COMP3121 Social and Collaborative Computing
Exam 1, Total marks: 70, Total time: 65 minutes

Name:

Student ID:

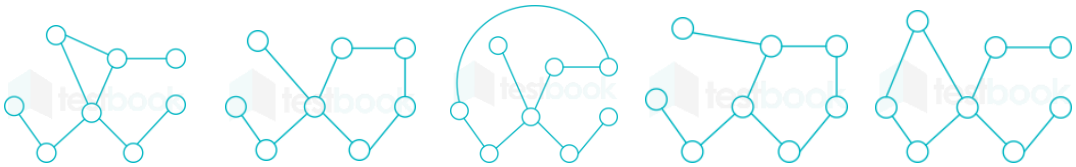
For each question, you have to show how you derive your answers. A simple answer bears no mark!

Name:

Student ID:

Question 1: Isomorphism

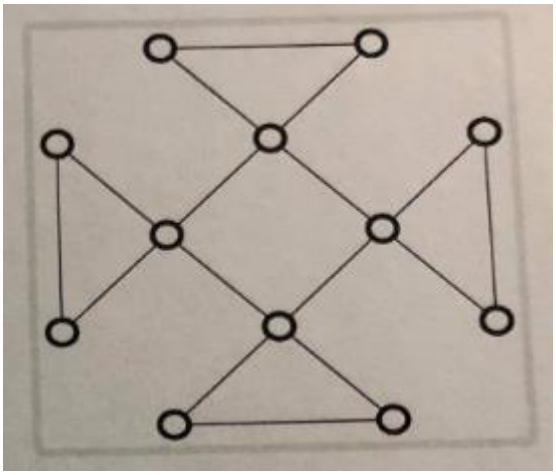
Which of the following five graphs are isomorphic?



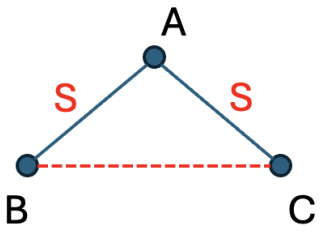
(a) (b) (c) (d) (e)

Answer: (b) and (c)

Question 2: Strong tie, weak tie, strong triadic closure property

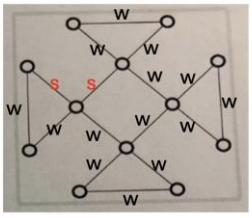


Find two ways to label the edge of the graph as “strong” and “weak”, so that the Strong Triadic Closure Property is satisfied.

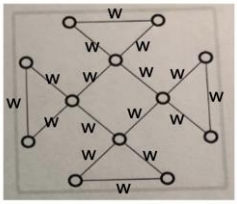


As shown in the figure above, the following situation is not allowed in your answer: A-B and A-C are strong ties, while there is no connection between B and C.

For example, the figure below is a wrong answer.



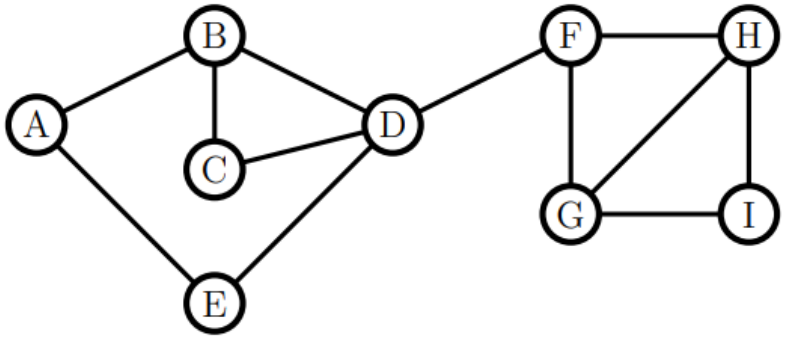
Here's an example of the correct answer. *Just give me two correct examples, don't give me the reason or I'll deduct points as appropriate.*



Question 3 [10pts]: Graph Theory

Consider the graph *G* shown below. The *diameter* of a network is the average shortest path distance between pairs of vertices.

(a) Compute the distances between all pairs of nodes in *G*.
(b) Compute the diameter of the graph *G*.



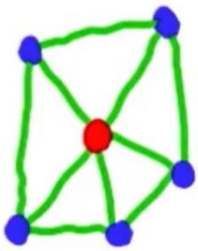
Answer:

(a)

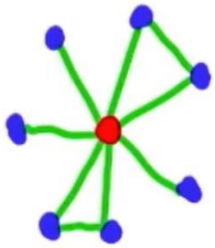
Distance	A	B	C	D	E	F	G	H	I
A	-	1	2	2	1	3	4	4	5
B	-	-	1	1	2	2	3	3	4
C	-	-	-	1	2	2	3	3	4
D	-	-	-	-	1	1	2	2	3
E	-	-	-	-	-	2	3	3	4
F	-	-	-	-	-	-	1	1	2
G	-	-	-	-	-	-	-	1	1
H	-	-	-	-	-	-	-	-	1
I	-	-	-	-	-	-	-	-	-
J	-	-	-	-	-	-	-	-	-

