Firm Quality Dynamics and the Slippery Slope of Credit Intervention

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Introduction

- After crisis: Government offers funding directly to nonfinancial firms.
- This paper analyzes the long-term consequences of such policies.
- In a laissez-faire economy, firms with high productivity are more likely to survive crises than those with low productivity.
- Government funding saves more firms, dampening cleansing effect of crises.
- The policy distortion is self-perpetuating: A downward bias in firm quality distribution necessitates interventions of greater scale in future crises.
- The mechanism is quantitatively important.

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Preferences and Technology

- Utility of households: $\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt} dc_t\right]$.
- Two Firm Types Productivity A^H and A^L ($A^H > A^L$).
- $Y_t = A^H K_t^H + A^L K_t^L = \left(A^H \omega_t + A^L (1-\omega_t)\right) K_t$.
- Capital quality: $\omega_t \equiv \frac{K_t^H}{K_t}$ and total capital: $K_t \equiv K_t^H + K_t^L$.
- Investment Problem: $\max_{\iota_t^j} \left(q_t^j \iota_t^j K_t^j - \Phi(\iota_t^j, K_t^j)\right)$ where $\Phi(\iota_t^j, K_t^j) = \left(\iota_t^j + \frac{\theta}{2}(\iota_t^j)^2\right) K_t^j$.
- Optimal investment rate: $\iota_t^j = \frac{q_t^j - 1}{\theta}$.

Crisis

- Crisis Arrival: Poisson process N_t with intensity λ (Gourio, 2012).
- Liquidity Crisis: Firms must raise x_t per unit capital to survive ($dN_t = 1$).
- Survival Probability: $F(x_t + \zeta)$, where $\zeta \sim H(\zeta)$ (i.i.d. across firms). Assume $F' > 0$, $F'' < 0$, $F(0) = 0$.
- Crisis Severity: Low ζ can be interpreted as severe liquidity shock (e.g., payment delays, credit suspension).
- Financing Decision: $x_t^j(\zeta)$ depends on $j \in \{H, L\}$ and shock ζ.
- $$dK_t^H = K_t^H \left(\iota_t^H - \delta\right) dt - K_t^H \left[\int_\zeta \left(1 - F\left(x_t^H(\zeta) + \zeta\right)\right) dH(\zeta)\right] dN_t$$
  $$dK_t^L = K_t^L \left(\iota_t^L - \delta\right) dt - K_t^L \left[\int_\zeta \left(1 - F\left(x_t^L(\zeta) + \zeta\right)\right) dH(\zeta)\right] dN_t + \eta K_t dt$$

Decision

- Social Planner's Optimum: $\max_x[F(x+\zeta)q_t^j - x]$ FOC: $F'(x_t^{*j}+\zeta)q_t^j = 1$.
- Credit Market Equilibrium: Firm's problem:

$$\max_{0\leq x\leq \bar{d}+\bar{g}} F(x+\zeta)[q_t^j - (1+r_t^j)x]$$

  Break-even rate: $F(x+\zeta)(1+r_t^j)x = x$.
- **Key Result**: Frictionless market $\Rightarrow$ first-best x_t^{*j}.
- Government Intervention: Expands credit limit: $\bar{d}+\bar{g}$ (vs private cap $\bar{d}$)
- Type Heterogeneity: Critical threshold: $F'(\bar{\zeta}_t^j)q_t^j = 1$ implies $\bar{\zeta}_t^H > \bar{\zeta}_t^L$.
- Financial Shocks: Credit constraints distinguish crises from normal times.

Moral Hazard Problem

- Strategic Default Condition: defaults iff $\beta > q_t^j - \left[1 + r_t^j(\zeta, x_t^j(\zeta))\right] x_t^j(\zeta)$.

  Indifference threshold: $q_t^j - \left[1 + r_t^j(\underline{\zeta}_t^j, x)\right] x = \beta$.
- Threshold Behavior: Unique solution $\underline{\zeta}_t^j$ exists when $\beta \in (0, q_t^j)$.
- Type Heterogeneity: $\underline{\zeta}_t^H < \underline{\zeta}_t^L$ due to $q_t^H > q_t^L \Rightarrow$ More type-L firms default.
- Default Drivers: High debt $x_t^j(\zeta) \Rightarrow$ high default incentive. Low capital value $q_t^j \Rightarrow$ high default likelihood.

