

U.S. Treasury Yield Term Structure Dynamics with Kalman Filter

Introduction:

Assume the dynamics of U.S. Treasury Yield term structure are driven by three factors, namely level, slope, and convexity. This project aims to extract the unobservable hidden states of the three factors from observed historical yield curve, with the help of Kalman Filter. Compared to simple linear regression approach, Kalman filter is able to establish a connection between curves at different time. Also, users can incorporate their understanding of the dynamics into the model by tweaking transition matrix and covariance matrix. Major drawback is that the optimization algorithm to fit parameters is slightly time consuming. Model is implemented in Python and an Excel interface is provided.

Source Code: <https://github.com/wyx-cornell/US-Treasury-Dynamics>

Model:

States:

$$\alpha_t = (\text{level}_t, \text{slope}_t, \text{convexity}_t)$$

Observation equations:

$$y_{t,m} = \text{level}_t + \text{slope}_t \times m + \text{convexity}_t \times m^2 + \varepsilon_{t,m}, \text{ where } m \text{ is the maturity}$$

Or stacked:

$$\mathbf{y} = \mathbf{Z} \cdot \alpha_t + \varepsilon_t, \text{ where } \mathbf{Z} = \begin{bmatrix} 1 & m_1 & m_1^2 \\ \vdots & \ddots & \vdots \\ 1 & m_n & m_n^2 \end{bmatrix}$$

Transition equation:

$$\alpha_t = T \cdot \alpha_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q)$$

Dependent Python packages: quandl, xlwings, pykalman (all packages need to be installed and set up properly in order for the buttons in Excel to work)

Excel Interface:

- “Dashboard” tab: Enable user to update data, fit Kalman filter, extract and visualize hidden state time series;
- “Data” tab: Contain the historical yield curve data from 1991/1/2. Data source: Quandl.
- “Kalman Filter Parameters” tab: Store the trained Kalman filter parameters, i.e. transition matrix, transition covariance, observation covariance. User can tweak parameters in this tab to incorporate his/her understanding of the dynamics.

Examples:

- Specify how far curve at t can deviate from curve at $t-1$ by tweaking transition variance;
 - Specify how changes in level, slope, and convexity interrelated by tweaking transition covariance;
 - Set larger observation variance for unreliable tenors;
 - Enforce mean reversion or other dynamics by setting transition matrix.
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- “Smoothed Factors” tab: Smoothed hidden state time series (conditional expectation for hidden states given complete realized yield curve historical data);
 - “Filtered Factors” tab: Filtered hidden state time series (conditional expectation for hidden states given realized yield curve historical data up to t);
 - “Config” tab: Model parameters, e.g. number of iterations used in EM algorithm.