)$ $\quad$ $A+(B+C)=C+(A+B)$

$A\cdot(A+B)=A$ $\quad$ $A+(A\cdot B)=A$

De Morgan's Laws

$\overline{A\cdot B\cdot C\cdot \ldots}=\overline{A}+\overline{B}+\overline{C}+\ldots$ $\quad$ $\overline{A+B+C+\ldots}=\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\ldots$

7.1.1 Basic Flowchart Shapes and Symbols



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8 Mathematical Notation – what the symbols mean

$\in$	is a member of. ($x \in \mathbb{R}$ means x is a member of $\mathbb{R}$)
$\mathbb{N}$	the set of natural numbers 1, 2, 3, ...
$\mathbb{Z}$	the set of all integers ....., -2, -1, 0, 1, 2, 3, ...
$\mathbb{Q}$	the set of rational numbers including $\mathbb{Z}$ and fractions $\frac{p}{q}; p, q \in \mathbb{Z}$
$\mathbb{R}$	the set of all real numbers. Numbers represented by drawing a continuous number line.
$\mathbb{C}$	the set of complex numbers. Numbers represented by drawing vectors.
敍	therefore
w.r.t.	with respect to
$*$	used as a multiplication sign ($\times$) (in computer algebra)
^	used as “power of” (x^y) in computer algebra
$\neq$	not equal to
$\approx$	approximately equal to
$>$	greater than. $x>2$ means x is greater than and not equal to 2
$\geq$	greater than or equal to.
$<$	less than. $a<2$ means a is less than and not equal to 2.
$\leq$	less than or equal to.
$a \leq x \leq b$	x is greater than or equal to a and less than or equal to b
ab	abbreviation for $a \times b$ or $a * b$ or $a \cdot b$
$a \times 10^n$	a number in scientific (or standard) form. ($3 \times 10^3 = 3000$) use EXP or $\times 10^x$ key on a calculator
$n!$	“ n factorial” $n \times (n-1) \times (n-2) \times (n-3) \times ... \times 1$

$A \propto B$	implies $A = k\,B$ where k is a constant (direct variation)
$\lvert x \rvert$	the modulus of x . The magnitude of the number x , irrespective of the sign. $\lvert -3 \rvert = 3 = \lvert 3 \rvert$
∞	infinity
$\Rightarrow$	implies

- -

8.1.1 Notation for Indices and Logarithms

a^n	abbreviation for $a \times a \times a \times a \ldots \times a$ (n terms). see 21 $x^{■}$ or ^ or x^y or y^x or a^b on a calculator.
$\sqrt{a}$	the positive square root of the number a . $\sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$
$\sqrt[k]{a}$	kth root of a number a . $\sqrt[3]{8} = 2$ $\sqrt[k]{a} = a^{\frac{1}{k}}$.
e^x	$\exp(x)$ (2.71828.... to the power of x). See 21.
$\log_e x$	$\ln(x)$ on a calculator. The logarithm of x to the base e
$\log_{10} x$	$\log(x)$ on a calculator. The logarithm of x to the base 10

- -

8.1.2 Notation for Functions

$f(x)$	a function of x . Also seen as $g(x)$, $h(x)$, $y(x)$
$f^{-1}(x)$	the inverse of the function labelled $f(x)$
$g \circ f$	the composite function - first f then g . or $g(f(x))$.

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9 Laws of Mathematics

Associative laws - for addition and multiplication

$a+(b+c)=(a+b)+c$ $\qquad$ $a(bc)=(ab)c$

Commutative laws - for addition and multiplication

$a+b=b+a$ but $a-b\neq b-a$

$ab=ba$ but $\frac{a}{b}\neq\frac{b}{a}$

Distributive laws - for multiplication and division

$a(b+c)=ab+ac$ $\qquad$ $\frac{b+c}{a}=\frac{b}{a}+\frac{c}{a}$

Arithmetical Identities

$x+0=x$ $\qquad$ $x\times 1=x$ $\qquad$ $(x\times 0=0)$

Algebraic Identities K Singh pp 73 – 75

$(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$ $\qquad$ $a^2-b^2=(a+b)(a-b)$

$(a+b)^3=(a+b)(a^2+2ab+b^2)=a^3+3a^2b+3ab^2+b^3$ see 21.1.6

Other useful facts

$a-b=a+(-b)$ $\qquad$ $\frac{a}{b}=a\div b=\frac{a}{1}\times\frac{1}{b}$

$a-(-b)=a--b=a+b$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ $\qquad$ $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ see 22.3.8, 4

$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ MC

$(a+b)(c+d)=ac+ad+bc+bd$ FOIL

MC

9.1 Algebra – sequence of operations

[K Singh pp 40 - 43]

Sequence of operations - the same sequence as used by scientific calculators.

Brackets ( ) *come before*

Of $x^{2,}$ $\sqrt{x}$, $\sin x$, e^{x}, *comes before*
*"square **of** x , sine **of** x*

Multiplication $\times$ *comes before*

Division $\div$ *comes before*

Addition $+$ *comes before*

Subtraction $-$

$3\sin(a\,x^2+b)-5$ would be read in this order

left bracket

x squared

times a

plus b

right bracket

sine **of** the result ($\sin(a\,x^2+b)$)

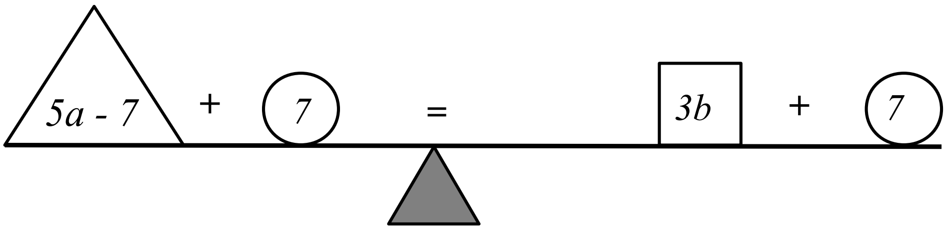
times 3

minus 5

10 Changing the subject of a Formula (Transposition)

[K Singh pp 53 - 66]

An equation or formula must always be **BALANCED** -
whatever mathematical operation you do to one side of an equals sign must be done to other side as well. (to **all** the terms)



You can't **move** a term (or number) from one side of the equals sign to the other.
You must **UNDO** it by using the correct **MATHEMATICAL** operation.

UNDO	$\times$	with	$\div$	**AND**	$\div$	with	$\times$
UNDO	$+$	with	$-$	**AND**	$-$	with	$+$
UNDO	$\sqrt{}$	with	x^2	**AND**	x^2	with	$\sqrt{}$
UNDO	x^n	with	$\sqrt[n]{}$	**AND**	$\sqrt[n]{}$	with	x^n
UNDO	$\sin x$	with	$\sin^{-1} x$	**AND**	$\sin^{-1} x$	with	$\sin x$
UNDO	e^x	with	$\ln x$	**AND**	$\ln x$	with	e^x
UNDO	10^x	with	$\log_{10} x$	**AND**	$\log_{10} x$	with	10^x
UNDO	$\frac{dy}{dx}$	with	$\int dx$	**AND**	$\int dx$	with	$\frac{dy}{dx}$

etc

Generally (but not always) start with the terms
FURTHEST AWAY from the new subject **FIRST.**

Think of the terms in the formula as layers of an onion
- take the layers off one by one.

$$((a((x^2)))+b)$$

Try http://www.mathsnet.net/algebra/equation.html for getting started.

MC

11 Simultaneous Equations with 2 variables

[K Singh p 90-98]

General method:

Write down both equations and label (1) and (2).

$a_1 x + b_1 y = c_1 \quad (1)$

$a_2 x + b_2 y = c_2 \quad (2)$

Multiply every term on both sides of (1) by a_2 and every term on both sides of (2) by a_1 and re-label as (3) and (4).

$a_2 a_1 x + a_2 b_1 y = a_2 c_1 \quad (3)$

$a_1 a_2 x + a_1 b_2 y = a_1 c_2 \quad (4)$

Multiply every term on both sides of (4) by -1 and re-label.

$a_2 a_1 x + a_2 b_1 y = a_2 c_1 \quad (3)$

$-a_1 a_2 x - a_1 b_2 y = -a_1 c_2 \quad (5)$

Add (3) to (5) to eliminate x

Calculate the value of y

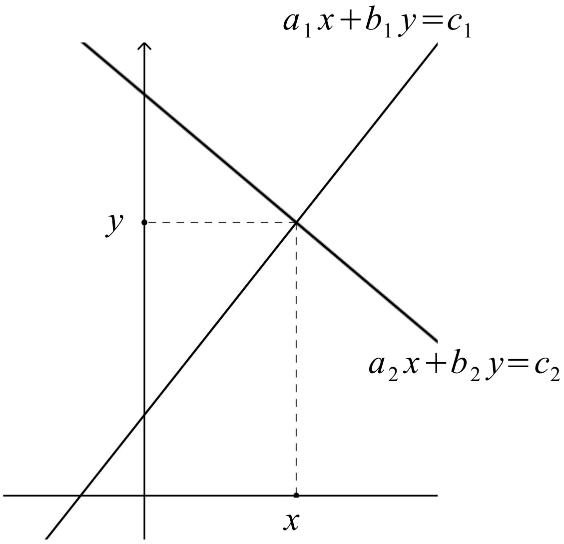
Substitute the value of y into equation (1)

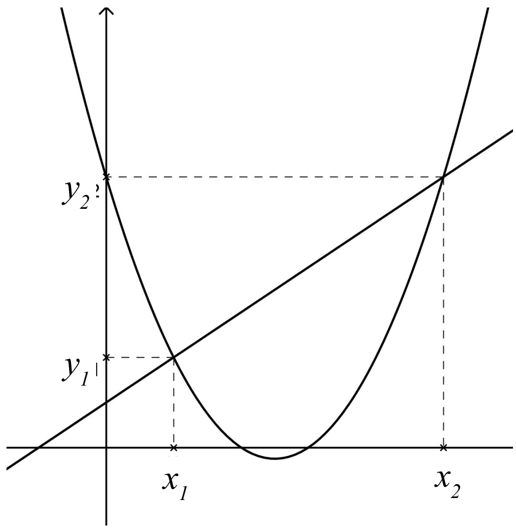
Calculate the value of x MC

Check by substituting the values of x and y into (2)

- -

Graphical Solution





If $f(x) = g(x)$ then $f(x) - g(x) = 0$ - also see 13 and 14

12 Matrices

[K Singh pp 507 – 566]

Notation:

$$\text{Identity} = \begin{bmatrix} 1 & 0 & 0 & .. \\ 0 & 1 & 0 & .. \\ 0 & 0 & 1 & .. \\ . & . & . & .. \end{bmatrix}$$

A $m \times n$ matrix has m rows and n columns.

a_{ij} an element in the i th row and j th column.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

then $A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$

and $A \times B = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$ $\quad \text{Columns}_A = \text{Rows}_B$

Solution of Equations 2 x 2

If $A X = B$ then $X = A^{-1} B$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then the inverse matrix,

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad (a d - b c \neq 0)$$

where $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

- -

Inverse Matrix, 3 x 3 or larger

Start with $\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right]$ carry out row operations to:

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 1 & 0 & b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 & b_{31} & b_{32} & b_{33} \end{array}\right]$ where $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = A^{-1}$

or for 3x3 $A^{-1} = \dfrac{1}{\det A} \times (\text{transpose of the co-factors of } A)$ **[place signs!!]**

where $det\ A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

or use Sarrus' Rule as below

− − −

$$det\ A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \left[\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right] \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

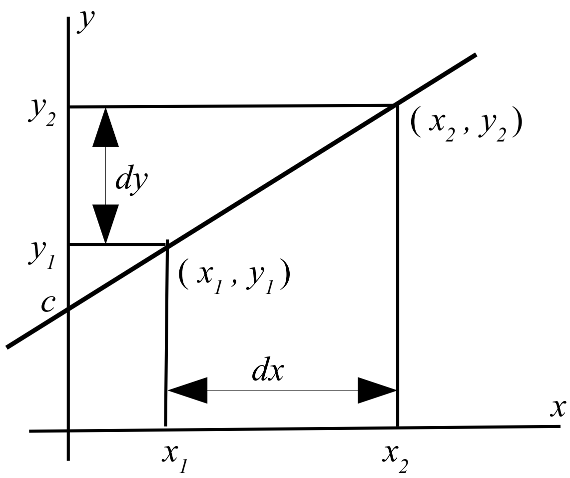
\+ \+ \+

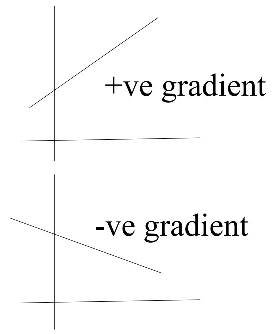
$$detA = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}$$
$$- a_{31} a_{22} a_{13} - a_{32} a_{23} a_{11} - a_{33} a_{21} a_{12}$$

Thanks to Richard Kaczkowski, Aberdeen College.