(b) diam(G) = 2.25

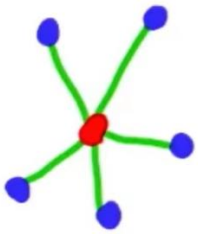
Question 4: Cluster Coefficient



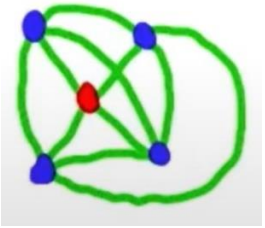
(a)



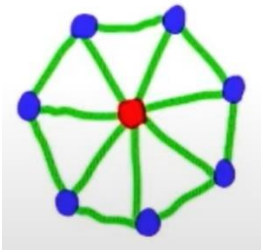
(b)



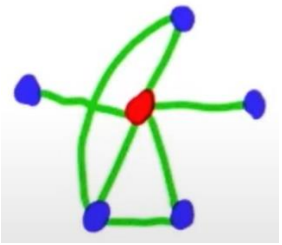
(c)



(d)



(e)



(f)

Order the cluster coefficient of the red node of each graph – from the lowest to the highest (please also show how you derive your answer)

Answer:

The cluster coefficient of each graph is:

(a) cluster coefficient $= \frac{5}{C_5^2} = \frac{1}{2}$

(b) cluster coefficient $= \frac{2}{C_7^2} = \frac{2}{21}$

(c) cluster coefficient $= \frac{0}{C_5^2} = 0$

(d) cluster coefficient $= \frac{6}{C_4^2} = 1$

(e) cluster coefficient $= \frac{7}{C_7^2} = \frac{1}{3}$

(f) cluster coefficient $= \frac{2}{C_5^2} = \frac{1}{5}$

Thus, the order of graphs is c, b, f, e, a, d.

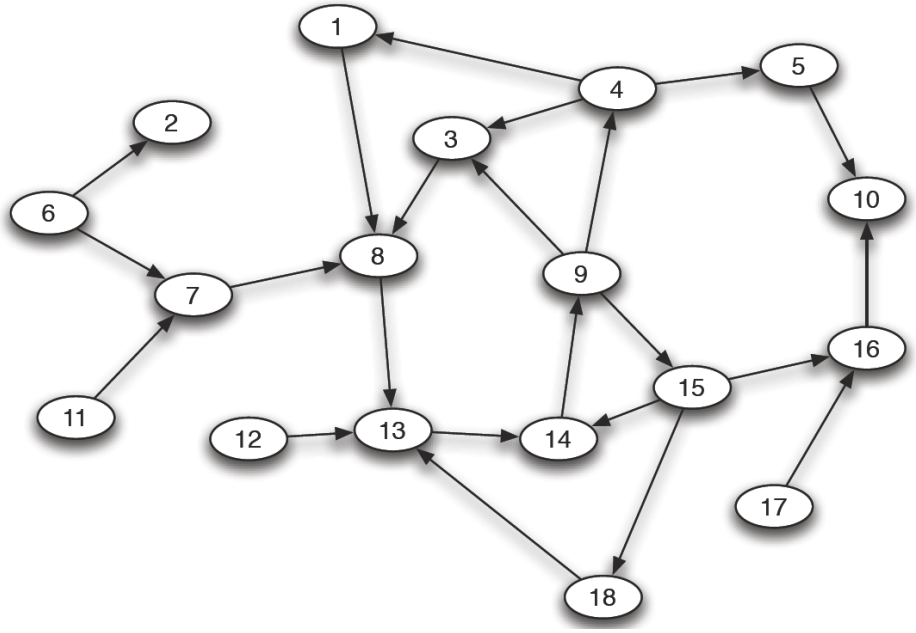
Question 5: Small World

In the basic "six degrees of separation" question, one asks whether most pairs of people in the world are connected by a path of six edges in the social network, where an edge joins any two people who know each other on a first-name basis. Now let's consider a variation on this question. Suppose that we consider the full population of the world, and suppose that from each person in the world we create a directed edge only to their ten closest friends (but not to anyone else they know on a first-name basis). In the resulting "closest-friend" version of the social network, is it possible that for each pair of people in the world, there is a path of six edges connecting this pair of people? Explain.

Answer:
No. There is no weak tie now.

Question 6: PageRank

Consider a toy small example of a directed web graph G = (V; E)



(1) Suppose we assign all 18 nodes equal initial page rank values of 1/18. Now suppose we apply the unscaled Page Rank algorithm to this graph. After one iteration (or round) of the algorithm, will any nodes have a page rank value of zero? If so, which ones and why? If not, briefly explain why not.

(2) Answer the same question as in part (1) for both two and three iterations of unscaled page rank.

