U.S. Treasury Yield Term Structure Dynamics with Kalman Filter

Introduction:

Assuming that the dynamics of U.S. Treasury Yield term structure are driven by three factors, namely level, slope, and convexity, this project aims to extract the unobservable hidden states of the factors from observed historical yield curve, by applying Kalman Filter. One advantage over simple regression approach is that, Kalman filter is able to establish a connection between curves at different time. Also, users can incorporate their understandings of the dynamics into the model by tweaking the transition matrix and covariance matrix. The drawback is that the optimization algorithm to fit parameters is slightly time consuming. The model is implemented in Python and an Excel interface is provided.

Source Code: https://github.com/wyx-cornell/US-Treasury-Dynamics

Model:

States:

$$\alpha_t = (level_t, slope_t, convexity_t)$$

Observation equations:

$$y_{t,m} = level_t + slope_t \times m + convexity_t \times m^2 + \varepsilon_{t,m}, \quad where\ m\ is\ the\ maturity$$

Or stacked:

$$\boldsymbol{y} = Z \cdot \alpha_t + \varepsilon_t, \ \ where\ Z = \begin{bmatrix} 1 & m_1 & m_1^2 \\ \vdots & \ddots & \vdots \\ 1 & m_n & m_n^2 \end{bmatrix}$$

Transition equation:

$$\alpha_t = T \cdot \alpha_{t-1} + \eta_t, \ \ \eta_t \sim N(0, Q)$$

Dependent Python packages: quandl, xlwings, pykalman (all packages need to be installed and set up properly in order for the buttons in Excel to work)

Excel Interface:

- "Dashboard" tab: Enables the user to update data, fit Kalman filter, extract and visualize hidden state time series;
- "Data" tab: Contains the historical yield curve data from 1991/1/2. Data source: Quandl.
- "Kalman Filter Parameters" tab: Stores the trained Kalman filter parameters, i.e. transition matrix, transition covariance, observation covariance. The user can tweak

parameters in this tab to incorporate his/her understanding of the dynamics. For example, the user can:

- Specify how far curve at t can deviate from curve at t-1 by tweaking transition variance;
- Specify how changes in level, slope, and convexity interrelated by tweaking transition covariance;
- Set larger observation variance for unreliable tenors;
- Enforce mean reversion or other dynamics by setting transition matrix.

- "Smoothed Factors" tab: Smoothed hidden state time series (conditional expectation for hidden states given complete realized yield curve historical data);
- "Filtered Factors" tab: Filtered hidden state time series (conditional expectation for hidden states given realized yield curve historical data up to t);
- "Config" tab: Model parameters, e.g. number of iterations used in EM algorithm.