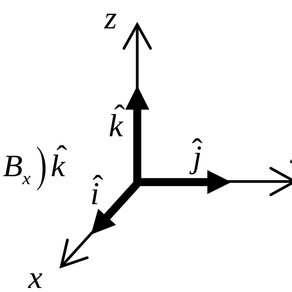
Formula Sheet – Physics 221

Vectors and math



$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad \vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta \qquad \vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

10^{-15}	femto-	(f)
10^{-12}	pico-	(p)
10^{-9}	nano-	(n)
10^{-6}	micro-	(μ)
10^{-3}	milli-	(m)
10^{-2}	centi-	(c)
10^{3}	kilo-	(k)
10^{6}	mega-	(M)
10^{9}	giga-	(G)
10^{12}	tera-	(T)

Geometry

circumference: $2\pi R$ area sphere: $4\pi R^2$

area circle: πR^2 volume sphere: $\frac{4}{3}\pi R^3$

1 revolution = 2π radians = 360°

Conversion factors (for barbaric units)

1 yard = 3 foot = 36 inches 1 inch = 2.54 cm

1 mile = 1.609 km 1 lb = 4.448 N

1 gallon = 3.788 liters $1\ \mathrm{m}^3 = 1000$ liters

1 atm = 1.01×10^5 Pa = 760 mm Hg

1 cal = 4.186 J 1 Cal = 1000 cal

Physical constants

$$g = 9.81\ \mathrm{m/s^2} \qquad G = 6.67\times10^{-11}\ \mathrm{Nm^2/kg^2}$$

$$R = 8.31\ \frac{\mathrm{J}}{\mathrm{mol\cdot K}} = 0.082\ \frac{\mathrm{atm\cdot l}}{\mathrm{mol\cdot K}} \qquad \sigma = 5.67\times10^{-8}\ \mathrm{W}\big/\left(\mathrm{m^2\cdot K^4}\right) \qquad k = \frac{R}{N_\mathrm{A}} = 1.38\times10^{-23}\ \mathrm{J/K}$$

$$\upsilon_{\text{sound, air}} = 343\ \mathrm{m/s} \qquad \text{STP: 1 atm, 273K} \qquad N_\mathrm{A} = 6.022\times10^{23}$$

General kinematics

$$\vec{\upsilon}_{\text{average}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{\upsilon} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{average}} = \frac{\Delta\vec{\upsilon}}{\Delta t} \qquad \vec{a} = \frac{d\vec{\upsilon}}{dt}$$

Constant acceleration

$$\vec{r} = \vec{r}_0 + \vec{\upsilon}_0 t + \frac{1}{2}\vec{a}t^2 \qquad \vec{\upsilon} = \vec{\upsilon}_0 + \vec{a}t \qquad \upsilon^2 - \upsilon_0^2 = 2\vec{a}\cdot\Delta\vec{r}$$

$$x = x_0 + \upsilon_{0x}t + \frac{1}{2}a_xt^2 \qquad \upsilon_x = \upsilon_{0x} + a_xt \qquad \upsilon_x{}^2 - \upsilon_{0x}^2 = 2a_x\Delta x$$

Circular motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad s = R\theta \qquad v = R\omega \qquad a_{\tan} = R\alpha$$

$$a_{\mathrm{rad}} = \frac{v^2}{R} = R\omega^2 \qquad a_{\tan} = \frac{d|\vec{v}|}{dt} \qquad \vec{a} = \vec{a}_{\mathrm{rad}} + \vec{a}_{\tan} \qquad \text{Constant } \omega\text{: } T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi R}{v}$$

Constant α: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\qquad \omega = \omega_0 + \alpha t$ $\qquad \omega^2 - \omega_0^2 = 2\alpha\Delta\theta$

Relative motion

$$\vec{r}_{\text{A relative to C}} = \vec{r}_{\text{A relative to B}} + \vec{r}_{\text{B relative to C}} \qquad \vec{\upsilon}_{\text{A relative to C}} = \vec{\upsilon}_{\text{A relative to B}} + \vec{\upsilon}_{\text{B relative to C}}$$

$$\vec{a}_{\text{A relative to C}} = \vec{a}_{\text{A relative to B}} + \vec{a}_{\text{B relative to C}}$$