13 The Straight Line

[K Singh pp 100 – 108]





The general equation of a straight line of gradient m cutting the y axis at $(0,c)$ is

$$y = mx + c$$

where the gradient

$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ or $\frac{dy}{dx} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$. See 22.1.1 and 17.3

or $y_1 = m x_1 + c$ (1)
$y_2 = m x_2 + c$ (2) then (1) – (2) and solve for m (then c)

Also:

A straight line, gradient m passing through (a, b) has the equation:

$$(y - b) = m(x - a)$$

Also see 27, back to 22.2.3, 22.5, 23.2.1, 22.3.10 MC

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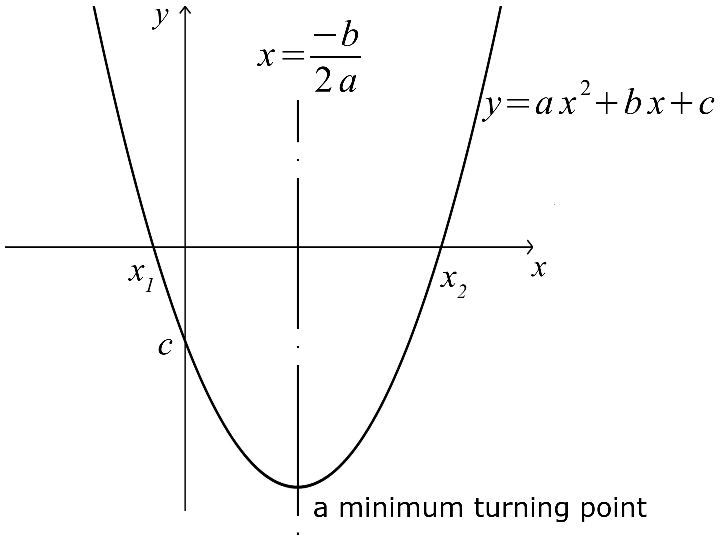
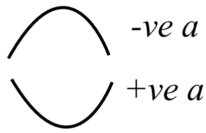
14 Quadratic Equations

[K Singh pp 88 - 90 & 109 - 113]





Geogebra quadratic

MC

The solutions (roots) x_1 and x_2 of the equation $ax^2+bx+c=0$ are the value(s) of x where $y=ax^2+bx+c$ crosses the x axis.

The solutions (roots) x_1 and x_2 of $ax^2+bx+c=0$ are given by the Quadratic Formula.

$$x=\frac{-b}{2a}\pm\frac{\sqrt{(b^2-4ac)}}{(2a)} \qquad \text{or} \qquad x=\frac{\left(-b\pm\sqrt{(b^2-4ac)}\right)}{(2a)}$$

Definition of a root: The value(s) of x which make y equal to zero.

....

Also:

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{\left(\frac{b}{a}\right)}{2}\right)^2+d^2=0$$

where $d^2=\frac{c}{a}-\left(\frac{\left(\frac{b}{a}\right)}{2}\right)^2$ see 22.4

If $y=k(x+A)^2+B$ the turning point is $(-A, B)$ Geogebra

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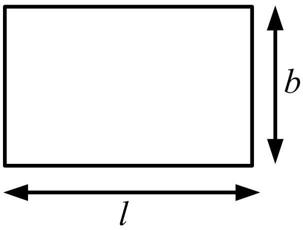
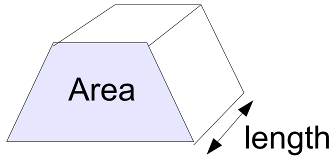
15 Areas and Volumes

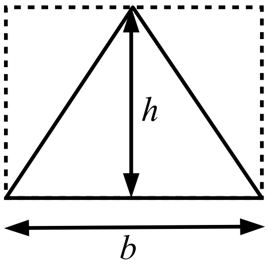


Rectangle $A = l\,b$

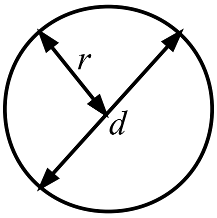
Volume of a Prism



$V = A\,l$

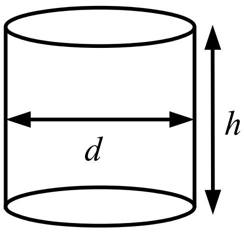


Triangle $A = \frac{1}{2} b\,h$



Circle $A = \frac{\pi d^2}{4} = \pi r^2$

$C = \pi d = 2\pi r$

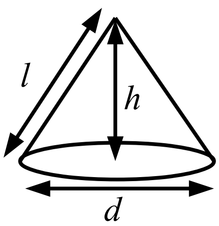


Cylinder Total surface area = $\pi\,d\,h + 2\frac{\pi d^2}{4}$

side + 2 ends

$2\pi r h + 2\pi r^2$

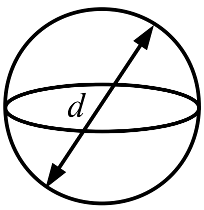
$V = \frac{\pi d^2 h}{4} = \pi r^2 h$



Cone Curved surface area = $\frac{\pi d l}{2} = \pi r l$

Total surface area = $\pi r l + \pi r^2$

$V = \frac{\pi d^2 h}{12} = \frac{\pi r^2 h}{3}$

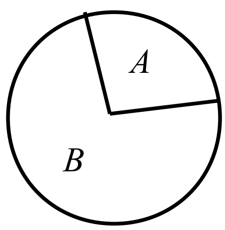


Sphere Total surface area = $\pi d^2 = 4\pi r^2$

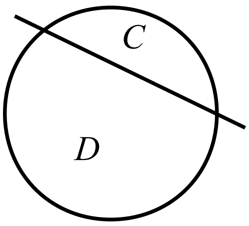
$V = \frac{\pi d^3}{6} = \frac{4\pi r^3}{3}$

16 The Circle

A Minor Sector



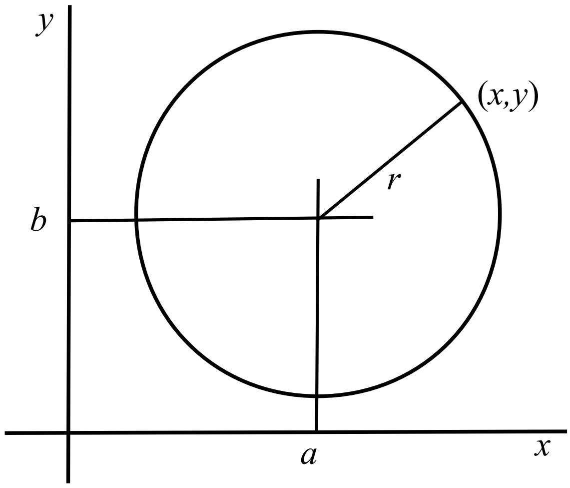
C Minor Segment



B Major Sector

D Major Segment

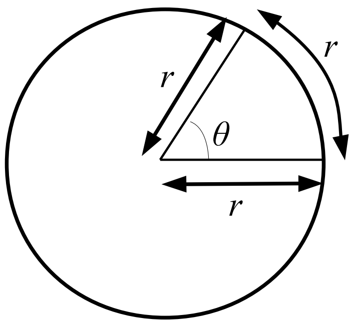
The equation $(x-a)^2+(y-b)^2=r^2$ represents a circle centre (a, b) and radius r.



16.1.1 Radian Measure

A radian: The angle θ subtended (or made by) an arc the same length as the radius of a circle. Notice that an arc is curved.

BE.com degrees and radians

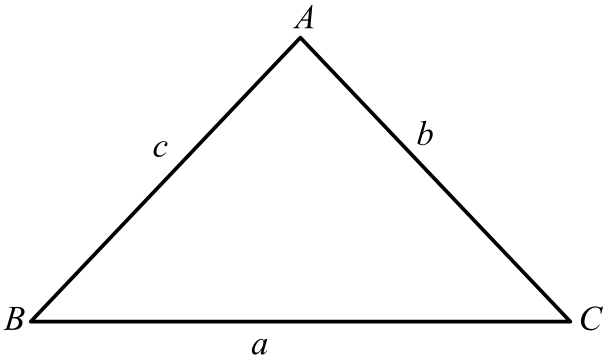


17 Trigonometry

[K Singh pp 168 - 176]

17.1.1 Notation for Trigonometry

Labelling of a triangle



$\sin\theta$ the value of the sine function of the angle θ

$\cos\theta$ the value of the cosine function of the angle θ

$\tan\theta$ the value of the tangent function of the angle θ

$\theta=\sin^{-1}b$ $\arcsin b$ the value of the basic angle θ whose sine function value is b. $(-90^o\le\theta^o\le 90^o)$ or $\left(\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}\right)$

$\theta=\cos^{-1}b$ $\arccos b$ the value of the basic angle θ whose cosine function value is b. $(0^o\le\theta^o\le 180^o)$ or $(0\le\theta\le\pi)$

$\theta=\tan^{-1}b$ $\arctan b$ the value of the basic angle θ whose tangent function value is b. $(-90^o\le\theta^o\le 90^o)$ or $\left(\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}\right)$

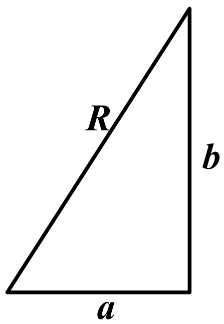
17.2 Pythagoras' Theorem

In a **right angled** triangle, with hypotenuse, length R ,
and the other two sides of lengths a and b ,
then

$$R^2=a^2+b^2$$

or

$$R=\sqrt{a^2+b^2}$$



use of Pythagoras' Theorem BE surprising uses
Pythagorean distance BE pythagorean distance
Interactive proof http://www.sunsite.ubc.ca/LivingMathematics/V001N01/UBCExamples/Pythagoras/pythagoras.html

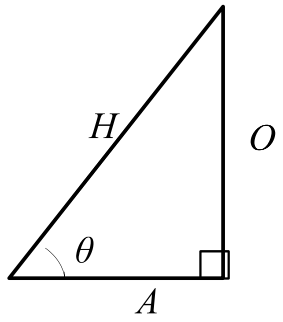
- -

The Triangle

In a **right angled** triangle, with hypotenuse, (which is the longest side), of length H ,

SOHCAHTOA

The other two sides have lengths A (adjacent, or next to angle θ) and O (opposite to angle θ) then



MC

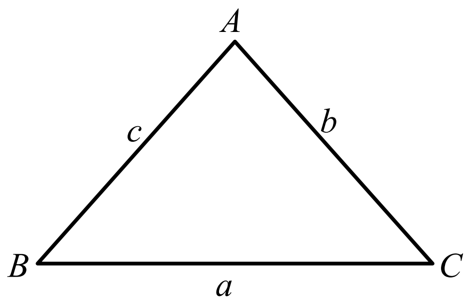
$$\sin\theta=\frac{O}{H} \qquad \cos\theta=\frac{A}{H} \qquad \tan\theta=\frac{O}{A}$$

see also 20

and 13

[K Singh pp 187 - 192]

In **any** triangle ABC, where A is the angle at A, B is the angle at B and C is the angle at C the following hold:



