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1. Huffman Coding

88	88	88	88	88	88	88	88
20	20	20	20	20	20	20	20
0	0	0	0	0	0	0	0
0	40	40	40	40	0	0	0
0	40	40	40	40	0	0	0
0	40	40	40	40	0	0	0
0	40	40	40	40	0	0	0
0	0	0	0	0	0	0	0

(a) What is the entropy η of the image below, where numbers (0, 20, 40, 88) denote the gray-level intensities? (10 marks)

According to Lecture 5, the Entropy of a source $S = \{s | s \in \{v_1, v_2, ..., v_n\}\}$ is

$$\eta = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i} = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- where p_i is the probability that discrete symbol v_i occurs in S.

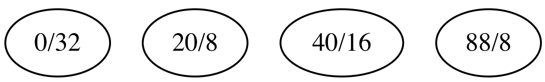
In our case, $v_1 = 0, p_1 = 0.5$; $v_2 = 20, p_2 = 0.125$; $v_3 = 40, p_3 = 0.25$; $v_4 = 88, p_4 = 0.125$.

Therefore, the entropy $\eta = 1.75$.

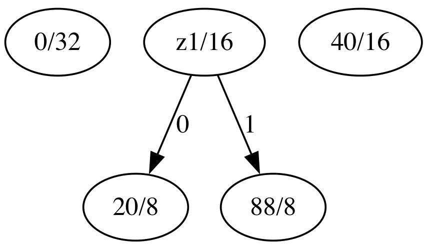
(b) Show step by step how to construct the Huffman tree to encode the above four intensity values in the image. Show the resulting code for each intensity. (10 marks)

Based on (a), we have $S = \{0/32, 20/8, 40/16, 88/8\}$

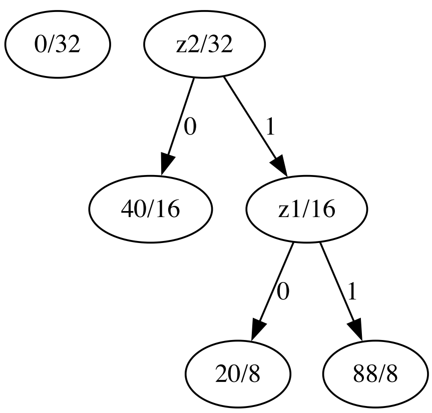
Step 0:



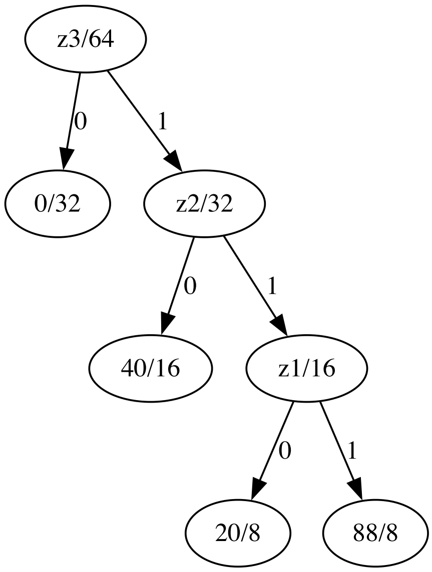
Step 1: Merge $20/8$ and $88/8$



Step 2: Merge $40/16$ and $z_1/16$



Step 3: Merge $0/32$ and $z_2/32$



Intensity	**Code**
0	0
20	110
40	10
88	111

(c) What is the average number of bits needed for each pixel, using your Huffman code? (10 marks)

Based on (a) and (b), we have:

Intensity	**Code**	**Number of bits**	**Count**
0	0	1	32
20	110	3	8
40	10	2	16
88	111	3	8
Total length			64

Average number of bits is

$$\frac{32 \times 1 + 8 \times 3 + 16 \times 2 + 8 \times 3}{64} = 1.75$$

2. DCT

(a) For an 8×8 grayscale image, each pixel is in the range [0, 255], after applying the above 2D-DCT, we obtain a new 8×8 matrix, what is the largest value an element of that matrix could be? and for what input image? and show all elements of the matrix for that image. (8+8+8 = 24 marks)