Endogenous Borrowing Constraint

- Strategic Defaulters: Maximize survival probability via $F(x_t^j(\zeta) + \zeta)\beta$ by borrowing as much as possible.
- Creditor Knowledge: Observe q_t^j and $\zeta \Rightarrow$ predict default thresholds $\underline{\zeta}_t^j \Rightarrow$ Set dual controls: interest rate $r_t^j(\zeta, x)$ + debt limit $\hat{d}_t^j(\zeta)$.
- Debt Capacity Equation: $\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta)$.
- Properties: 1. Unique solution $\hat{d}_t^j(\zeta)\ \forall\zeta$. 2. $\hat{d}_t^j(\zeta)$ strictly $\nearrow$ and concave in ζ. 3. $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$.

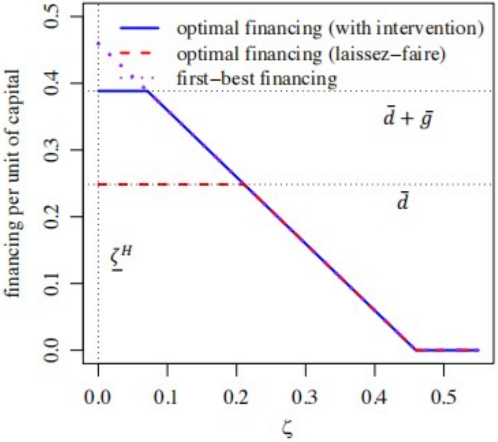
Summary

- In summary, a type- j firm's optimal choice of x is given by

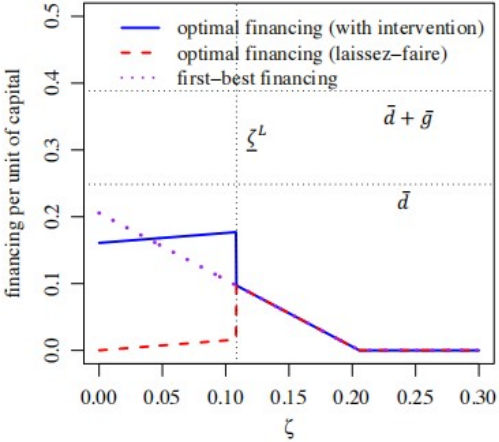
$$x_t^j(\zeta) = \underbrace{\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min \left\{ \left(\bar{\zeta}_t^j - \zeta \right)^+, \bar{d} + \bar{g} \right\}}_{\text{no strategic default}} + \underbrace{\mathbf{1}_{\zeta < \underline{\zeta}_t^j} \left(\min \left\{ \hat{d}_t^j(\zeta), \bar{d} \right\} + \bar{g} \right)}_{\text{strategic default}},$$

- Private funding crowding-in: For any ζ and $j \in \{H, L\}$, and given q_t^j, government financing crowds in private lending, $\partial \hat{d}_t^j(\zeta)/\partial \bar{g} > 0$, and reduces interest rate, $\partial r_t^j(\zeta)/\partial \bar{g} \leq 0$.

Intervention and Efficiency



(a) Type-H firm financing



(b) Type-L firm financing

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Quality Dynamics

- The capital quantity K_t is

$$\frac{dK_t}{K_{t-}} = \underbrace{\left[-\delta + \left(\omega^H_{t-l} + (1-\omega_{t-})\,\iota^L_{t-}\right) + \eta\right]}_{\mu^K_t(\omega_{t-})} dt + \underbrace{\left(\omega_{t-}\kappa^H_t + (1-\omega_{t-})\,\kappa^L_t - 1\right)}_{\Delta^K_t(\omega_{t-})} dN_t,$$

where κ^j_t, the fraction of type- j capital that survives a crisis

$$\kappa^j_t \equiv \int F\left(x^j_t(\zeta) + \zeta\right) dH(\zeta)$$

- Quality dynamics:

$$d\omega_t = \underbrace{\omega_{t-}\,(1-\omega_{t-})\left(\iota^H_{t-} - \iota^L_{t-} - \frac{\eta}{1-\omega_{t-}}\right)}_{\mu^\omega_t(\omega_{t-})} dt + \underbrace{\left(\frac{\omega_{t-}\kappa^H_t}{\omega_{t-}\kappa^H_t + (1-\omega_{t-})\,\kappa^L_t} - \omega_{t-}\right)}_{\Delta^\omega_t(\omega_{t-})} dN_t$$

