Control-Oriented Data-Driven Dynamics Modeling for Underwater Robots

Project uwnav_dynamics

January 24, 2026

Contents

1 Notation

1.1 Reference Frames

- $\mathcal{F}_b$: body-fixed frame attached to the robot.
- $\mathcal{F}_n$: navigation frame, defined as ENU (East–North–Up).

$\boldsymbol{R}_{nb} \in SO(3)$ denotes the rotation matrix from body frame to navigation frame.

1.2 State and Control

The control-oriented system state is defined as

$$\boldsymbol{x} = \begin{bmatrix} v_x & v_y & v_z & \omega_z & a_x & a_y & a_z & \psi \end{bmatrix}^\top, \tag{1}$$

where

- (v_x, v_y, v_z): linear velocity expressed in the navigation frame $\mathcal{F}_n$,
- ω_z: yaw angular rate,
- (a_x, a_y, a_z): linear acceleration,
- ψ: yaw angle.

The control input is denoted by

$$\boldsymbol{u} \in \mathbb{R}^m, \tag{2}$$

representing normalized thruster commands.

1.3 Temporal Parameters

- Control frequency: $f_c = 50\,\mathrm{Hz}$.
- History window length: L.
- Prediction horizon: H.

1.4 Uncertainty

For any predicted variable y, the model outputs both mean $\hat{y}$ and standard deviation $\hat{\sigma}_y$.

2 Modeling Assumptions

2.1 Time-Scale Assumptions

The system is considered at a discrete-time resolution aligned with the control loop. Hydrodynamic effects are assumed to be locally stationary within a short prediction horizon.

2.2 Observability Assumptions

- Planar velocity is intermittently observable through DVL measurements.
- Attitude and angular velocity are provided by the onboard IMU fusion algorithm.
- Vertical motion and roll/pitch dynamics are not explicitly modeled.

2.3 Applicability Domain

The proposed model targets low-to-moderate speed underwater robots operating in structured environments. Highly aggressive maneuvers and full 6-DOF coupling are outside the current scope.

3 Sensor Models

3.1 Inertial Measurement Unit (IMU)

The accelerometer measures the specific force:

$$\tilde{\boldsymbol{a}} = \boldsymbol{a} - \boldsymbol{R}_{bn}\boldsymbol{g} + \boldsymbol{b}_a + \boldsymbol{n}_a, \tag{3}$$

where $\boldsymbol{g}$ is the gravity vector, $\boldsymbol{b}_a$ the accelerometer bias, and $\boldsymbol{n}_a$ measurement noise.

The gyroscope measurement is given by

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} + \boldsymbol{b}_\omega + \boldsymbol{n}_\omega. \tag{4}$$

Bias terms are modeled as random walks.

3.2 Doppler Velocity Log (DVL)

The DVL provides low-frequency planar velocity observations:

$$\tilde{\boldsymbol{v}} = \boldsymbol{v} + \boldsymbol{n}_v, \tag{5}$$

where $\boldsymbol{n}_v$ is non-Gaussian and subject to dropouts. Quality indicators are used to gate valid observations.

4 Control-Oriented Dynamics Abstraction

Rather than identifying full hydrodynamic parameters, we model the system response as

$$\boldsymbol{x}_{k+1} = f_\theta(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{h}_k), \tag{6}$$

where $\boldsymbol{h}_k$ represents latent hydrodynamic memory states.

The latent dynamics are defined as

$$\boldsymbol{h}_{k+1} = \boldsymbol{\alpha} \odot \boldsymbol{h}_k + g_\theta(\boldsymbol{x}_k, \boldsymbol{u}_k), \tag{7}$$

with element-wise decay factors $\boldsymbol{\alpha} \in (0, 1)$ ensuring stability.

This structure captures wake effects, added mass, and other unmodeled fluid interactions.

5 Heteroscedastic Uncertainty Modeling

Sensor noise and environmental disturbances are state-dependent and non-stationary. We therefore model predictive uncertainty explicitly.

For a predicted variable y, the network outputs $(\hat{y}, \hat{\sigma}_y)$. Training minimizes the negative log-likelihood:

$$\mathcal{L}_{\text{NLL}} = \frac{\|y - \hat{y}\|^2}{\hat{\sigma}_y^2} + \log \hat{\sigma}_y^2. \tag{8}$$

This formulation allows the model to adaptively down-weight unreliable observations and provides uncertainty estimates usable by downstream controllers.

6 Learning Objective

The model is trained for multi-step prediction over a horizon H.

6.1 Prediction Loss

$$\mathcal{L}_{\text{pred}} = \sum_{j=1}^{H} w_j \|\hat{\boldsymbol{x}}_{k+j} - \boldsymbol{x}_{k+j}\|^2. \tag{9}$$

6.2 Consistency Constraint

Velocity and acceleration predictions are constrained by

$$\hat{\boldsymbol{v}}_{k+1} \approx \hat{\boldsymbol{v}}_k + \hat{\boldsymbol{a}}_k \Delta t. \tag{10}$$

6.3 Total Loss

$$\mathcal{L} = \mathcal{L}_{\text{pred}} + \lambda_c \mathcal{L}_{\text{cons}} + \lambda_u \mathcal{L}_{\text{NLL}}. \tag{11}$$

7 Multi-Step Rollout and Stability

Single-step accuracy is insufficient for control. Errors compound during rollout and may destabilize predictive controllers.

The latent-state formulation with bounded decay ensures internal stability, while uncertainty estimates grow with prediction horizon, reflecting reduced confidence.

8 Interface to Control Algorithms

At each control step, the model provides

$$(\hat{\boldsymbol{x}}_{k+1:k+H}, \hat{\Sigma}_{k+1:k+H}), \tag{12}$$

where $\hat{\Sigma}$ denotes predictive uncertainty.

This interface is compatible with MPC and model-based RL algorithms, which may incorporate uncertainty for risk-sensitive decision making.

9 Discussion

The proposed framework prioritizes control usability over full physical fidelity. While not replacing high-fidelity CFD models, it offers a practical compromise between interpretability, stability, and deployment efficiency.

Future work includes extension to full 6-DOF dynamics and online adaptation.

A Additional Derivations

This appendix will include supplementary derivations and implementation details.