1. **the largest value an element of that matrix could be**

$$F(0,0)=\frac{C(0)C(0)}{4}\sum_{i=0}^{7}\sum_{j=0}^{7}\cos\frac{((2i+1)0\pi)}{16}\cos\frac{((2j+1)0\pi)}{16}f(i,j),\text{where } f(i,j)=255,\forall i,j$$

$$=\frac{\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}}{4}\sum_{i=0}^{7}\sum_{j=0}^{7}1\times 1\times 255=\frac{1}{8}\times 64\times 255=2040$$

2. **input image should be a white image, where all pixel values are 255.**
3. **all elements of the matrix for that image:**

$$F(u,v)=\frac{C(u)C(v)}{4}\sum_{i=0}^{7}\sum_{j=0}^{7}(\cos\frac{((2i+1)u\pi)}{16}\cos\frac{((2j+1)v\pi)}{16}\times 255)$$

Approximate values for all elements of the matrix are as follows, where the DC is 2040, and all of the ACs are near zero.

```
[[ 2.04000000e+03  1.55753172e-13 -1.70826060e-13  1.55753172e-13
   4.01943669e-14  5.67745433e-13 -3.96919374e-13 -2.41166202e-13]
 [ 7.03401421e-14  7.10542736e-15  0.00000000e+00  7.10542736e-15
   7.10542736e-15  1.42108547e-14 -3.55271368e-15 -1.24344979e-14]
 [-9.04373256e-14  5.68434189e-14  0.00000000e+00 -7.10542736e-15
  -7.10542736e-15 -1.42108547e-14  0.00000000e+00 -5.32907052e-15]
 [ 2.81360569e-13  4.26325641e-14 -7.10542736e-15  7.10542736e-15
   0.00000000e+00 -1.77635684e-14  0.00000000e+00  5.32907052e-15]
 [ 8.03887339e-14  2.84217094e-14  7.10542736e-15 -1.42108547e-14
   2.13162821e-14  1.77635684e-14 -7.10542736e-15  7.10542736e-15]
 [ 5.12478179e-13 -7.81597009e-14 -1.42108547e-14 -3.55271368e-14
   3.55271368e-15 -1.06581410e-14 -8.88178420e-15 -2.57571742e-14]
 [-3.76822190e-13  3.55271368e-15 -1.77635684e-14  1.06581410e-14
  -7.10542736e-15 -1.95399252e-14 -8.88178420e-15  0.00000000e+00]
 [-2.61263385e-13  4.08562073e-14 -5.32907052e-15 -5.32907052e-15
   1.06581410e-14 -1.50990331e-14  8.88178420e-16  8.88178420e-15]]
```

(b) For that image in (a), if we first subtract the value 128 from each pixel of the whole image and then carry out the 2D-DCT, what is the effect on the DCT value F[2,3]? (6 marks)

The original DCT value F[2,3] is `-7.10542736e-15`. If we first subtract the value 128 from each pixel of the whole image, the $F(u,v)$ becomes

$$F(u,v)=\frac{C(u)C(v)}{4}\sum_{i=0}^{7}\sum_{j=0}^{7}(\cos\frac{((2i+1)u\pi)}{16}\cos\frac{((2j+1)v\pi)}{16}\times 127)$$

Therefore, the DCT value F[2,3] becomes `-1.06581410e-14`. Compared with the original value, the effect on the F[2,3] is small, since the F[2,3] is high frequency AC coefficient, the subtraction of 128 will not affect the value significantly.

3. Implementation of LZ

To compress the following string of zeros/ones, we consider the dictionary-based LZW com- pression algorithm. Suppose the initial dictionary is {‘0’:0, ‘1’:1}. Implement the LZW compression and decompression algorithms using two separate functions.

```
011001100111000
```

(a) Show the constructed dictionaries for both compression and decompression stages. (10 marks)

- constructed dictionaries for compression:

```
{'0': 0, '1': 1, '01': 2, '11': 3, '10': 4, '00': 5, '011': 6, '100': 7, '0111': 8, '1000': 9}
```

- constructed dictionaries for decompression:

```
{0: '0', 1: '1', 2: '01', 3: '11', 4: '10', 5: '00', 6: '011', 7: '100', 8: '0111', 9: '1000'}
```

(b) Calculate the compression rate. (5 marks)

- Original string: `0110 0110 0111 000` (15 length)
- Compress result: `0110 2467 0` (9 length)
- compression rate: $\frac{15}{9} = 1.67$

(c) Show the full and correct python code in a separate .py file. (25 marks)

- **Please refer to the file.**