

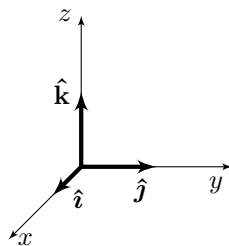
Formula Sheet - Physics 222

Vectors:

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$= (A_x, A_y, A_z)$$

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Unit vector along A

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|}, \quad |\hat{\mathbf{A}}| = 1$$

Scalar (dot) product and Vector (cross) product:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

$$|\mathbf{A} \times \mathbf{B}| = AB \sin \theta$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

Algebra, Calculus & Trigonometry:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{dx^n}{dx} = nx^{n-1} \quad \frac{d \cos x}{dx} = -\sin x \quad \frac{d \sin x}{dx} = \cos x$$

$$\frac{d \tan x}{dx} = \sec^2 x \quad \frac{d \cot x}{dx} = -\csc^2 x \quad \frac{d}{dx} \left(\frac{f}{g} \right) = \frac{\frac{df}{dx}g - f \frac{dg}{dx}}{g^2}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \quad 2 \sin^2 x + \cos(2x) = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \quad 2 \cos^2 x - \cos(2x) = 1$$

$$2 \cos x \cos y = \cos(x - y) + \cos(x + y) \quad \sin^2 x + \cos^2 x = 1$$

$$2 \sin x \sin y = \cos(x - y) - \cos(x + y) \quad \sec^2 x - \tan^2 x = 1$$

$$2 \sin x \cos y = \sin(x - y) + \sin(x + y) \quad \csc^2 x - \cot^2 x = 1$$

Conversion factors:

1 yard = 3 ft	1 ft = 12 in
1 in = 2.54 cm	1 mi = 1609 m
1 eV = 1.6×10^{-19} J	1 m ³ = 1000 L
1 Tesla = 10^4 Gauss	1 Å = 10^{-10} m
= 1 Weber/m ²	1 lb = 4.448 N

Geometry

Circle: Perimeter = $2\pi r$

Area = πr^2

Sphere: Surface Area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Cylinder: Surface Area = $2\pi r^2 + 2\pi rh$

Volume = $\pi r^2 h$

femto- (f) = 10^{-15}

pico- (p) = 10^{-12}

nano- (n) = 10^{-9}

micro- (μ) = 10^{-6}

milli- (m) = 10^{-3}

centi- (c) = 10^{-2}

kilo- (k) = 10^3

mega- (M) = 10^6

giga- (G) = 10^9

tera- (T) = 10^{12}

Physical Constants & Quantities:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2 \quad k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$$

$$e = 1.60 \times 10^{-19} \text{ C} \quad c = 3.00 \times 10^8 \text{ m/s}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg} \quad m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A} \quad 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

$$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3 \quad \rho_{\text{air}} = 1.20 \text{ kg/m}^3$$

Circular motion

$$\omega = \frac{d\theta}{dt} \quad s = R\theta \quad v = R\omega \quad a_{\text{tan}} = R\alpha$$

$$\alpha = \frac{d\omega}{dt} \quad a_{\text{rad}} = \frac{v^2}{R} = \omega^2 R \quad a_{\text{tan}} = \frac{dv}{dt}$$

$$F_{\text{net,rad}} = m \frac{v^2}{r} \quad F_{\text{net,tan}} = ma_{\text{tan}}$$

Uniform Circular Motion: $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi R}{v}$

Fluids

$$p = \frac{F}{A} \quad p = p_0 + \rho gh \quad \rho = \frac{m}{V}$$

$$F_b = \rho_f V_{\text{displ}} g \quad dV/dt = Av$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \text{const} \quad p + \rho gy + \frac{1}{2} \rho v^2 = \text{const}$$

Electrostatics

$$\mathbf{F}_{\text{on 2 by 1}} = \frac{k q_1 q_2}{r_{1 \rightarrow 2}^2} \hat{r}_{1 \rightarrow 2} \quad \mathbf{E} = \frac{\mathbf{F}}{q} \quad \vec{p} = q \mathbf{d}_{- \rightarrow +}$$

$$\mathbf{E}_{\text{at P by q}} = \frac{kq}{r_{q \rightarrow P}^2} \hat{r}_{q \rightarrow P} \quad U_{\text{dip}} = -\vec{p} \cdot \mathbf{E} \quad \vec{\tau}_{\text{dip}} = \vec{p} \times \mathbf{E}$$

$$\mathbf{E}_{\text{sphere}} = \frac{kq}{r^2} \hat{r} \quad \mathbf{E}_{\text{cylinder}} = \frac{2k\lambda}{r} \hat{r} \quad \mathbf{E}_{\text{plane}} = \pm \frac{\sigma}{2\epsilon_0} \hat{i}$$