17.2.1 Sine Rule

Sine Rule $$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

or $$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

http://www.ies.co.jp/math/java/trig/seigen/seigen.html

17.2.2 Cosine Rule

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

or $$a^2=b^2+c^2-2bc\cos A$$

http://www.ies.co.jp/math/java/trig/yogen1/yogen1.html

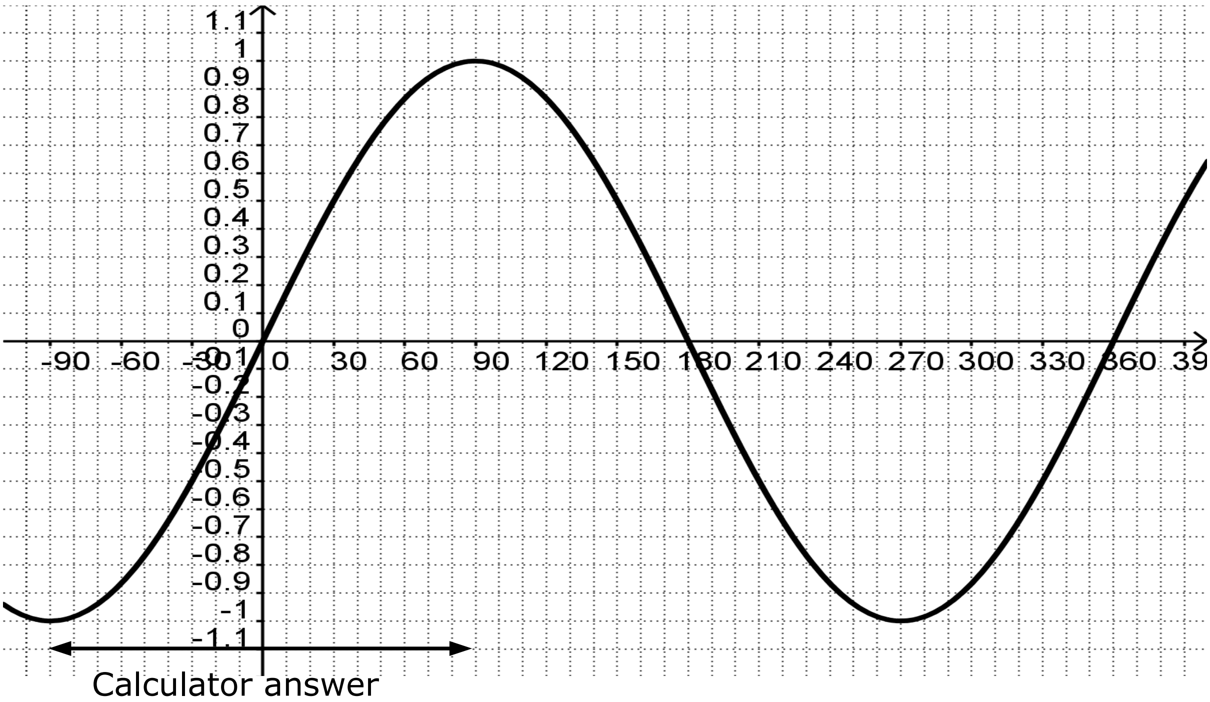
17.2.3 Area formula

Area = $$\frac{bc\sin A}{2}$$

17.3 Trigonometric Graphs

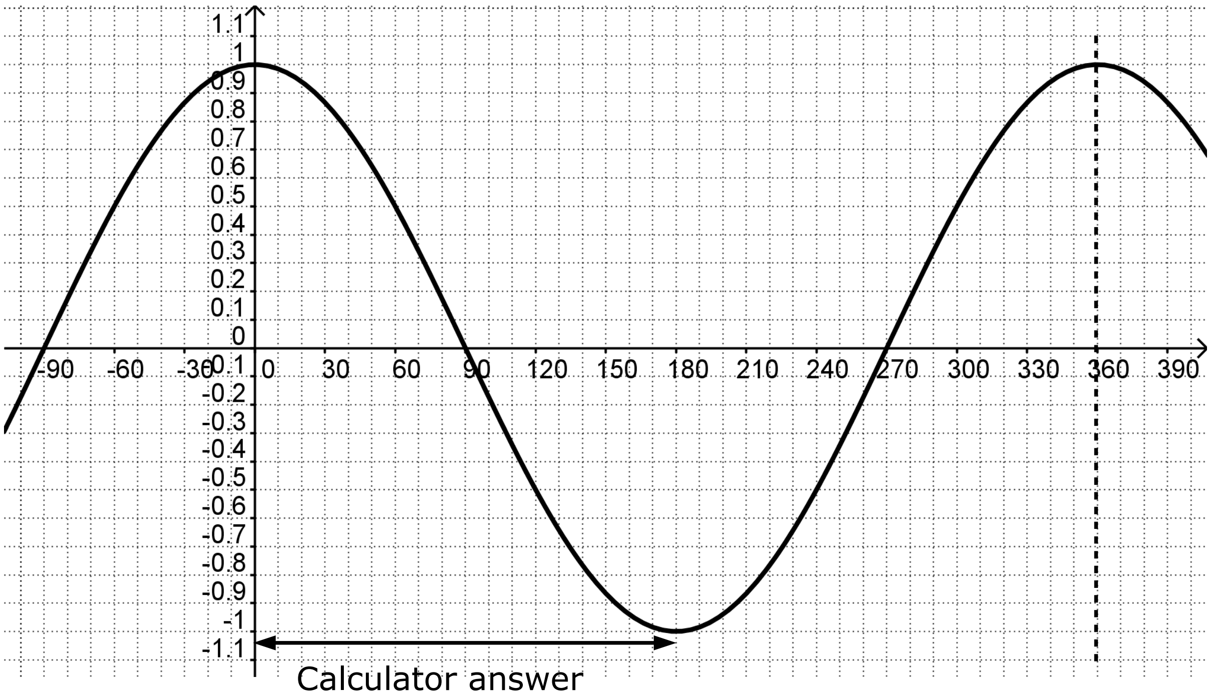
[K Singh pp 177 - 187]

$$y = \sin x^o$$



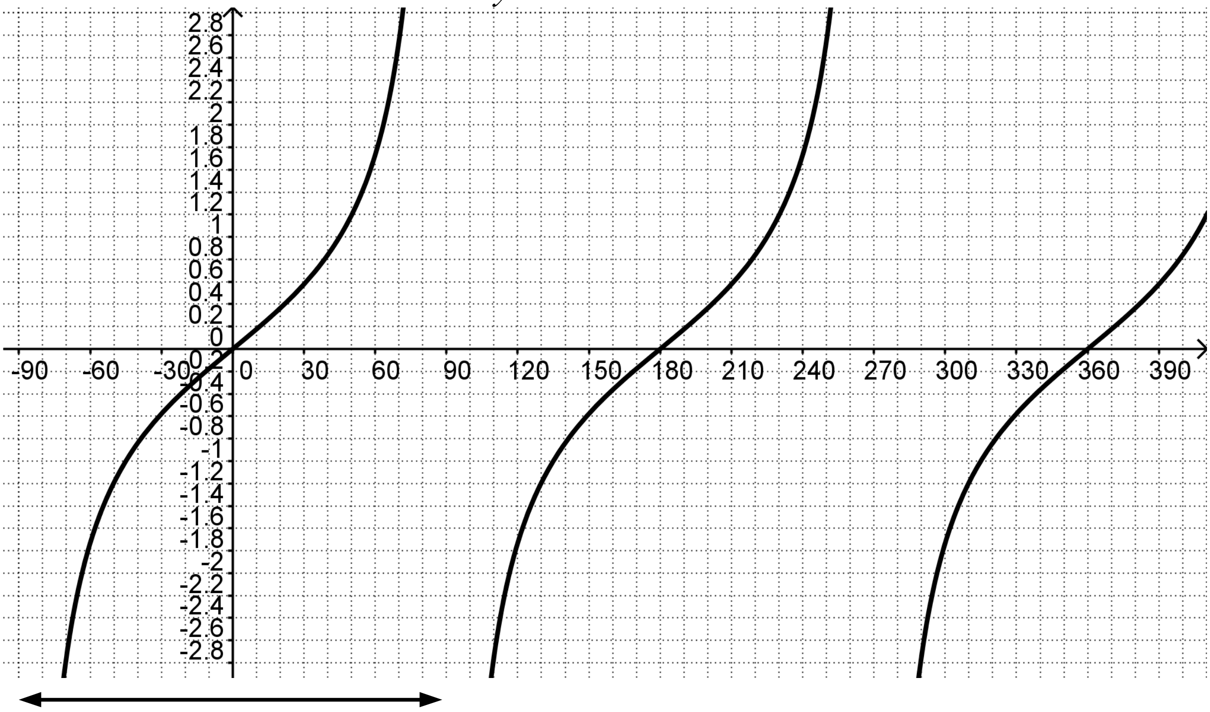
[Geogebra Sine wave slider](http://www.ies.co.jp/math/java/trig/graphSinX/graphSinX.html) http://www.ies.co.jp/math/java/trig/graphSinX/graphSinX.html

$$y = \cos x^o$$



Geogebra Cosine wave slider http://www.ies.co.jp/math/java/trig/graphCosX/graphCosX.html

$y = \tan x^o$



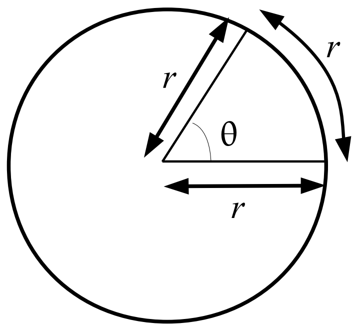
Calculator answer

17.3.1 Degrees - Radians Conversion

0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Degrees to radians $\quad x^o \div 180 \times \pi = \theta \text{ rad}$

Radians to degrees $\quad \theta\, rad \div \pi \times 180 = x^o$



$\theta = 1 \text{ radian}$

Geogebra Radians
BE degrees and radians

see 5.1.2

17.4 Trigonometric Identities

[K Singh pp 203 - 213]

$\tan A = \dfrac{\sin A}{\cos A}$ $\qquad$ $\cot A = \dfrac{1}{\tan A} = \dfrac{\cos A}{\sin A}$, (the cotangent of A)

- -

$\sec A = \dfrac{1}{\cos A}$, (the secant of A), $\operatorname{cosec} A = \dfrac{1}{\sin A}$, (the cosecant of A)

- -

$\sin^2 A + \cos^2 A = 1$ $\qquad$ entered as $(\sin A)^2 + (\cos A)^2$

- -

$\sin(-\theta) = -\sin\theta$ $\qquad$ (an ODD function)

$\cos(-\theta) = +\cos(\theta)$ $\qquad$ (an EVEN function)

17.5 Multiple / double angles

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(2A) = 2\sin A \cos B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\cos(2A) = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$\cos^2 A = \frac{1}{2}(\cos(2A) + 1)$$
$$\cos(2A) = 1 - 2\sin^2 A$$
$$\sin^2 A = \frac{1}{2}(1 - \cos(2A))$$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(2A) = \dfrac{2\tan A}{1 - \tan^2 A}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

- -

Products to Sums

$$\sin A\cos B=\frac{1}{2}(\sin(A+B)+\sin(A-B))$$

$$\cos A\sin B=\frac{1}{2}(\sin(A+B)-\sin(A-B))$$

$$\cos A\cos B=\frac{1}{2}(\cos(A+B)+\cos(A-B))$$

$$\sin A\sin B=\frac{1}{2}(\cos(A-B)-\cos(A+B))$$

Sums to Products

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

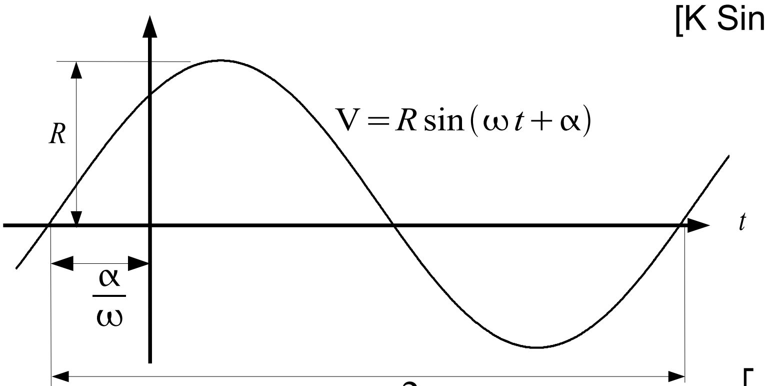
$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

17.6 Sinusoidal Wave

[K Singh pp 195 - 202]



see 22.6, 4.4

$$\text{Period}=\frac{2\pi}{\omega}$$

$$\left[\text{Frequency}=\frac{\omega}{2\pi}\right]$$

α = phase angle $\qquad \frac{\alpha}{\omega}$ = phase shift

Thanks to Mark Perkins, Bedford College

18 Complex Numbers

[K Singh pp 464 - 506]

Notation for Complex Numbers

BE - imaginary numbers

j symbol representing $\sqrt{-1}$. (i used on most calculators)

$a+jb$ a complex number in Cartesian (or Rectangular) form ($x+yi$ on a calculator). $a, b \in \mathbb{R}$, jb imaginary part.

z a complex number $z=a+jb$ (or $x+yi$)

$r\angle\theta$ a complex number in polar form

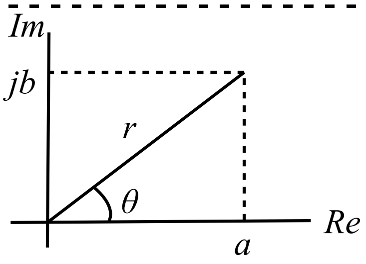
$\bar{z}$ complex conjugate of the complex number
If $z=a+jb$ then the complex conjugate $\bar{z}=a-jb$
or if $z=r\angle\theta$ then the complex conjugate $\bar{z}=r\angle-\theta$

$z=a+jb=r(\cos\theta+j\sin\theta)=r\angle\theta=re^{j\theta}$ where $j^2=-1$

Modulus, (or magnitude) $r=|z|=\sqrt{a^2+b^2}$ see 17.2, 17.2

Argument, (or angle) $\theta=\arg z=\tan^{-1}\left(\frac{b}{a}\right)$

BE - Complex arithmetic - better explained



Argand Diagram

Addition $(a+jb)+(c+jd)=(a+c)+j(b+d)$

Multiplication $(a+jb)(c+jd)$ Division $\frac{(a+jb)(c-jd)}{(c+jd)(c-jd)}$

Polar Multiplication $z_1 z_2=r_1\angle\theta_1\times r_2\angle\theta_2=r_1 r_2\angle(\theta_1+\theta_2)$

Polar Division $\frac{z_1}{z_2}=\frac{r_1\angle\theta_1}{r_2\angle\theta_2}=\frac{r_1}{r_2}\angle(\theta_1-\theta_2)$

See also: 20 Co-ordinate conversion MC

De Moivre's Theorem

$(r\angle\theta)^n=r^n\angle n\theta=r^n(\cos n\theta+j\sin n\theta)$ $\left(\sqrt{r\angle\theta}=\sqrt{r}\angle\frac{\theta}{2}\right)$

http://www.justinmullins.com/home.htm

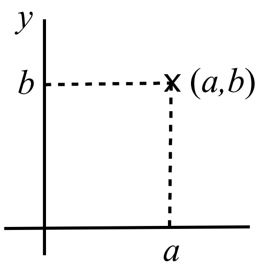
19 Vectors

Notation for Graphs and Vectors [K Singh pp 568 - 600]

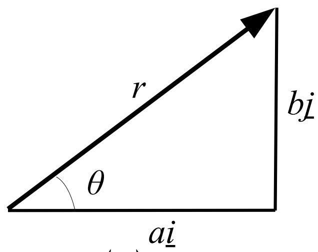
(x, y)	the co-ordinates of a point, where x is the distance from the y axis and y is the distance from the x axis
$\underline{v}$	a vector. Always underlined in written work
$\vec{AB}$	a vector
$a\underline{i}+b\underline{j}$	a vector in Cartesian form (Rectangular form)
$r\angle\theta$	a vector in polar form (where $r=\lvert\underline{v}\rvert$))
$\begin{pmatrix} a \\ b \end{pmatrix}$	a vector in Component form (Rectangular Form)
$\lvert\underline{v}\rvert$	modulus or magnitude of vector $\underline{v}$.