Cleansing Effect

- In the first-best economy where firms spend at the optimal level, crises have a cleansing effect, i.e.,$\Delta_t^{\omega}(\omega_{t-}) > 0$.
- Given any measure of ζ where both types of firms strategically default and the private-sector debt limit binds before the private sector funding supply constraint (i.e., $\hat{d}_t^j < \bar{d}$), a cleansing effect emerges in crises. Credit intervention weakens this channel.
- If q^H is sufficiently large and q^L is sufficiently low, crises feature capital destruction, $\Delta_t^K < 0$, and have a cleansing effect, $\Delta_t^{\omega} > 0$.

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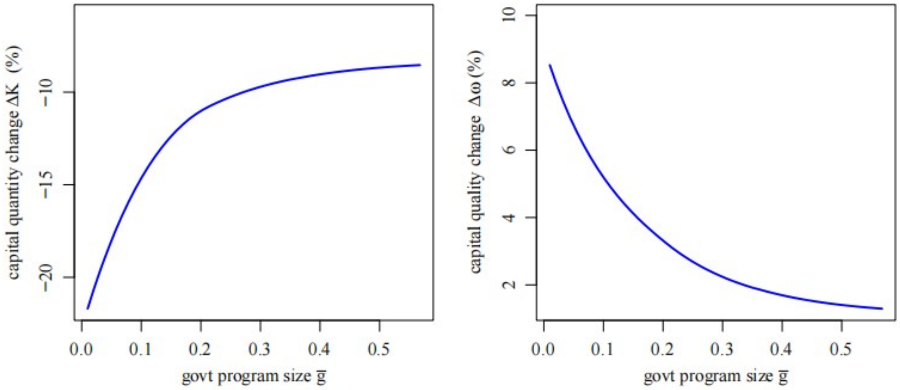
Parameter	Description	Value	Data Source or Targeted Moment
θ	Investment cost	9.2	Average investment/capital ratio
λ_F	Survival rate	5.8	Average GDP drop in crises
l_ζ	Average ζ	0.16	Impact of credit intervention on firm survival
λ	Crisis frequency	0.06	Crisis frequency in the data
β	Debt restructuring rent	0.43	Average creditor recovery rate
δ	Capital depreciation rate	0.2	Capital depreciation and firm exit rate
r	Real discount rate	0.06	Average real bond return plus equity premium
A^H	Productivity of type-H firms	0.57	Average output-to-capital ratio
A^L	Productivity of type-L firms	0.15	TFP inter-quartile ratio
η	Entry rate of new firms	0.062	Firm entry rate
$\bar{d}$	Private-sector credit availability	0.25	Private-sector debt/GDP ratio
$\bar{g}$	Government credit support	0.14	Covid-19 credit support in the U.S.

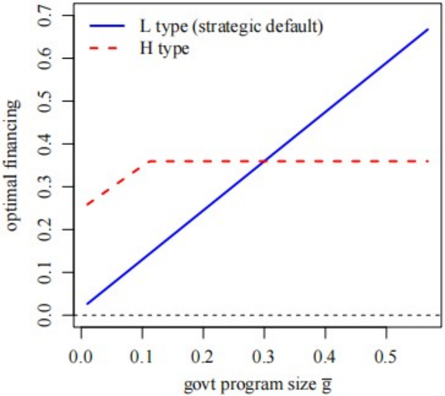
Moment Description	Model	Data
Average investment-to-capital ratio	0.1	0.1
Average GDP drop in crises	-9.3%	-9.3%
Average impact of credit intervention on firm survival likelihood	10%	10%
Average creditor recovery rate	49%	49%
Average output-to-capital ratio	45%	45%
TFP ratio between 90% and 10% percentiles	3.7	3.7
Average private-sector debt/GDP ratio	36%	36%