$$E_{\text{dip}}(r \gg d) = \frac{p}{2\pi\epsilon_0 r^3} \quad \Phi_E = \int_S \mathbf{E} \cdot d\mathbf{A} \quad \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$\rho = \frac{dq}{dV} \quad \sigma = \frac{dq}{dA} \quad \lambda = \frac{dq}{dl}$$

$$V_B - V_A = - \int_A^B \mathbf{E} \cdot d\vec{l} \quad \Delta V = \pm E_{\text{const}} d \quad V_{q \text{ at } P} = \frac{kq}{r_{q \rightarrow P}}$$

$$\mathbf{E} = -\vec{\nabla} V \quad U = qV \quad W = -q\Delta V$$

Capacitors

$$C = \frac{Q}{V} \quad C_{\text{rectangular}} = \frac{\epsilon_0 A}{d} \quad C_{\text{sphere}} = 4\pi\epsilon_0 a$$

$$C_{\text{spherical}} = \frac{4\pi\epsilon_0 ab}{b-a} \quad C_{\text{cylindrical}} = \frac{2\pi\epsilon_0 L}{\ln(a/b)}$$

$$C_{\text{parallel}} = C_1 + C_2 + \dots \quad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

$$\epsilon = \kappa\epsilon_0 \quad C = \kappa C_0 \quad E = \frac{E_0}{\kappa} \quad u = \frac{1}{2} \epsilon E^2$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C} \quad \sigma_{\text{ind}} = \sigma_0 \left(1 - \frac{1}{\kappa} \right)$$

Electric current and resistors

$$I = \frac{dQ}{dt} \quad \vec{J} = ne\vec{v}_d \quad J = \frac{dI}{dA_\perp} \quad \vec{v}_d = \frac{e\tau}{m_e} \mathbf{E}$$

$$\vec{E} = \rho \vec{J} \quad V = RI \quad \rho = \frac{m_e}{e^2 n \tau} \quad R = \frac{\rho L}{A_\perp}$$

$$P = IV = I^2 R = \frac{V^2}{R} \quad \frac{\Delta \rho}{\rho_0} = \alpha \Delta T \quad \frac{\Delta R}{R_0} = \alpha \Delta T$$

$$R_{\text{series}} = R_1 + R_2 + \dots \quad \frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$I_{\text{series}} = I_1 = I_2 = \dots \quad I_{\text{parallel}} = I_1 + I_2 + \dots$$

$$V_{\text{series}} = V_1 + V_2 + \dots \quad V_{\text{parallel}} = V_1 = V_2 = \dots$$

Kirchhoff's Rules

$$\sum_{\text{junction}} I = 0 \quad \sum_{\text{loop}} V = 0$$

R-C DC circuits

$$q(t) = Q_\infty (1 - e^{-t/\tau}) \quad q(t) = Q_0 e^{-t/\tau}$$

$$v_C(t) = \mathcal{E} (1 - e^{-t/\tau}) \quad v_C(t) = \mathcal{E} e^{-t/\tau}$$

$$i(t) = I_0 e^{-t/\tau} \quad \tau = RC$$

Magnetostatic

$$\mathbf{F}_{\text{on } q \text{ by } \mathbf{B}} = q \vec{v} \times \mathbf{B} \quad \mathbf{B}_{\text{at } P \text{ by } q} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{\mathbf{r}}_{q \rightarrow P}}{r_{q \rightarrow P}^2}$$

$$d\mathbf{F}_{\text{on } I \text{ by } \mathbf{B}} = I d\vec{\ell} \times \mathbf{B} \quad d\mathbf{B}_{\text{at } P \text{ by } I} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{\mathbf{r}}_{I \rightarrow P}}{r_{I \rightarrow P}^2}$$

$$\mathbf{B}_{\text{long wire}} = \frac{\mu_0 I}{2\pi r} \quad \mathbf{B}_{\text{loop}} = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \quad \frac{\mathbf{F}}{\ell} = \frac{\mu_0 I I'}{2\pi r}$$

$$\mathbf{B}_{\text{Solenoid}} = \mu_0 \frac{N}{\ell} I \quad \mathbf{B}_{\text{Toroid}} = \frac{\mu_0 N I}{2\pi r} \quad R = \frac{mv}{|q|B}$$

$$\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} \quad \Phi_B = \int_S \mathbf{B} \cdot d\mathbf{A} \quad \oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\vec{\mu} = IA \quad \vec{\tau}_\mu = \vec{\mu} \times \mathbf{B} \quad U_\mu = -\vec{\mu} \cdot \mathbf{B}$$