Forces

$$\sum \vec{F} = m\vec{a} \qquad \vec{F}_g (\equiv \vec{W}) = m\vec{g} \qquad f_s \le \mu_s N \qquad f_k = \mu_k N \qquad F_{\mathrm{Hooke}} = -k\Delta x$$

Work and energy

$$W = \int \vec{F} \cdot d\vec{l} \qquad \left(W = F\Delta x \cos\theta\right) \qquad KE = \frac{1}{2}m\upsilon^2 = \frac{p^2}{2m} \qquad W_{\mathrm{net}} = \Delta KE$$

$$P_{\mathrm{inst}} = \frac{dW}{dt} = \vec{F} \cdot \vec{\upsilon}$$

$$W_{\mathrm{conservative}} = -\Delta U \qquad U(\vec{r}\,) - U(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{l} \qquad \vec{F} = -\vec{\nabla} U \qquad \left(F_x = -\frac{\partial U}{\partial x}, \text{ etc}\right)$$

$$U = \frac{1}{2}kx^2 + C \qquad U = mgy + C$$

$$E = KE + U \qquad \Delta E = W_{\text{non-conservative}} \qquad \left(\text{When only conservative forces do work: } \Delta E = 0\right)$$

Momentum, impulse. Systems of particles.

$$\vec{p} = m\vec{\upsilon} \qquad \vec{J} = \Delta\vec{p} = \int \vec{F} dt = \vec{F}_{\mathrm{ave}} \Delta t$$

$$\vec{r}_{\mathrm{CM}} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \qquad \vec{\upsilon}_{\mathrm{CM}} = \frac{\sum_i m_i \vec{\upsilon}_i}{\sum_i m_i} \qquad \vec{a}_{\mathrm{CM}} = \frac{\sum_i m_i \vec{a}_i}{\sum_i m_i} \qquad KE_{\mathrm{lab}} = KE_{\mathrm{CM}} + KE_{\text{relative to CM}}$$

$$\vec{p}_{\mathrm{total}} = m_{\mathrm{total}} \vec{\upsilon}_{\mathrm{CM}} \qquad \vec{F}_{\mathrm{net}} = \frac{d\vec{p}_{\mathrm{total}}}{dt} = m_{\mathrm{total}} \vec{a}_{\mathrm{CM}} \qquad \left(\text{When } \vec{F}_{\mathrm{net}} = 0,\ \vec{p}_{\mathrm{total,i}} = \vec{p}_{\mathrm{total,f}}\right)$$

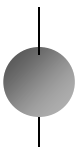
$$\upsilon_{\mathrm{A}x} = \frac{m_{\mathrm{A}} - m_{\mathrm{B}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} \upsilon_{0x} \qquad \upsilon_{Bx} = \frac{2m_{\mathrm{A}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} \upsilon_{0x} \qquad \upsilon_{\mathrm{A,i},x} - \upsilon_{\mathrm{B,i},x} = -(\upsilon_{\mathrm{A,f},x} - \upsilon_{\mathrm{B,f},x})$$

Rigid-body motion

$$KE_{\text{total}} = KE_{\text{translation}} + KE_{\text{rotation}} \qquad KE_{\text{translation}} = \frac{1}{2} m v_{\text{CM}}^2 \qquad KE_{\text{rotation}} = \frac{1}{2} I \omega^2$$

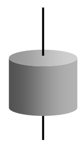
$$I = \sum_i m_i r_i^2 \qquad I = I_{\text{CM}} + md^2 \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = I\vec{\alpha} \qquad \vec{L} = \vec{r} \times \vec{p} \qquad L_z = I\omega_z$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \qquad (\text{When } \tau_{\text{net}} = 0, \quad \vec{L}_{\text{total,i}} = \vec{L}_{\text{total,f}}) \qquad W = \int \vec{\tau} \cdot d\vec{\theta} \qquad P = \vec{\tau} \cdot \vec{\omega}$$