- -

Vectors



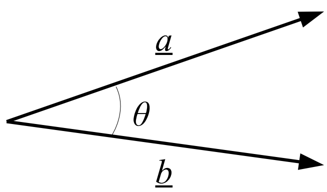
A point (a, b)



A vector $\underline{v}=\begin{pmatrix} a \\ b \end{pmatrix}$ or $\underline{v}=r\measuredangle\theta$

Vector Addition $\begin{pmatrix} a \\ b \end{pmatrix}+\begin{pmatrix} c \\ d \end{pmatrix}=\begin{pmatrix} a+c \\ b+d \end{pmatrix}$ Geogebra

see also 20 Co-ordinate Conversion



Scalar Product $\underline{a}\times\underline{b}=\lvert\underline{a}\rvert\lvert\underline{b}\rvert\cos\theta$

Dot Product $\underline{a}\cdot\underline{b}=a_1 b_1+a_2 b_2+a_3 b_3...$

where $\underline{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ . \end{pmatrix}$ and $\underline{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ . \end{pmatrix}$

---

20 Co-ordinate Conversion using Scientific Calculators

R to P — Rectangular to Polar — $\begin{pmatrix} x \\ y \end{pmatrix}$ to $r\angle\theta$ ($(x+jy)$ to $r\angle\theta$)

P to R — Polar to Rectangular — $r\angle\theta$ to $\begin{pmatrix} x \\ y \end{pmatrix}$ ($r\angle\theta$ to $(x+jy)$)

see also 17.3

Casio Natural Display **Edit keystrokes for your calculator**

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	$=$	r θ out
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	$=$	x y out

Casio S-VPAM and new Texet **Edit keystrokes for your calculator**

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	$=$	r out	RCL	tan	θ out
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	$=$	x out	RCL	tan	y out

Sharp *ADVANCED* D.A.L. **Edit keystrokes for your calculator**

R to P	x	2ndF	,	y	2ndF	$\rightarrow r\,\theta$	r out	$\rightarrow$	θ out
or				MATH	1	r out	2ndF	$\leftarrow\cdot\rightarrow$	θ out
P to R	r	2ndF	,	θ	2ndF	$\rightarrow x\,y$	x out	$\rightarrow$	y out
or				MATH	2	x out	2ndF	$\leftarrow\cdot\rightarrow$	y out

Old Casio *fx* & VPAM

R to P	x	SHIFT	$R\rightarrow P$	y	$=$	r out	SHIFT	$X\rightarrow Y$	θ out
P to R	r	SHIFT	$P\rightarrow R$	θ	$=$	x out	SHIFT	$X\rightarrow Y$	y out

Texet - albert 2

R to P	x	INV	$x \leftrightarrow y$	y	$R \to P$	r out	INV	$x \leftrightarrow y$	θ out
P to R	r	INV	$x \leftrightarrow y$	θ	$P \to R$	x out	INV	$x \leftrightarrow y$	y out

Casio Graphics (1)

R to P	SHIFT	Pol(	x	SHIFT	,	y	)	EXE	r out	ALPHA	J	EXE	θ out
P to R	SHIFT	Rec(	r	SHIFT	,	θ	)	EXE	x out	ALPHA	J	EXE	y out

Casio Graphics (2)

R to P	FUNC	4	MATH	4	COORD	1	Pol(	x	,	y	)	EXE	r	ALPHA	J	EXE	θ
P to R	FUNC	4	MATH	4	COORD	1	Rec(	r	,	θ	)	EXE	x	ALPHA	J	EXE	y

Casio Graphics (7 series)

R to P	OPTN	▶	F2	▶	▶	Pol(	x	,	y	)	EXE	r, θ out
R to P	OPTN	▶	F2	▶	▶	Rec(	r	,	θ	)	EXE	x, y out

Old Texet and old Sharp and some £1 calculators
You must be in Complex Number mode.

2ndF	CPLX

R to P	x	a	y	b	2ndF	a	r out	b	θ out
P to R	r	a	θ	b	2ndF	b	x out	b	y out

Texas - 36X

R to P	x	$x \leftrightarrow y$	y	3rd	$R \to P$	r out	$x \leftrightarrow y$	θ out
P to R	r	$x \leftrightarrow y$	θ	2nd	$P \to R$	x out	$x \leftrightarrow y$	y out

Texas Graphics (TI 83)

R to P	2nd	Angle	$R \to Pr$ (	x, y	)	ENTER	r out
	2nd	Angle	$R \to P\theta$ (	x, y	)	ENTER	θ out
P to R	2nd	Angle	$P \to Rx$ (	r, θ	)	ENTER	x out
	2nd	Angle	$P \to Ry$ (	r, θ	)	ENTER	y out

Sharp Graphics

R to P	MATH	(D)CONV	(3) $xy \to r$ (	x	y	)	ENTER	r out
	MATH	(D)CONV	(4) $xy \to \theta$ (	x	y	)	ENTER	θ out
P to R	MATH	(D)CONV	(5) $r\,\theta \to x$ (	r	θ	)	ENTER	x out
	MATH	(D)CONV	(6) $r\,\theta \to y$ (	r	θ	)	ENTER	y out

Insert the keystrokes for your calculator here (if different from above)

R to P										
P to R										

Degrees to Radians $\div 180 \times \pi$ **Radians to degrees** $\div \pi \times 180$

21 Indices and Logs

21.1.1 Rules of Indices:

[K Singh pp 224 - 245] notation 8.1.1

MC

1. $a^m \times a^n = a^{(m+n)}$

2. $\dfrac{a^m}{a^n} = a^{(m-n)}$

3. $(a^m)^n = a^{mn}$

4. $a^{\left(\frac{m}{n}\right)} = \sqrt[n]{a^m}$ $\qquad a^{\left(\frac{1}{n}\right)} = \sqrt[n]{a}$

5. $k\,a^{-n} = \dfrac{k}{a^n}$

Also,

$a^0 = 1$ $\qquad \sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$ and $\sqrt[2]{a} = \sqrt{a}$

$a^1 = a$ $\qquad \sqrt[n]{a} = b \Leftrightarrow b^n = a$

21.1.2 Definition of logarithms

If $N = a^n$ then $n = \log_a N$

21.1.3 Rules of logarithms:

MC

1. $\log(A \times B) = \log A + \log B$

2. $\log\left(\dfrac{A}{B}\right) = \log A - \log B$

3. $\log A^n = n \log A$

4. $\log_a N = \dfrac{\log_b N}{\log_b a}$

$\exp x \equiv e^x$ $\qquad \log_e x \equiv \ln x$ $\qquad \log_{10} x \equiv \lg x$

21.1.4 Infinite Series

[K Singh pp 338 - 346]

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ldots \qquad \text{for } |x| < \infty$$

BE exponential functions better explained

$$\sin x = \frac{e^{jx} - e^{-jx}}{j2} \quad \left(= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \right) \qquad \text{for } |x| < \infty$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad \left(= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \right) \qquad \text{for } |x| < \infty$$

$$\ln x = \frac{x-1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \ldots \qquad \text{for } 0 < x \le 2$$

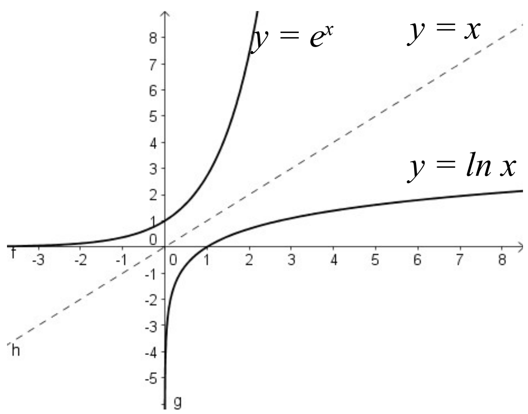
BE- demystifying the natural logarithm

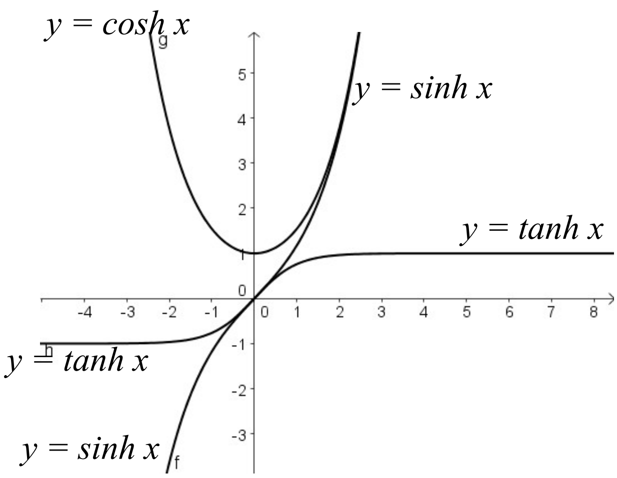
21.1.5 Hyperbolic Functions

- definitions [K Singh p 246]

MC

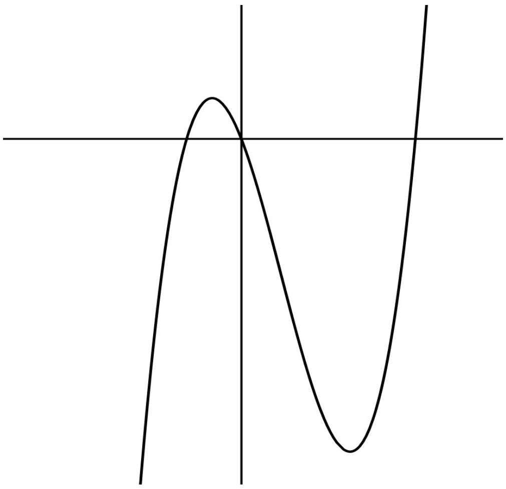
	pronunciation
$\sinh x = \frac{e^x - e^{-x}}{2} \quad \left(= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots \right)$	“shine x”
$\cosh x = \frac{e^x + e^{-x}}{2} \quad \left(= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots \right)$	“cosh x”
$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	“thaan x”