Magnetic Material

$$\mathbf{M} = \frac{\vec{\mu}_{\text{total}}}{V} \quad \mathbf{B} = \mathbf{B}_{\text{ext}} + \mu_0 \mathbf{M} \quad \mu = \kappa_M \mu_0$$

$$\chi = \kappa_M - 1 \quad \chi_{\text{dia}} < 0, \chi_{\text{para}} > 0 \quad \mathbf{M} = \frac{\chi}{\mu_0} \mathbf{B}_{\text{ext}}$$

Induction

$$\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad \mathcal{E} = vB\ell \quad \mathcal{E} = \oint (\vec{v} \times \mathbf{B}) \cdot d\vec{\ell}$$

$$\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 (i_C + i_D)_{\text{encl}} \quad i_D = \epsilon_0 \frac{d\Phi_E}{dt} \quad \mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Maxwell's equations

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (\text{Gauss' law for } \mathbf{E} \text{ field})$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad (\text{Gauss' law for } \mathbf{B} \text{ field})$$

$$\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law})$$

$$\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \quad (\text{Ampere's law})$$

Inductance

$$\mathcal{E}_{\text{in 2 by 1}} = -M \frac{di_1}{dt} \quad \mathcal{E} = -L \frac{di}{dt} \quad L = \frac{N\Phi_B}{i}$$

$$\mathcal{E}_{\text{in 1 by 2}} = -M \frac{di_2}{dt} \quad M = \frac{N_1 \Phi_{B1}}{i_2} = \frac{N_2 \Phi_{B2}}{i_1}$$

R-L circuits

$$i(t) = I_\infty (1 - e^{-t/\tau}) \quad v_L(t) = \mathcal{E} e^{-t/\tau} \quad U = \frac{1}{2} LI^2$$

$$i(t) = I_0 e^{-t/\tau} \quad \tau = \frac{L}{R} \quad L = \mu_0 n^2 \ell A \quad u = \frac{B^2}{2\mu_0}$$

$$L_{\text{series}} = L_1 + L_2 + \dots \quad \frac{1}{L_{\text{parallel}}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots$$

R-L-C circuit

$$q(t) = Q_0 e^{-\frac{R}{2L}t} \cos(\omega' t + \phi) \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

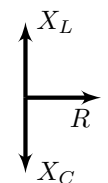
AC R-L-C circuit

$$i = I \cos(\omega t - \phi) \quad I_{\text{rms}} = \frac{I}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V}{\sqrt{2}}$$

$$v = V \cos \omega t \quad I_{\text{rav}} = \frac{2}{\pi} I \quad V_{\text{rms}} = I_{\text{rms}} Z$$

$$V_R = IR \quad V_L = IX_L \quad X_L = \omega L$$

$$V = IZ \quad V_C = IX_C \quad X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \tan \phi = \frac{X_L - X_C}{R}$$


$$P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos \phi = \frac{1}{2} IV \cos \phi \quad p = iv$$

$$\text{Transformers : } \frac{V_2}{V_1} = \frac{N_2}{N_1} \quad V_1 I_1 = V_2 I_2$$

Complex impedance

$$\mathbf{Z}_{\text{series}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \dots \quad \frac{1}{\mathbf{Z}_{\text{parallel}}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \dots$$

$$\mathbf{Z} = R + jX \quad X = X_L - X_C \quad \cos \phi = R/Z$$

$$\mathbf{Z} = Ze^{j\phi} \quad Z = \sqrt{R^2 + X^2} \quad \tan \phi = X/R$$

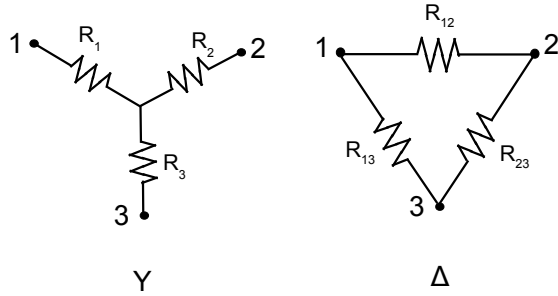
$$\mathbf{V} = V_0 e^{j\omega t} \quad \mathbf{I} = \mathbf{V}/\mathbf{Z} = I_0 e^{j(\omega t - \phi)} \quad v_X = \text{Re}(\mathbf{I}_X \mathbf{Z}_X)$$

$$z = a + jb \quad z^* = a - jb \quad \frac{1}{z} = \frac{z^*}{Z^2} \quad Z^2 = a^2 + b^2$$

$$z_1 z_2 = a_1 a_2 - b_1 b_2 + j(a_1 b_2 + a_2 b_1) = Z_1 Z_2 e^{j(\phi_1 - \phi_2)}$$

$$\frac{z_1}{z_2} = \frac{z_1 z_2^*}{Z_2^2} = \frac{a_1 b_1 + b_1 b_2 + j(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2} = Z_1 Z_2 e^{j(\phi_1 - \phi_2)}$$