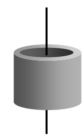


$$I_{\text{solid sphere}} = \frac{2}{5} mr^2$$

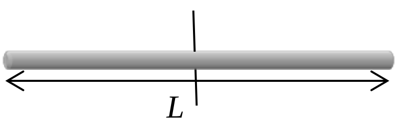
$$I_{\text{hollow sphere}} = \frac{2}{3} mr^2$$



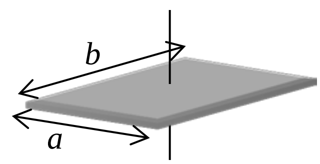
$$I_{\text{solid cylinder}} = \frac{1}{2} mr^2$$



$$I_{\substack{\text{hollow cylinder} \\ \text{with thin walls}}} = mr^2$$



$$I_{\text{rod}} = \frac{1}{12} mL^2$$



$$I_{\text{rectangle}} = \frac{1}{12} m\left(a^2 + b^2\right)$$

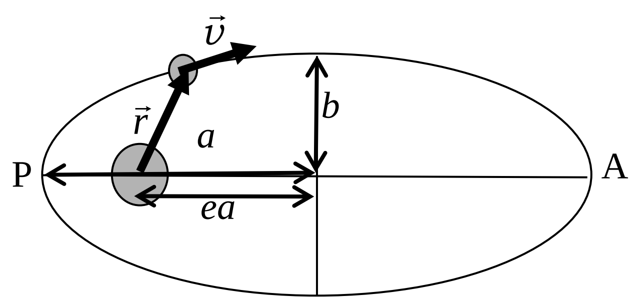
Gravitation

$$\left|\vec{F}_{\text{Newton}}\right| = G\frac{Mm}{r^2} \qquad g = G\frac{M}{r^2} \qquad U = -G\frac{Mm}{r} \qquad v_{\text{circular orbit}} = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\vec{r} \times \vec{v} = \text{constant}$$

$$r_{\text{A}} v_{\text{A}} = r_{\text{P}} v_{\text{P}}$$



$$g = 9.81 \text{ m/s}^2 \qquad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$M_{\text{E}} = 5.97 \times 10^{24} \text{ kg} \qquad R_{\text{E}} = 6.38 \times 10^6 \text{ m}$$

Fluids

$$p = \frac{F}{A} \qquad \Delta p = \rho g \Delta h \qquad \rho = \frac{dm}{dV} \qquad F_{\text{buoyancy}} = \rho_{\text{fluid}} V_{\text{displaced}}\, g$$

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ mm Hg}$$

Simple harmonic motion

$$\frac{d^2x}{dt^2}+\omega^2x=0 \qquad x=A\cos(\omega t+\varphi) \qquad \upsilon=-A\omega\sin(\omega t+\varphi) \qquad a=-A\omega^2\cos(\omega t+\varphi)$$

$$T=\frac{1}{f}=\frac{2\pi}{\omega} \qquad \omega=\sqrt{\frac{k}{m}} \qquad \omega=\sqrt{\frac{\kappa}{I}} \qquad \omega=\sqrt{\frac{g}{l}} \qquad \omega=\sqrt{\frac{mgd}{I}}$$

$$\vec{F}_{\text{damping}}=-b\vec{\upsilon} \qquad x=A(t)\cos(\omega' t+\varphi) \qquad A(t)=Ae^{-\frac{b}{2m}t} \qquad \omega'=\sqrt{\omega_0^2-\left(\frac{b}{2m}\right)^2}$$

$$A=\frac{\frac{F_{\max}}{m}}{\sqrt{(\omega_0^2-\omega_{\text{d}}^2)^2+\left(\frac{b\omega_{\text{d}}}{m}\right)^2}}$$