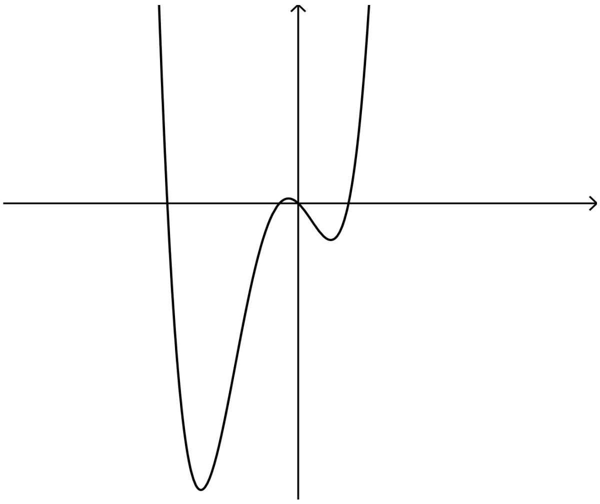


$k e^{ax}$ **slider** $k \ln(ax)$ slider

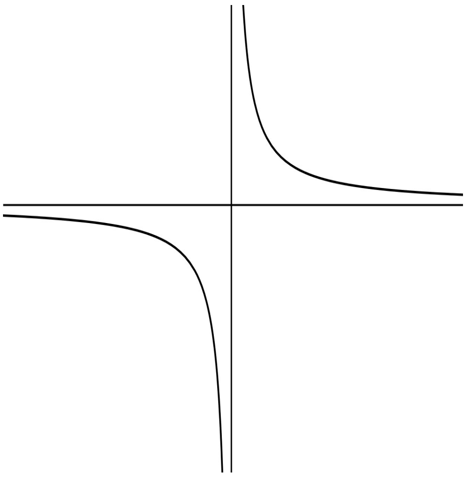
21.1.6 Graphs of Common Functions



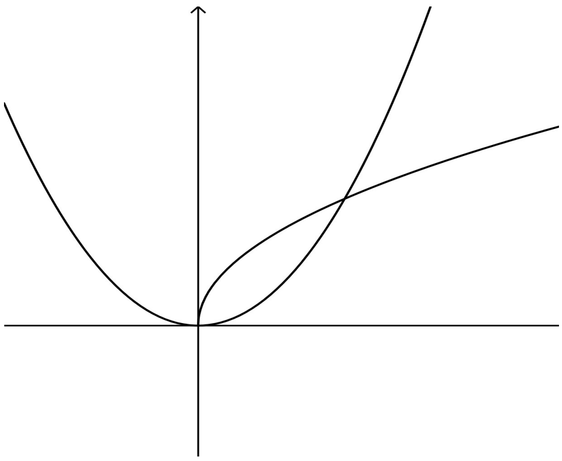
$y = a\,x^3 + b\,x^2 + c\,x + d$



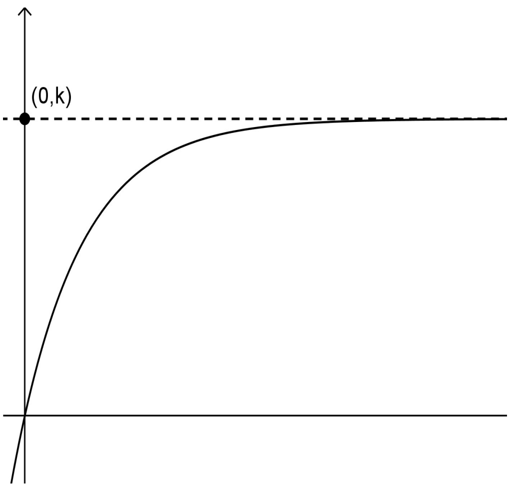
$y = a\,x^4 + b\,x^3 + c\,x^2 + d\,x + f$



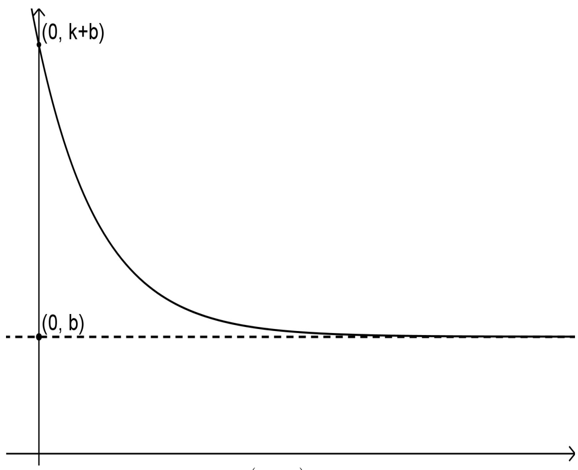
$y = \frac{a}{x} + b$



$y = x^2$ and $y = \sqrt{x}$



$y = k(1 - e^{(-\alpha t)})$



$y = k\,e^{(-\alpha t)} + b$

22 Calculus

22.1.1 Notation for Calculus

see also section 8

Differentiation

$\frac{dy}{dx}$	the first derivative of y where y is a function of x (Leibniz) Also see 13
$f'(x)$	the first derivative of $f(x)$. (as above). (Euler)
$\dot{v}$	the first derivative of v w.r.t. time. (Newtonian mechanics)
$D(u)$	the first derivative of u
$\frac{d^2y}{dx^2}$	the second derivative of y w.r.t x. The $\frac{dy}{dx}$ of $\frac{dy}{dx}$
$f''(x)$	the second derivative of $f(x)$. ($f^2(x)$ is also used)
$\ddot{v}$	the second derivative of v w.r.t. time. (Newtonian mechanics)
$\frac{\partial z}{\partial x}$	the partial derivative of z w.r.t. x. (∂ "partial d")
δx	a small change (increment) in x . (δ "delta")

Integration

$\int$	the integral sign (Summa)
$\int f(x)dx$	the indefinite integral of $f(x)$ (the anti-differential of $f(x)$)
$\int_a^b f(x)dx$	the definite integral of $f(x)$ from $x=a$ to $x=b$ the area under $f(x)$ between $x=a$ and $x=b$
$F(x)$	the primitive of $f(x)$ ($\int f(x)dx$ without the c)
$L[f(t)]$	the Laplace operator (with parameter s)

22.2 Differential Calculus - Derivatives

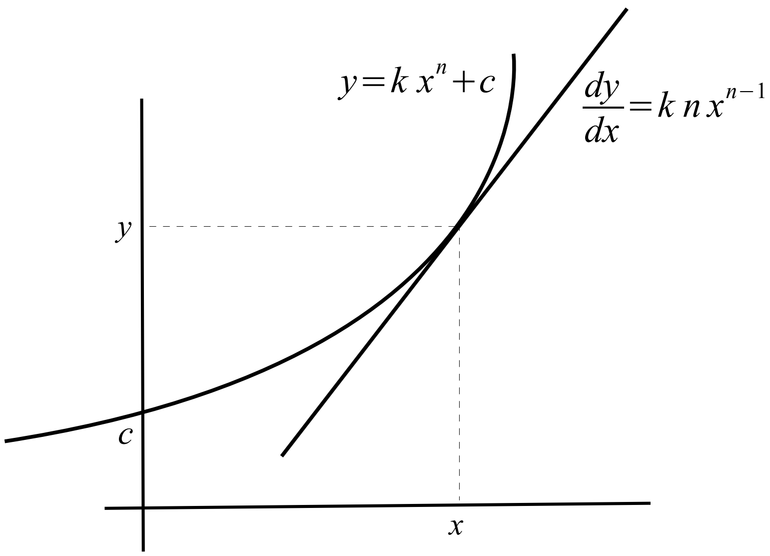
[K Singh pp 258 - 358] $\frac{dy}{dx}$

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$
x^n	$n x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
e^x	e^x
$\ln x$	$\frac{1}{x}$
k	0
$k x^n$	$k n x^{n-1}$
$\sin a x$	$a \cos a x$
$\cos a x$	$-a \sin a x$
$e^{a x}$	$a e^{a x}$
$\ln a x$	$\frac{a}{a x} = \frac{1}{x}$
$k(a x+b)^n$	$k n a (a x+b)^{n-1}$
$k \sin(a x+b)$	$k a \cos(a x+b)$
$k \cos(a x+b)$	$-k a \sin(a x+b)$
$k \tan(a x+b)$	$k a\, sec^2(a x+b) = \frac{k a}{\cos^2(a x+b)}$
$k e^{(ax+b)}$	$k a e^{(ax+b)}$ e^x gradient slider
$k \ln(a x+b)$	$\frac{k a}{(a x+b)}$

Further Standard Derivatives

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$
$\ln[f(x)]$	$\dfrac{f'(x)}{f(x)}$
$\sin^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{1}{\sqrt{a^2-x^2}}, \quad x^2<a^2$
$\cos^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{-1}{\sqrt{a^2-x^2}}, \quad x^2<a^2$
$\tan^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{a}{a^2+x^2}$
$\sinh(ax+b)$	$a\cosh(ax+b)$
$\cosh(ax+b)$	$a\sinh(ax+b)$
$\tanh(ax+b)$	$a\,sech^2(ax+b)$
$\sinh^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{1}{\sqrt{x^2+a^2}}$
$\cosh^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{1}{\sqrt{x^2-a^2}}, \quad x^2>a^2$
$\tanh^{-1}\left(\dfrac{x}{a}\right)$	$\dfrac{a}{a^2-x^2}, \quad x^2<a^2$

Differentiation as a gradient function (tangent to a curve).

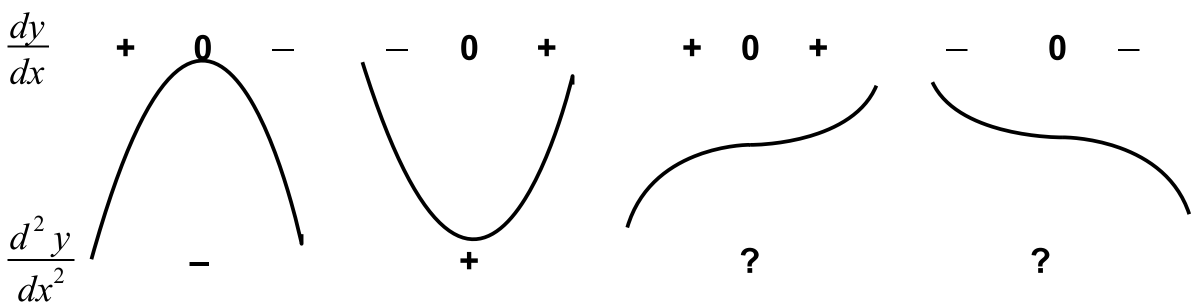


22.2.1 Maxima and Minima

(Stationary Points) [K Singh pp 308 - 325]

If $y = f(x)$ then at any turning point or stationary point $\dfrac{dy}{dx} = f'(x) = 0$

Determine the nature (max, min or saddle) of the turning points by evaluating gradients locally (i.e. close to turning point). MC



22.2.2 Differentiation Rules

[K Singh pp 274 – 285]

For D read *differentiate* $D[k\,f(x)] = k\,f'(x)$, k a constant

- -

Function of a function rule $D[f(g(x))] = f'(g(x)) \times g'(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$ **MC**

- -

If u and v are functions of x then:

Addition Rule $D(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx} = u' + v'$

- -

Product Rule $D(uv) = v\dfrac{du}{dx} + u\dfrac{dv}{dx} = v\,u' + u\,v'$ MC

- -

Quotient Rule $D\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \dfrac{vu' - uv'}{v^2}$ **MC**

22.2.3 Formula for the Newton-Raphson Iterative Process

[K Singh pp 352 - 356]

Set $f(x)=0$ with guess value x_0 (from graph) see 13

Test for Convergence $\left|\frac{f(x_0)f''(x_0)}{[f'(x_0)]^2}\right|<1$ see 8 - modulus

x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

(where $f'(x_n)\neq 0$)

$f(x)=0$ when $x_{n+1}=x_n$ to accuracy required.

http://archives.math.utk.edu/visual.calculus/3/newton.5/1.html

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22.2.4 Partial Differentiation

[K Singh pp 695 - 704]

If $z=f(x,y)$ then a small change in x, named δx (delta x) and a small change in y, named δy etc. will cause a small change in z, named δz such that $\delta z\simeq\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y+...$ where $\frac{\partial z}{\partial x}$ is the partial derivative of z w.r.t. x and $\frac{\partial z}{\partial y}$ is the partial derivative of z w.r.t y. see 8

22.2.5 Implicit Differentiation

If $z=f(x,y)$ then $\frac{dy}{dx}=\frac{\left(\frac{\partial z}{\partial x}\right)}{\left(\frac{\partial z}{\partial y}\right)}$ Also $\frac{dy}{dx}=\frac{1}{\left(\frac{dx}{dy}\right)}$

22.2.6 Parametric Differentiation

[K Singh pp 291 - 296]

If $x=f(t)$ and $y=g(t)$

$\frac{dx}{dt}=f'(t)$ and $\frac{dy}{dt}=g'(t)$

$\frac{dy}{dx}=\frac{g'(t)}{f'(t)}$ or $\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$ $\left(f'(t),\frac{dx}{dt}\neq 0\right)$ MC

22.3 Integral Calculus - Integrals

[K Singh pp 359 - 462] $\int$

$\frac{dy}{dx}$ or $f(x)$	y or $\int f(x)dx$ or $F(x)$ + **c**	
x^n	$\frac{x^{n+1}}{n+1}$	$n \neq -1$
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
e^x	e^x	
$\frac{1}{x} = x^{-1}$	$\ln x$	(when $n=-1$)
k	$k x$	
$k x^n$	$\frac{kx^{n+1}}{n+1}$	$n \neq -1$
$\sin a x$	$\frac{-\cos a x}{a}$	
$\cos a x$	$\frac{\sin a x}{a}$	
e^{ax}	$\frac{e^{ax}}{a}$	
$\frac{k}{x} = k x^{-1}$	$k \ln x$	(where $n=-1$)
$k(a x+b)^n$	$\frac{k(a x+b)^{n+1}}{(n+1)a}$	$n \neq -1$
$k \sin(a x+b)$	$\frac{-k \cos(a x+b)}{a}$	
$k \cos(a x+b)$	$\frac{k \sin(a x+b)}{a}$	
$k\, sec^2(a x+b)$	$\frac{k \tan(a x+b)}{a}$	
$k\, e^{(a x+b)}$	$\frac{k\, e^{(a x+b)}}{a}$	
$\frac{k}{(a x+b)}$	$\frac{k \ln(a x+b)}{a}$	$n=-1$