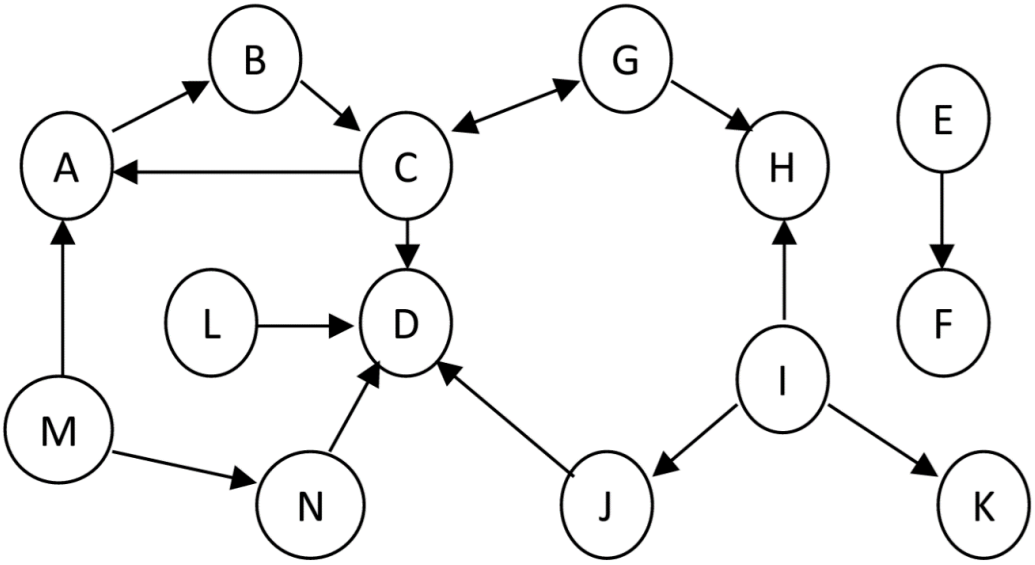
Answer:

(1) Nodes 6, 11, 12, and 17 will give up their initial page rank after the first iteration (since they all have outgoing edges). Since their in-degree is 0, they will receive no page rank from any other node; hence they will have page rank zero.

(2) After two iterations, the nodes above will still have page rank zero. In addition, node 7 have page rank zero, since all of its incoming edges are from nodes that lost their page rank after the first iteration. No other nodes have page rank zero after two iterations (all other nodes have an in-path of length 2). No further nodes will lose their page rank after the third iteration.

Question 7: Web Structure

Suppose the following graph structure represented all of the pages and links in a new experiment of the web. In which components would each of the pages be placed?



IN:

SCC:

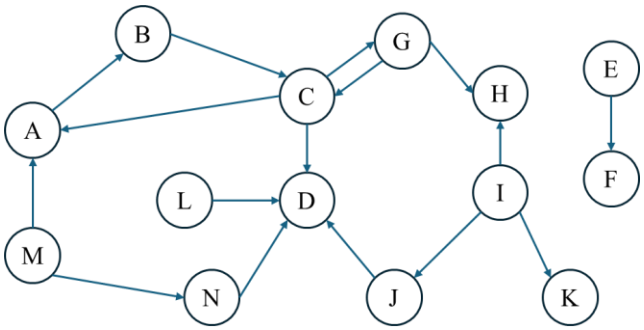
OUT:

Tendrils:

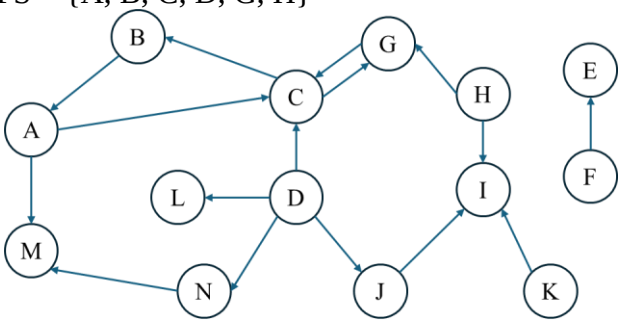
Disconnect:

Show how you derive your answer.

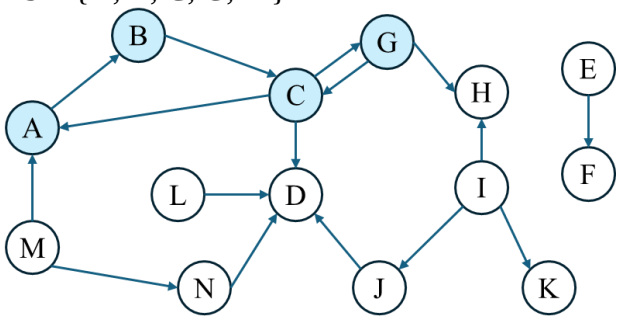
Answer:



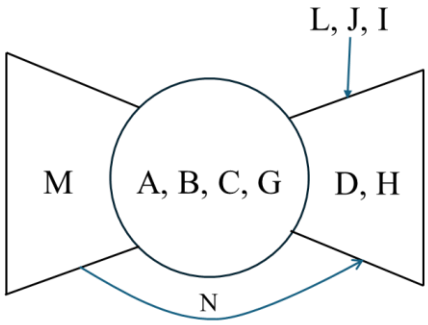
FS = {A, B, C, D, G, H}



BS = {A, B, C, G, M}



IN = BS – SCC = {M}
OUT = FS – SCC = {D, H}
SCC = FS ∩ BS = {A, B, C, G}



Tendrils: {L, J, I, N}
Disconnect: {K, E, F}