Notes During PhD

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This document is an ongoing collection of my writing during my PhD. Inspired by the Tufte-Handout Style [1], this handout is built using `tufte-latex`[2].

[1] Edward R. Tufte!

[2] `https://github.com/Tufte-LaTeX/tufte-latex`

Contents

LaTeX Project Management

The goal is to have a consistent boilerplate for LaTeXprojects, I choose AOS for regular article.[3]

- `chapters/01-*.tex`: individual files
- `fig`: figures to reproduce
  - External figures by R or Python, DPS ≥ 300, `.pdf`
  - TikZ: `.tex` and `.pdf`
  - Asymptote: `.asy` and `.pdf`

Below is an example project hosted on Github or Overleaf:

```
├── chapters
│   ├── 01-preface.tex
│   ├── 02-intro.tex
├── fig
│   ├── hilbertrecursive.tex
│   ├── hilbertrecurves.pdf
│   ├── helix.asy
│   ├── helix.pdf
├── latexmkrc
├── main.bib
├── main.pdf
├── main.tex
└── tex
    ├── macro.tex
    ├── tufte-book.cls
    ├── tufte-common.def
    ├── tufte-handout.cls
    └── tufte.bst
```

[3] https://vtex-soft.github.io/texsupport.ims-aos/

Mathematical Notation

It has always been a hassle to organise mathematical notation across different sources, in fact, I would go so far as to argue that it is the most annoying thing when one starts reading a book or an article.

However, there *must be* some notational conflicts beyond primary school simply due to the fact that the limited number of alphabets (**26**). For example, "$\mathbb{E}$" might be energy in physics while it could refer to expectation or scores in probability.

Another difficulty is that the authors often assume some familiarity in the topics *also* I am expected to read in some logical or chronological order. In reality, I am constantly jumping back and forth between one literature to another.

Symbol	Context	Meaning
$\mathbb{Z}$		set of integers
$\mathbb{R}$		set of real numbers $(-\infty, \infty)$

Reverse Inverse Projections (RIPr)

Here we assume a family $\mathcal{Q}$ (singleton or composite) of parametric family.

And we assume the distribution of null hypothesis is well supported P and begin with single element for simplicity.

Li's Algorithm

Originally developed by @liEstimationMixtureModels1999, the following algorithm is written based on the notes from [GdK24] @haoEvalues*k*sampleTests2024.

The gist is to obtain the RIPr in a greedy manner where the KL divergence between distribution Q onto the convex hull of a set of distributions $\mathcal{Q}$ (composite null) is minimized. It is assumed that the KL divergence between Q and any distribution $Q \in \mathcal{Q}$ is finite.

Algorithm 1: Li's Algorithm

$Q_{(1)} = \arg\min_{Q \in \mathcal{Q}} D(P \| Q)$
for $j = 2, 3, \ldots, K$ **do**
  $Q_{(m)} := \alpha Q_{(m-1)} + (1-\alpha) Q'$

Here, the distribution Q' and α is chosen (coupled) such that the divergence $D(P \| \alpha Q_{(m-1)} + (1-\alpha)Q')$ is minimized. The minimizer need not be unique (*I think*).

Regularity condition on $\mathcal{Q}$

Li's algorithm is apparently greedy with high fluctuation in initial steps.

Additionally, this task is computationally expensive, and it is not clear of the convexity.

Is the returned mixture in the convex hull?

Correction: The returned $Q_{(m)}$ is in the convex hull by definition. The first step returned a single element in $\mathcal{Q}$ with the smallest KL divergence. Iteratively, the linear combination is still in the convex hull, e.g.

$$\begin{aligned}
Q_{(2)} &= \alpha_1 \cdot Q_{(1)} + (1-\alpha_1) \cdot Q'_{(1)} \\
Q_{(2)} &= \alpha_2 \cdot Q_{(2)} + (1-\alpha_2) \cdot Q'_{(2)} \\
&= \alpha_2 \cdot \alpha_1 \cdot Q_{(1)} + \alpha_2 \cdot (1-\alpha_1) \cdot Q'_{(1)} + (1-\alpha_2) \cdot Q'_{(2)}
\end{aligned}$$

It is clear that $Q_{(2)}$ or $Q_{(m)}$ would still be a convex combination of elements in $\mathcal{Q}$.

Another way to look at Li's is that we have. It remains unclear how the minimization is actually being done in practice.

Requirement

To quote Brinda, Li's inequality requires the family $\mathcal{Q}$ to have a uniformly bounded density ratio.

Cisszar Algorithm

Originally proposed by

References

[GdK24] Peter Grünwald, Rianne de Heide, and Wouter Koolen. Safe testing. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 86(5):1091–1128, November 2024.