Further Standard Integrals

$\frac{dy}{dx}$ or $f(x)$	y or $\int f(x)\,dx$ or $F(x)$ + **c**
$\left(\frac{\frac{dy}{dx}}{y}\right)$ $\frac{f'(x)}{f(x)}$	$\ln(f(x))$ $(\ln(y))$
$\frac{1}{\sqrt{a^2-x^2}}$, $x^2<a^2$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\sinh(ax+b)$	$\frac{1}{a}\cosh(ax+b)$
$\cosh(ax+b)$	$\frac{1}{a}\sinh(ax+b)$
$\operatorname{sech}^2(ax+b)$	$\frac{1}{a}\tanh(ax+b)$
$\frac{1}{\sqrt{x^2+a^2}}$, $x^2>a^2$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}$, $x^2>a^2$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2})$
$\frac{1}{a^2-x^2}, x^2<a^2$	$\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$ or $\frac{1}{2a}\ln\left\|\frac{(a+x)}{(a-x)}\right\|$
$\frac{1}{x^2-a^2}, x^2>a^2$	$\frac{-1}{a}\coth^{-1}\left(\frac{x}{a}\right)$ or $\frac{1}{2a}\ln\left\|\frac{(x-a)}{(x+a)}\right\|$

Addition Rule $\int f(x)+g(x)\,dx=\int f(x)\,dx+\int g(x)\,dx$

22.3.1 Integration by Substitution

[K Singh p 368]

$\int f(g(x))\,dx$ **MC**

$\int f(u)\,du$ where $u=g(x)$ then $\frac{du}{dx}=g'(x)$ and $dx=\frac{du}{g'(x)}$

Note change of limits $\int_{x=a}^{x=b} f(g(x))\,dx$ to $\int_{u\text{ when }x=a}^{u\text{ when }x=b} f(u)\,du$

du is a function of u or $du\in\mathbb{R}$

22.3.2 Integration by Parts

[K Singh pp 388 - 395]

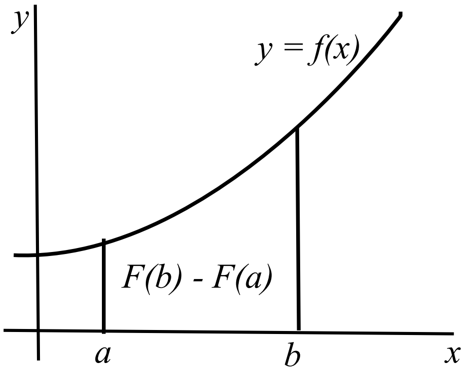
see 22.6

$\int u\,dv=uv-\int v\,du$ **MC**

22.3.3 Indefinite Integration

$$\frac{dy}{dx} = f(x)$$
$$dy = f(x)\,dx$$
$$\int 1\,dy = \int f(x)\,dx$$
$$y = F(x) + c$$

22.3.4 Area under a Curve - Definite Integration [K Singh p 442]

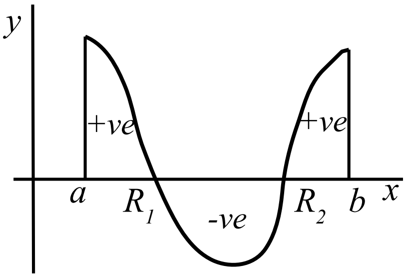


$$\int_a^b f(x)\,dx$$
$$= [F(x) + c]_a^b$$
$$= (F(b) + \cancel{c}) - (F(a) + \cancel{c})$$

Hyperlink to interactive demo of areas by integration
http://surendranath.tripod.com/Applets/Math/IntArea/IntAreaApplet.html

Procedure

Plot between limits - a and b
Check for roots (R_1 , R_2 .. R_n) and evaluate
See Newton Raphson 22.2.3
Integrate between left limit, a , and R_1
then between R_1 and R_2 and so on to
last root R_n and right limit b
Add moduli of areas. (areas all +ve)

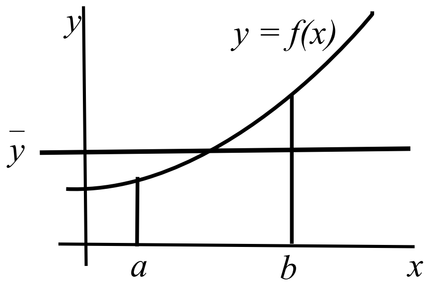


22.3.5 Mean Value

[K Singh p 445]

If $y = f(x)$ then $\bar{y}$,
the mean (or average) value of y
over the interval $x = a$ to $x = b$ is

$$\bar{y} = \frac{1}{(b-a)} \int_a^b y\,dx$$



22.3.6 Root Mean Square (RMS)

$$y_{rms} = \sqrt{\frac{1}{b-a} \int_a^b y^2\,dx} \qquad \text{where } y = f(x)$$

22.3.7 Volume of Revolution around the x axis

[J Bird pp 207-208]

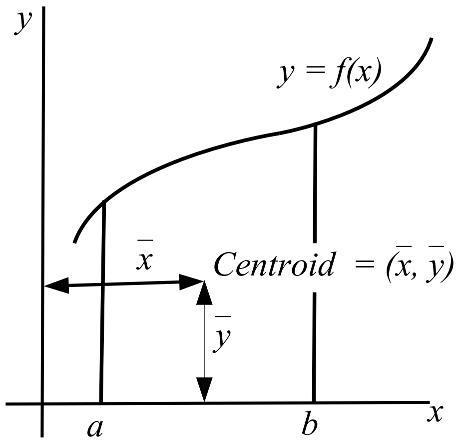
$$V = \pi \int_a^b y^2\,dx \quad \text{where } y = f(x)$$

22.3.8 Centroid

[J Bird pp 208 - 210]

The centroid of the area of a lamina bounded by a curve $y = f(x)$ and limits $x = a$ and $x = b$ has co-ordinates $(\bar{x}, \bar{y})$.

$$\bar{x} = \frac{\int_a^b x\,y\,dx}{\int_a^b y\,dx} \quad \text{and} \quad \bar{y} = \frac{\frac{1}{2}\int_a^b y^2\,dx}{\int_a^b y\,dx}$$



22.3.9 Partial Fractions

[K Singh pp 397 - 402]

$$\frac{f(x)}{(x+a)(x+b)} \equiv \frac{A}{(x+a)} + \frac{B}{(x+b)}$$ see 8

$$\frac{f(x)}{(x+a)^2(x+b)} \equiv \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}$$

$$\frac{f(x)}{(x^2+a)(x+b)} \equiv \frac{Ax}{(x^2+a)} + \frac{B}{(x^2+a)} + \frac{C}{(x+b)}$$

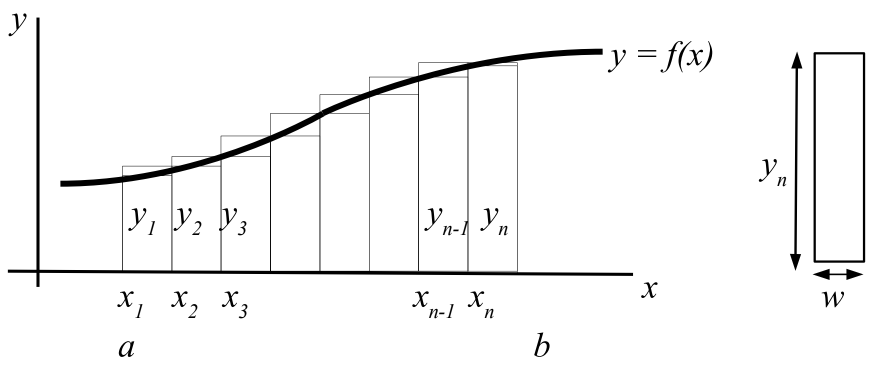
MC

22.3.10 Approximation of Definite Integrals

[K Singh p 434]

22.3.10.1 Simpson's Rule

$$w = \frac{b-a}{n}$$



$$\int_a^b f(x)\,dx \approx Area \approx \frac{w}{3}\left(y_1 + 4y_2 + 2y_3 + \ldots + 2y_{n-1} + 4y_n + y_{n+1}\right)$$

(n is even)

$$\int_a^b f(x)\,dx \approx \frac{w}{3}\left[\text{first} + \text{last} + 4\left(\sum \text{evens}\right) + 2\left(\sum \text{odds}\right)\right]$$

n	x_n	y_n	Multiplier m	Product $m\,y_n$
1	a	y_1	1	$1 \times y_1$
2	$a+w$	y_2	4	$4 \times y_2$
3	$a+2w$	y_3	2	$2 \times y_3$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$n-1$	.	y_{n-1}	2	$2 \times y_{n-1}$
n	.	y_n	4	$4 \times y_n$
$n+1$	b	y_{n+1}	1	$1 \times y_{n+1}$
			Sum =	
			$\times w$ =	
			$\div 3$ =	

22.3.10.2 Trapezium Method

$$\int_a^b f(x)\,dx \approx \frac{w}{2}\left(y_1 + 2y_2 + 2y_3 + \ldots\ldots + 2y_n + y_{n+1}\right)$$

22.4 Laplace Transforms

[J Bird pp 582 – 604] $\mathscr{L}[f(t)]$

Table of Laplace Transforms

$L[f(t)]$ is defined by $\int_0^\infty f(t)e^{-st}\,dt$ and is written as $F(s)$

	$f(t)$	$L[f(t)]$
1	1	$\frac{1}{s}$ $\quad (L[0]=0)$
2	t	$\frac{1}{s^2}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$1-e^{-at}$	$\frac{a}{s(s+a)}$
6	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
10	$1-\cos(\omega t)$	$\frac{\omega^2}{s(s^2+\omega^2)}$
11	$\omega t\sin(\omega t)$	$\frac{2\omega^2 s}{(s^2+\omega^2)^2}$
12	$\sin(\omega t)-\omega t\cos(\omega t)$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
13	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$ see 14
14	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$e^{-at}(\cos(\omega t)-\frac{a}{\omega}\sin(\omega t))$	$\frac{s}{(s+a)^2+\omega^2}$
16	$\sin(\omega t+\varphi)$	$\frac{s\sin\phi+\omega\cos\phi}{s^2+\omega^2}$
17	$e^{-at}+\frac{a}{\omega}\sin(\omega t)-\cos(\omega t)$	$\frac{a^2+\omega^2}{(s+a)(s^2+\omega^2)}$

	$f(t)$	$L[f(t)]$
18	$\sinh(\beta t)$	$\frac{\beta}{s^2-\beta^2}$
19	$\cosh(\beta t)$	$\frac{s}{s^2-\beta^2}$
20	$e^{-at}\sinh(\beta t)$	$\frac{\beta}{(s+a)^2-\beta^2}$
21	$e^{-at}\cosh(\beta t)$	$\frac{s+a}{(s+a)^2-\beta^2}$

First order differential equation:

$L\left[\frac{dy}{dt}\right]=s\,L[y]-y(0)$ where $y(0)$ is the value of y at $t=0$

see also 26.1 Diff Eq

Second order differential equation:

$L\left[\frac{d^2y}{dt^2}\right]=s^2L[y]-s\,y(0)-y'(0)$ where $y'(0)$ is the value of $\frac{dy}{dt}$ at $t=0$

MC **Efunda Calculator** **Efunda - Laplace**

22.5 Approximate numerical solution of differential equations

[K Singh pp 630 - 655] and section 26.1

Eulers' method

$y_1=y_0+h(y')_0$ 13 Range $x=a(h)b$

where h is the step size
a and b are limits

x_0	y_0	$(y')_0$	y_1

Plot the graph of y against x from values in first 2 columns.
See also 26.1 – Runge-Kutta.

See also K Singh pp 601 - 693 - Differential Equations

22.6 Fourier Series.

[J Bird pp 611 - 657] and next page and 26 and 26.1

For period T, the smallest period of $f(t)$. (determine from a graph)

Fundamental angular frequency $\omega = \frac{2\pi}{T}$

$$f(t) = a_0 + a_1\cos(\omega t) + a_2\cos(2\omega t) + a_3\cos(3\omega t) + \ldots$$
$$+ b_1\sin(\omega t) + b_2\sin(2\omega t) + b_3\sin(3\omega t) + \ldots$$

a_n, b_n constants

or

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$$

where

$$a_0 = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\,dt$$

mean value of $f(t)$ over period T

see 22.3.9

$$a_n = \frac{2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\cos(n\omega t)\,dt \qquad n = 1, 2, 3 \ldots$$

$$b_n = \frac{2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)\sin(n\omega t)\,dt \qquad n = 1, 2, 3 \ldots$$

Alternatively written as:

$$f(t) = a_0 + c_1\sin(\omega t + \alpha_1) + c_2\sin(2\omega t + \alpha_2) + \ldots + c_n\sin(n\omega t + \alpha_n)$$

a_0 constant, $c_n = \sqrt{(a_n^2 + b_n^2)}$ and $\alpha_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$

$f(t)$ = constant + first harmonic + second harmonic + ...