Mechanical waves

$$\upsilon=\lambda f \qquad \omega=2\pi f \qquad f=\frac{1}{T} \qquad k=\frac{2\pi}{\lambda} \qquad \omega=\upsilon k$$

$$\frac{\partial^2 y(x,t)}{\partial x^2}=\frac{1}{\upsilon^2}\frac{\partial^2 y(x,t)}{\partial t^2} \qquad y(x,t)=A\cos(kx-\omega t)$$

$$\upsilon=\sqrt{\frac{\text{Tension}}{\mu}} \qquad \mu=\frac{dm}{dl} \qquad P_{\text{average}}=\frac{1}{2}\mu v\omega^2A^2$$

Sound

$$v_{\text{sound, air}}=343\text{ m/s} \qquad I=\frac{P}{A} \qquad \beta=(10\text{ dB})\log\frac{I}{I_0} \qquad I_0=1.0\times10^{-12}\text{ W/m}^2 \qquad f_{\text{beat}}=f_a-f_b$$

Temperature and heat

$$T_{\text{K}}=T_C+273.15\text{ K} \qquad T_{\text{F}}=\frac{9}{5}T_C+32^\circ\text{C} \qquad Q=mc\Delta T \qquad Q=nC_{\text{molar}}\Delta T \qquad Q=\pm mL$$

$$\Delta L=\alpha L_0\Delta T \qquad \Delta A\approx 2\alpha A_0\Delta T \qquad \Delta V=\beta V_0\Delta T\approx 3\alpha V_0\Delta T$$

$$H=\frac{dQ}{dt}=kA\frac{T_{\text{H}}-T_C}{L} \qquad \frac{dQ}{dt}=Ae\sigma\left(T_{\text{object}}^4-T_{\text{surrounding}}^4\right) \qquad \sigma=5.67\times10^{-8}\text{ W}\big/\left(\text{m}^2\cdot\text{K}^4\right)$$

Ideal gas

$$pV = nRT \qquad p_x = \frac{n_x}{n_{\text{all}}} p_{\text{all}} \qquad n = \frac{N}{N_\text{A}} \qquad N_\text{A} = 6.022 \times 10^{23}$$

STP: 1 atm, 273K $\qquad R = 8.31 \ \dfrac{\text{J}}{\text{mol} \cdot \text{K}} = 0.082 \ \dfrac{\text{atm} \cdot \text{l}}{\text{mol} \cdot \text{K}} \qquad k = \dfrac{R}{N_\text{A}} = 1.38 \times 10^{-23} \ \text{J/K}$

$$\left\langle K_{\text{trans, one particle}} \right\rangle = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT \qquad \lambda = \frac{V}{4\pi\sqrt{2} r^2 N}$$

$C_V = \dfrac{3}{2} R$ (monoatomic ideal gas) $\qquad C_V = \dfrac{5}{2} R$ (diatomic ideal gas)

$C_V = 3R$ (monoatomic solid crystal)

Thermodynamics

$$\Delta U = Q - W \qquad W = p\Delta V \qquad W = \int_{V_i}^{V_f} p dV \qquad dU = nC_V dT$$

$$dQ = nC_V dT \qquad dQ = nC_P dT \qquad C_P = C_V + R \qquad \gamma = \frac{C_P}{C_V}$$

$$TV^{\gamma - 1} = \text{constant} \qquad pV^\gamma = \text{constant} \qquad W_{\text{adiab}} = -\frac{\Delta(pV)}{\gamma - 1}$$

$$e = \frac{W}{Q_H} \qquad K_{\text{refrigerator}} = \frac{Q_C}{|W|} \qquad K_{\text{heat pump}} = \frac{|Q_H|}{|W|} \qquad W = Q_C + Q_H \qquad e_{\text{Otto}} = 1 - \frac{1}{r^{\gamma - 1}}$$

For Carnot cycle: $\dfrac{T_C}{T_H} = -\dfrac{Q_C}{Q_H} \qquad e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H} \qquad K_{\text{Carnot refrigerator}} = \dfrac{T_C}{T_H - T_C}$

$$dS = \frac{dQ_{\text{reversible}}}{T}$$