

## Assignment

1. Derive the complexity in Big- $\mathcal{O}$  notation for  $\langle \psi | \hat{H} | \psi \rangle$  with an MPS  $\psi$  of bond dimension  $\chi$  and an MPO  $\hat{H}$  of bond dimension  $D$  on chain of  $N$  sites and open index dimension  $d$ .
2. To find the  $m$ -th excited state one can add penalty terms  $\omega_i$  to the Hamiltonian

$$\hat{H}' = \hat{H} + \sum_{n=0}^{m-1} \omega_n |\Psi_n\rangle\langle\Psi_n| \quad (6.1)$$

such that minimizing this  $\langle \hat{H}' \rangle$  naturally avoids previously found states and gives the targeted excited state. Prove the criterion of  $\omega_0$  for the first excited states by discussing the three cases, (a)  $\omega_1 < \Delta\varepsilon$ , (b)  $\omega_1 > \Delta\varepsilon$ , (c)  $\omega_1 = \Delta\varepsilon$ , where  $\Delta\varepsilon = \varepsilon_1 - \varepsilon_0$  is the energy gap between the first excited and the ground state.

3. Write a code in any programming language as you prefer to implement the W states  $|W_3\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$  on three qubits as MPS using matrices and verify it by computing overlap to all the basis states, such as  $\langle 000|W\rangle$ ,  $\langle 001|W\rangle$ , and so forth.
4. The Hamiltonian of the one-dimensional transverse-field Ising model on  $N$  sites is given by

$$H = -J \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=1}^N \sigma_i^x, \quad (6.2)$$

where  $J$  is strength of local interaction and  $h$  is the field strength. Consider a quenched time evolution where the initial state  $|\psi_0\rangle$  is the ground state of the Hamiltonian with  $h \ll J$  and starting at  $t = 0$  the system is applied by a quenched Hamiltonian with  $h > J$ . One may imagine this is like we turn on the background field at  $t = 0$ . You can investigate any of the following by writing a code in **iTensor** to prepare the state and simulate the dynamics:

1. What happens to the order parameter, such as  $M_z = \frac{1}{N} \sum_i \langle \sigma_i^Z \rangle$ ,  $M_x = \frac{1}{N} \sum_i \langle \sigma_i^x \rangle$ ,  $M_z^2 = \frac{1}{N^2} \sum_{i,j} \langle \sigma_i^Z \sigma_j^Z \rangle$  as time evolves?
2. How does the correlation  $C_r(i) = \langle \sigma_i^Z \sigma_{i+r}^Z \rangle$  look like at different distance  $r$ ? You may plot the expectation  $C_r(t)$  as heatmap.
3. How does the overlap of the states  $\langle \psi_0 | \psi(t) \rangle$  look like?
4. or any other observable you find interesting.