See 17.7 of this book.

See Fourier series applet http://www.falstad.com/fourier/index.html

- -

22.6.1 Fourier Series - wxMaxima method.

Close wxMaxima and start again **F6 for text**

Write down the values of T, $\frac{T}{2}$, $\frac{2}{T}$, $\frac{1}{T}$ and ω

!! use ω (type as w) in input, not a number.

---

a_n Input $\frac{2}{T} f(t)\cos(n w t)$

Integrate between $\frac{-T}{2}$ and $\frac{T}{2}$

For piecewise functions $\frac{-T}{2}$ and 0 and 0 and $\frac{T}{2}$ or smaller intervals

Add the parts of a_n

---

b_n Input $\frac{2}{T} f(t)\sin(n w t)$

Integrate between $\frac{-T}{2}$ and $\frac{T}{2}$

For piecewise functions as above

Add the parts of b_n

Make up the sum $a_n\cos(n w t)+b_n\sin(n w t)$

Sum **Calculus**; **Calculate Sum** Start with 6 terms (n from 1 to 6) but you may need more.

Substitute in the value for w

Trial plot

---

a_0 By observation OR

Input $\frac{1}{T} f(t)$

Integrate between $\frac{-T}{2}$ and $\frac{T}{2}$

For piecewise functions as above, but your interval may have to be 0 to $\frac{T}{2}$

Add a_0 to the Sum

Plot You will have to adjust horizontal range to be able to see the result.

23 Statistics

[K Singh pp 726 - 796]

23.1.1 Notation for Statistics

n	sample size
x x_i	a **sample** statistic (a data value) OR the variate
X	a **population** statistic
$\bar{x}$	the arithmetic mean point of a **sample** set of data
s	standard deviation of a **sample**
μ	the mean value of a **population**
σ	standard deviation of a **population**
$\sum$	the sum of all terms immediately following
f	frequency
Q	quartile. (Q_1 lower; Q_2 median; Q_3 upper)
d f	degrees of freedom $(n-1)$ of a sample.
$P=(X-x)$	the probability that the population statistic equals the sample statistic
$x!$	$x\times(x-1)\times(x-2)\times(x-3)\times...\times1,\ \ x\in\mathbb{N}$

Range	maximum value – minimum value
Quartiles	in a set of ordered data, Median, Q_2 : the middle value. Lower, Q_1 : the middle value between minimum and Q_2 . Upper, Q_3 : the middle value between Q_2 and the maximum. Percentile: the kth percentile is in position $\frac{k}{100}\times n+\frac{1}{2}$.
Mode	in a set of data the mode is the most frequently occurring value.

23.2 Statistical Formulae

Mean, $\bar{x} = \dfrac{\sum f\,x}{\sum f}$ or $\bar{x} = \dfrac{\sum x_i}{n}$ where x_i is the variate,

f is frequency

n is the sample size

BE - averages

- -

Population Standard Deviation $\sigma = \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n}}$

$\sigma = \sqrt{\dfrac{\sum f\,d^2}{\sum f}}$ $\quad d = x_i - \bar{x}$

- -

Sample Standard Deviation $s = \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n-1}}$

where n is the sample size

- -

Table for the calculation of Sample Mean and Standard Deviation

x_i	f	$f\,x_i$	$x - \bar{x}$	$f(x - \bar{x})^2$
.	.	.	.	.
.	.	.	.	.
		$\sum f\,x_i =$		$\sum f(x - \bar{x})^2 =$
		$\bar{x} = \dfrac{\sum f\,x_i}{n} =$		$s = \sqrt{\dfrac{\sum f(x - \bar{x})^2}{n-1}} =$

- -

Coefficient of Variation

of a sample (as a %) $\dfrac{s}{\bar{x}} \times 100$

Semi-interquartile Range $SIR = \dfrac{Q_3 - Q_1}{2}$

23.2.1 Regression Line

- see 13 and 27

For the line $y = a + bx$ where b is the gradient and a is the y intercept and n is the number of pairs of values.

$$a = \frac{\sum y - b \sum x}{n} \qquad b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - \left(\sum x\right)^2}$$

- -

Product moment coefficient of Correlation (r value)

$$r = \frac{\left(n \sum xy - \sum x \sum y\right)}{\sqrt{\left(\left(n \sum x^2 - \left(\sum x\right)^2\right)\left(n \sum y^2 - \left(\sum y\right)^2\right)\right)}} \qquad -1 \le r \le 1$$

Z Scores $Z = \frac{x - \mu}{\sigma}$

Poisson Distribution - the probability of the occurrence of a rare event

Geogbra Poisson slider $P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$

T Test **1 sample**

Standard Error of the Mean $SE(\bar{x}) = \frac{s}{\sqrt{n}}$

T test (1 sample test) $t = \frac{x - \mu}{SE(\bar{x})}$

- -

2 sample for $n > 30$ (d f = $n_1 + n_2 - 2$)

Standard Error of Mean $SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1}{n_1} + \frac{s_2}{n_2}}$

T test (2 sample test) $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE(\bar{x}_1 - \bar{x}_2)}$

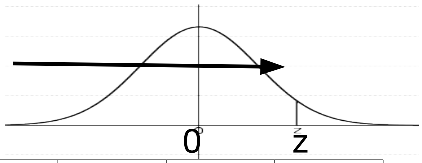
- -

2 sample for $n < 30$

Pooled Standard Deviation $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

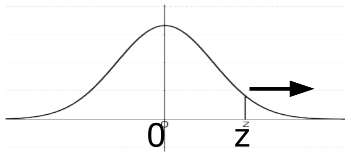
Standard Error of Mean $SE(\bar{x}_1 - \bar{x}_2) = s_p \sqrt{\frac{1}{n_1} + \frac{2}{n_2}}$

23.2.2 Tables of the Normal Distribution



Probability Content from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990	1



Far Right Tail Probabilities

Z	P{Z to ∞}	Z	P{Z to ∞}	Z	P{Z to ∞}	Z	P{Z to ∞}
2.0	0.02275	3.0	0.001350	4.0	0.00003167	5.0	2.867E-7
2.1	0.01786	3.1	0.0009676	4.1	0.00002066	5.5	1.899E-8
2.2	0.01390	3.2	0.0006871	4.2	0.00001335	6.0	9.866E-10
2.3	0.01072	3.3	0.0004834	4.3	0.00000854	6.5	4.016E-11
2.4	0.00820	3.4	0.0003369	4.4	0.000005413	7.0	1.280E-12
2.5	0.00621	3.5	0.0002326	4.5	0.000003398	7.5	3.191E-14
2.6	0.004661	3.6	0.0001591	4.6	0.000002112	8.0	6.221E-16
2.7	0.003467	3.7	0.0001078	4.7	0.000001300	8.5	9.480E-18
2.8	0.002555	3.8	0.00007235	4.8	7.933E-7	9.0	1.129E-19
2.9	0.001866	3.9	0.00004810	4.9	4.792E-7	9.5	1.049E-21

These tables are public domain. http://www.math.unb.ca/~knight/utility/NormTble.htm
They are produced by APL programs written by the author, William Knight

23.2.3 Critical Values of the t Distribution

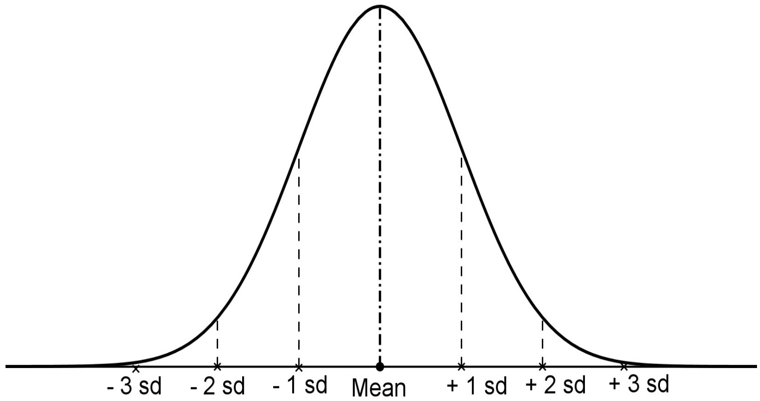
df	2-tailed testing 0.1	0.05	0.01		1-tailed testing 0.1	0.05	0.01
5	2.015	2.571	4.032		1.476	2.015	3.365
6	1.943	2.447	3.707		1.440	1.943	3.143
7	1.895	2.365	3.499		1.415	1.895	2.998
8	1.860	2.306	3.355		1.397	1.860	2.896
9	1.833	2.262	3.250		1.383	1.833	2.821
10	1.812	2.228	3.169		1.372	1.812	2.764
11	1.796	2.201	3.106		1.363	1.796	2.718
12	1.782	2.179	3.055		1.356	1.782	2.681
13	1.771	2.160	3.012		1.350	1.771	2.650
14	1.761	2.145	2.977		1.345	1.761	2.624
15	1.753	2.131	2.947		1.341	1.753	2.602
16	1.746	2.120	2.921		1.337	1.746	2.583
17	1.740	2.110	2.898		1.333	1.740	2.567
18	1.734	2.101	2.878		1.330	1.734	2.552
19	1.729	2.093	2.861		1.328	1.729	2.539
20	1.725	2.086	2.845		1.325	1.725	2.528
21	1.721	2.080	2.831		1.323	1.721	2.518
22	1.717	2.074	2.819		1.321	1.717	2.508
23	1.714	2.069	2.807		1.319	1.714	2.500
24	1.711	2.064	2.797		1.318	1.711	2.492
25	1.708	2.060	2.787		1.316	1.708	2.485
26	1.706	2.056	2.779		1.315	1.706	2.479
27	1.703	2.052	2.771		1.314	1.703	2.473
28	1.701	2.048	2.763		1.313	1.701	2.467
29	1.699	2.045	2.756		1.311	1.699	2.462
30	1.697	2.042	2.750		1.310	1.697	2.457
40	1.684	2.021	2.704		1.303	1.684	2.423
50	1.676	2.009	2.678		1.299	1.676	2.403
60	1.671	2.000	2.660		1.296	1.671	2.390
80	1.664	1.990	2.639		1.292	1.664	2.374
100	1.660	1.984	2.626		1.290	1.660	2.364
120	1.658	1.980	2.617		1.289	1.658	2.358
140	1.645	1.960	2.576		1.282	1.645	2.327

2 sample test d f = $(n_1-1)+(n_2-1)=n_1+n_2-2$

23.2.4 Normal Distribution Curve

$$y=\frac{1}{\sigma\sqrt{2\pi}}e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$$

$$\pm 1\,\text{sd} \approx 68\%$$
$$\pm 2\,\text{sd} \approx 95\%$$
$$\pm 3\,\text{sd} \approx 99.7\%$$



Geogebra Normal Dist slider
Geogebra Skewed Dist

23.2.5 Binomial Theorem

$$(x+y)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k \qquad \text{where} \qquad \binom{n}{k}=\frac{n!}{k!(n-k)!}$$

$$(x+y)^n=x^n+\frac{n!}{1!(n-1)!}x^{n-1}y^1+\frac{n!}{2!(n-2)!}x^{n-2}y^2+\ldots+\frac{n!}{(n-1)!1!}x^1y^{n-1}+y^n$$

23.2.6 Permutations and Combinations

The number of ways of selecting r objects from a total of n

BE - permutations and combinations

Permutations

Repetition allowed	${}^nP_r=n^r$	order does matter
No repetition	${}^nP_r=\frac{n!}{(n-r)!}$	order does matter

Combinations

No repetition	${}^nC_r=\frac{n!}{r!(n-r)!}$	order doesn't matter
Repetition allowed	${}^nC_r=\frac{(n+r-1)!}{r!(r-1)!}$	order doesn't matter

Thanks to Gillian Cunningham, Aberdeen College.

24 Financial Mathematics

Notation for Financial Mathematics

i	Interest rate (per time period) expressed as a fraction. (usually written as r)
d	Discount rate (per time period) expressed as a fraction.
n	Number of time periods (sometimes written as i)
P	Principal
A	Accrued amount
a	Amount
S_n	Sum to the n th term (of a geometric progression)
NPV	Net Present Value (of an accrued amount)
irr	Internal Rate of Return (when $NPV=0$)

Financial Mathematics Formulae

$$r=1+i$$

$$A=P(1+i)^n \qquad A=P(1-d)^n$$

$$S_n=\frac{a(r^n-1)}{r-1} \quad \text{or} \quad S_n=\frac{a(1-r^n)}{1-r}$$

(annuities) $$P=\frac{a(1-r^{-n})}{r-1}$$

BE - visual guide to interest rates

Efunda Calculator

---

25 Recommended Computer Programs

wxMaxima free (Open Source) MS Windows and Linux

http://wxmaxima.sourceforge.net/wiki/index.php/Main_Page
Windows: download maxima 5.24.0 (or later version)
http://portableapps.com/node/18166 (portable application)

A open source free download computer algebra system. It is being constantly updated.
You are not allowed implicit multiplication.

$$5e^{2t}+3\sin\left(\frac{\pi}{4}\right) \text{ typed as } 5*\%e\hat{}(2*t)+3*\sin(\%\mathrm{pi}/4)$$

The % sign designates special functions. (numerical values of letters)

Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms and ordinary differential equations. Also, Maxima can plot functions and data in two and three dimensions.