Y-Δ transform



$$R_1 = \frac{R_{12}R_{13}}{R_{12} + R_{13} + R_{23}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{13} + R_{23}}$$

$$R_3 = \frac{R_{13}R_{23}}{R_{12} + R_{13} + R_{23}}$$

$$R_{12} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

$$R_{13} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_{23} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

EM waves and optics

$$\mathbf{E}(x, t) = \hat{\mathbf{j}} E_{\max} \cos(kx - \omega t) \quad E = cB \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\mathbf{B}(x, t) = \hat{\mathbf{k}} B_{\max} \cos(kx - \omega t) \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2 \mu_0} B^2 = \epsilon_0 E^2 = \frac{B^2}{\mu_0} \quad \frac{1}{A} \frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

$$I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2 \mu_0} = \frac{E_{\max}^2}{2 c \mu_0} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{\max}^2 = \frac{1}{2} \epsilon_0 c E_{\max}^2$$

$$c = \lambda f = \frac{\omega}{k} \quad k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f$$

$$n = c/v \quad \theta_i = \theta_r \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$I = I_{\max} \cos^2 \phi \quad \sin \theta_c = n_2/n_1 \quad \tan \theta_p = n_2/n_1$$

Interference and diffraction

$$d \sin \theta = m \lambda \quad d \sin \theta = \left(m + \frac{1}{2}\right) \lambda \quad y = m R \frac{\lambda}{d}$$

$$I = I_{\max} \cos^2 \frac{\phi}{2} \quad \phi = 2\pi \frac{r_2 - r_1}{\lambda} = 2\pi \frac{d \sin \theta}{\lambda}$$

$$2t = m \lambda_n \quad 2t = \left(m + \frac{1}{2}\right) \lambda_n \quad \lambda_n = \lambda/n$$

$$a \sin \theta = m \lambda \quad \beta = 2\pi \frac{a \sin \theta}{\lambda} \quad I = I_{\max} \frac{\sin^2 \left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{2}\right)^2}$$

$$R = \frac{\lambda}{\Delta \lambda} \quad \sin \theta_1 = 1.22 \frac{\lambda}{D} \quad 2d \sin \theta = m \lambda$$

Data Analysis - Lab Formulas

Basic statistical estimators

$$\text{Mean value : } \bar{x} = \langle x \rangle = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{Standard deviation : } s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad \text{Standard error : } SE = \frac{s}{\sqrt{n}}$$

Propagation of uncertainty

Assume that x and y are independent variables and that their mean values and corresponding standard errors are:

$$x = x_0 \pm \varepsilon_x \quad \text{and} \quad y = y_0 \pm \varepsilon_y$$

Function f	Standard error	Limit of error
$x \pm y$	$\sqrt{\varepsilon_x^2 + \varepsilon_y^2}$	$\varepsilon_x + \varepsilon_y$
xy	$ x_0 y_0 \sqrt{\left(\frac{\varepsilon_x}{x_0}\right)^2 + \left(\frac{\varepsilon_y}{y_0}\right)^2} = \sqrt{(x_0 \varepsilon_y)^2 + (y_0 \varepsilon_x)^2}$	$ x_0 y_0 \left(\frac{\varepsilon_x}{ x_0 } + \frac{\varepsilon_y}{ y_0 }\right) = x_0 \varepsilon_y + y_0 \varepsilon_x$
$\frac{x}{y}$	$\left \frac{x_0}{y_0}\right \sqrt{\left(\frac{\varepsilon_x}{x_0}\right)^2 + \left(\frac{\varepsilon_y}{y_0}\right)^2}$	$\left \frac{x_0}{y_0}\right \left(\frac{\varepsilon_x}{ x_0 } + \frac{\varepsilon_y}{ y_0 }\right)$
x^n	$ n x_0^{n-1} \varepsilon_x$	$ n x_0^{n-1} \varepsilon_x$
$\sin(x)$	$ \cos(x_0) \varepsilon_x \quad (\varepsilon_x \text{ must be in radians})$	$ \cos(x_0) \varepsilon_x \quad (\varepsilon_x \text{ must be in radians})$
$\cos(x)$	$ \sin(x_0) \varepsilon_x \quad (\varepsilon_x \text{ must be in radians})$	$ \sin(x_0) \varepsilon_x \quad (\varepsilon_x \text{ must be in radians})$