Geogebra free (Open Source) MS Windows and Linux
http://www.geogebra.org

This program can be accessed over the web i.e. you do not need to download it although you do usually need to be running Java Runtime Environment (free download). GeoGebra is a dynamic mathematics software that joins geometry, algebra and calculus. An expression in the algebra window corresponds to an object in the geometry window and vice versa.

Mathcad (£1000 approx.) MS Windows

This is the tool of choice for most engineering mathematics. Notes available.

Graph
free (Open Source) MS Windows

A useful graphing tool which is easy to use. **http://www.padowan.dk/graph/**

Casio Calculator Manuals (in pdf format)
http://world.casio.com/calc/download/en/manual/

26 Computer Input

wxMaxima and Geogebra are recommended .
Most of this also applies to spreadsheets and online maths sites.
Spreadsheet programs are not recommended (except for statistical calculations).

Calculator key	Computer (Keyboard) entry		
	Geogebra (3)	Mathcad (2)	wxMaxima (5)
$\times$	$*$	$*$ (Shift 8)	$*$
$\div$	/	/	/
x^2	^2	^2 (Shift 6 then 2)	^2 (Shift 6 then 2)
$x^{\blacksquare}$ or ^ or x^y or y^x	^	^	^
$\sqrt{\ }$	**sqrt**() (also on drop down list)	\	**sqrt( )**
$\sqrt[x]{\ }$	^ $\frac{1}{x}$	$\sqrt[n]{\ }$ Calculator toolbar	^ $\frac{1}{x}$
$5\sin(x^o+30^o)$ (1)	$5\sin(x^o+30^o)$ o symbol from drop down list	$5\sin(x\,deg+30\,deg)$	5*sin(x/180*%pi+ 30/180*%pi)
e^x	e from drop down list then ^ or exp( )	e^x	%e^( ) or exp( )
ln	ln	ln	log
π	pi	CTRL g	%pi
$10\times\pi\times0.7$	10 pi *0.7	10 CTRL g*0.7	10*%pi*0.7
$\sin^{-1}(0.5)$ means arcsin(0.5)	asin(0.5)	asin(0.5)	asin(0.5)

(1) As all programs work in radians by default you must change every input into degrees (if you have to work in degrees).
(2) Also available on toolbars.
(3) Only x allowed as variable
(4) See also 17.5
(5) In wxMaxima typing pi will produce π as a variable NOT 3.1415... The same is true for e .

Back to **2 Web Sites**

26.1 wxMaxima Input

*Note: From version 0.8.1 use **Shift+Enter** to enter expressions to change behaviour go to **Edit: Configure***

See wxMaxima Introduction at http://ubuntuone.com/p/x77

See http://www.math.hawaii.edu/~aaronts/maximatutorial.pdf a simple introduction.

See **http://www.neng.usu.edu/cee/faculty/gurro/Maxima.html** but put in expression first!

Note: Implicit multiplication is **NOT** allowed. $3x$ is **always** typed as $3*x$

Insert Text Box F6

Zoom in Alt I Zoom out Alt O

Copy as an Image to a Spreadsheet File

Edit - Select All
Right click – Copy as Image...
Paste onto a worksheet

Assign w:3.7 (means $w=3.7$) f(x):=3*x (means $f(x)=3\text{x}$)

Matrix multiplication $\begin{bmatrix} & \end{bmatrix} \cdot \begin{bmatrix} & \end{bmatrix}$ Use . Do not use $*$

26.1.1 Newton Raphson

load(newton1)

newton $(f(x), x, x_{0,} p)$. Start with precision $p=0.1$ and then $p=0.01$ etc. until outputs are identical to significant figures required

---

26.1.2 Differential Equations

see also 22.4 (2nd page)

$\frac{dy}{dx}$ typed as 'diff(y,x) *note the apostrophe ' before* diff

$\frac{d^2 y}{d x^2}$ typed as 'diff(y,x,2)

Equations; Solve ODE. Equations; Initial value problem (1) or (2).

26.1.3 Runge-Kutta

$rk(f(x,y), y, y_{0,}[x, x_{0,} x_{end}, h])$ *See Euler's Method* 22.5

To plot result: wxplot2d([discrete,%o#],[style,points])

you can replace points *with* line. %o# *is a previous output line.*

26.2 Mathcad Input

Applied Maths

Definition of variables and functions

variable := number and units (:= use colon **:**)

Example: x:3kg will read as $x := 3\,\mathrm{kg}$ and a:5 m/s^2 as $a := 5\,\dfrac{m}{s^2}$

Function $f(x)$:= function in terms of x

Example: f(x): x*a will be interpreted as $f(x) := x \cdot a$

= gives numerical answer

Example $f(x) =$ will produce the answer $15\,N$ ■

You can type a different unit in place of the box and the number will change to satisfy the units chosen.

- -

Symbolic Maths

$f(x)$ = use Boolean **(bold)** equals

$\rightarrow$ symbolic units

Implicit multiplication: This is allowed but only with variables that cannot be confused with units.

For example, $3x$ is fine but $3s$ must be typed as $3*s$.

When editing expressions use the $\boxed{\mathrm{Ins}}$ key to change from editing to the left to editing to the right of cursor.

Also see Mathcad Notes

---

27 Using a Spreadsheet to find the 'best fit' formula for a set of data.

see 13 and 26 and 23.2.1

Data presented as

x	x_1	x_2	x_3	x_4	etc
y	y_1	y_2	y_3	y_4	etc

Basic Procedure:-

	Put **x** data in column A and **y** data in column B		
Highlight	All data		
Select	Insert	(or chart symbol)	
	Chart		
Chart Type	XY (Scatter)		
Titles	Give graph and axes titles		
(Chart Location	As New Sheet (optional)		E)
(Right click on plot	Format Plot Area		E)
(Area	Click to white		E)
Right click on data point	Add Trendline		
Type	Choose most appropriate		
Options	Display equation on chart		
	Display R^2 value on chart		

R^2 value should lie between 0.95 and 1. The closer to 1 the better. Right click on trendline to change to a better type.

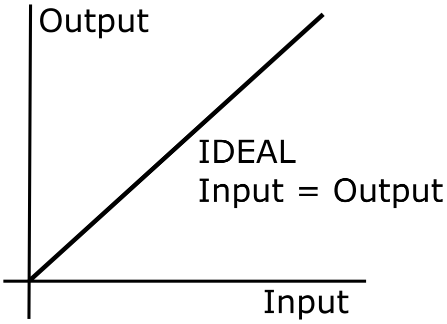
The equation displayed is the formula for the data

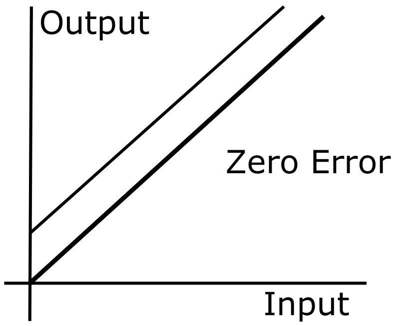
All instructions necessary for MS Excel (E). Open Office Calc will provide the same answers but in a slightly different format.
Mathcad and Maxima can be used but are more complicated mathematically but will be more accurate. Geogebra can be used to match a line to data.

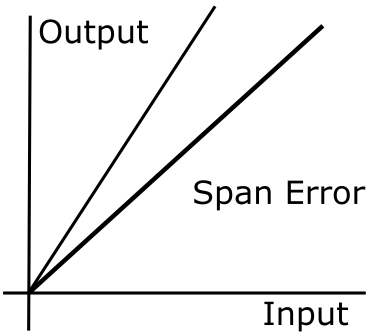
Note: **EXCEL is NOT recommended for any mathematical or engineering calculation where accuracy or consistency is vital**.

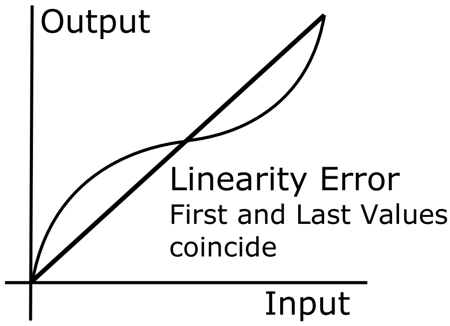
---

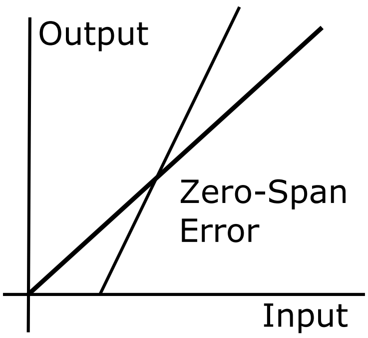
28 Calibration Error

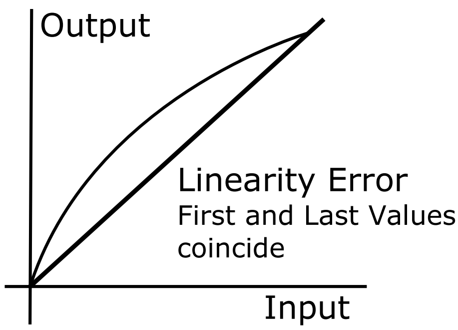












Thanks to Olaniyi Olaosebikan, Aberdeen College

29 SI Units - Commonly used prefixes

meaning	multiple	prefix	symbol
$\times 1000000000000000$	$\times 10^{15}$	Peta	P
$\times 1000000000000$	$\times 10^{12}$	Tera	T
$\times 1000000000$	$\times 10^{9}$	Giga	G
$\times 1000000$	$\times 10^{6}$	Mega	M
$\times 1000$	$\times 10^{3}$	kilo	k
$\times 1$	$\times 10^{0}$		
$\div 1000$	$\times 10^{-3}$	milli	m
$\div 1000000$	$\times 10^{-6}$	micro	μ
$\div 1000000000$	$\times 10^{-9}$	nano	n
$\div 1000000000000$	$\times 10^{-12}$	pico	p

30 Electrical Tables

Table of Resistivities

Material	Resistivity (ρ) $(\Omega\, m)$ at $20^o C$
Silver (Ag)	15.9×10^{-9}
Copper (Cu)	17.2×10^{-9}
Gold (Au)	24.4×10^{-9}
Tungsten (W)	56.0×10^{-9}
Nickel (Ni)	69.9×10^{-9}
Iron (Fe)	100×10^{-9}
Lead (Pb)	220×10^{-9}
Carbon (C)	35000×10^{-9}

Relative Static Permittivity

Material	Dielectric Constant ε_r
Vacuum	1
Air	1.00054
Diamond (C)	5.5 - 10
Salt (NaCl)	3 - 15
Graphite (C)	10 - 15
Silicon (Si)	11.68

Permeability Values for some Common Materials

Material	Permeability (μ) (H/m)
Electrical Steel	5000×10^{-6}
Ferrite (Nickel Zinc) (Ni Zn)	$20-800\times 10^{-6}$
Ferrite (Manganese Zinc) (Mn Zn)	$>800\times 10^{-6}$
Steel	875×10^{-6}
Nickel (Ni)	125×10^{-6}
Aluminium (Al)	1.26×10^{-6}

Thanks to Satej Shirodkar, Aberdeen College.

31 THE GREEK ALPHABET

UPPER CASE	lower case	Pronunciation	Examples of use
A	α	Alpha	angles, angular acceleration
B	β	Beta	angles
Γ	γ	Gamma	shear strain, heat capacity, kinematic viscosity
Δ	δ	Delta	DIFFERENTIAL, the change in... (Calculus)
E	ε	Epsilon	linear strain, permittivity
Z	ζ	Zeta	impedance, damping ratio
H	η	Eta	efficiency, viscosity
Θ	θ	Theta	angles, temperature, volume strain
I	ι	Iota	inertia
K	κ	Kappa	compressibility
Λ	λ	Lambda	wavelength, thermal conductivity, eigenvalues
M	μ	Mu	micro (10^{-6}), coefficient of friction
N	ν	Nu	velocity
Ξ	ξ	Xi	damping coefficient
O	o	Omicron	
Π	π	Pi	PRODUCT, 3.141592654...., $C = \pi d$
P	ρ	Rho	density, resistivity
Σ	σ	Sigma	SUM; standard deviation, normal stress
T	τ	Tau	shear stress, torque, time constant
Υ	υ	Upsilon	admittance
Φ	ϕ	Phi	angles, flux, potential energy, golden ratio
X	χ	Chi	PEARSON'S χ^2 TEST , angles
Ψ	ψ	Psi	helix angle (gears), phase difference
Ω	ω	Omega	RESISTANCE; angular velocity