Trade-offs

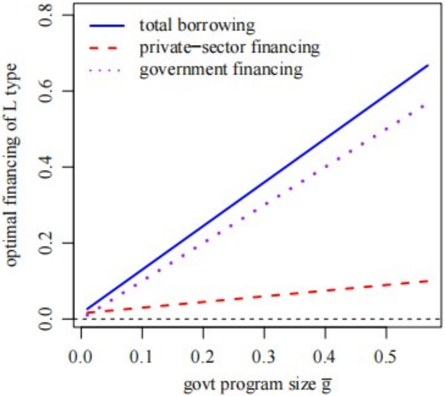


(a) Credit intervention and capital quantity (b) Credit intervention and capital quality

Optimal and Actual Policy

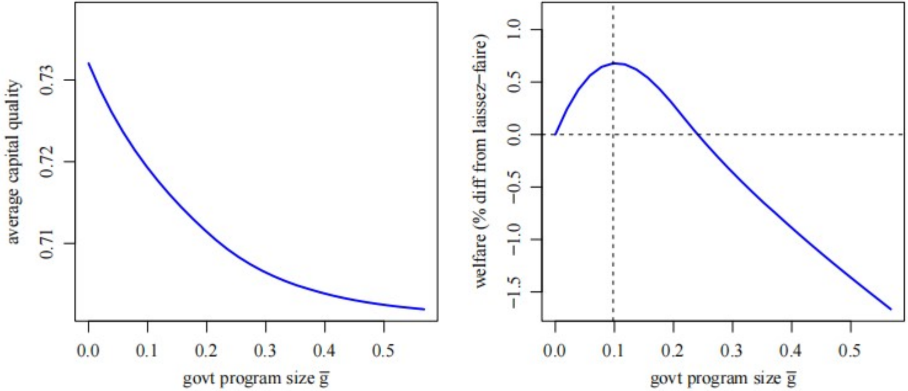


(a) Firm financing: $x_t^L(\zeta)$ and $x_t^H(\zeta)$



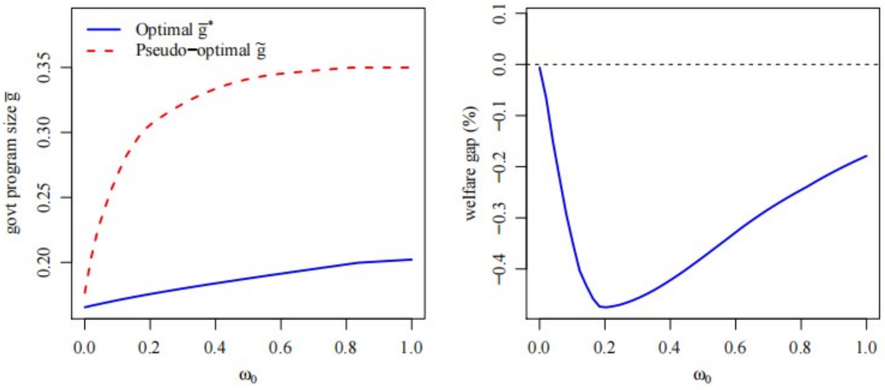
(b) Decomposing type-L firm financing

Welfare



(a) Average capital quality (b) Welfare improvement

Ignorance of Cleaning Effects



(a) Optimal government intervention

(b) Difference in optimal welfare

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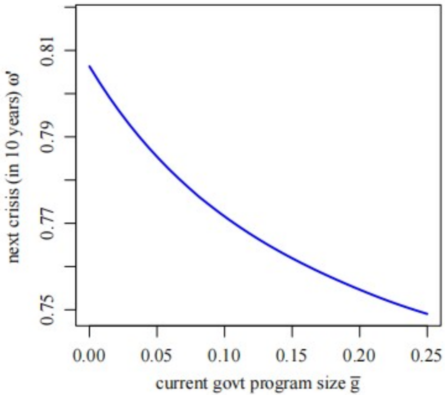
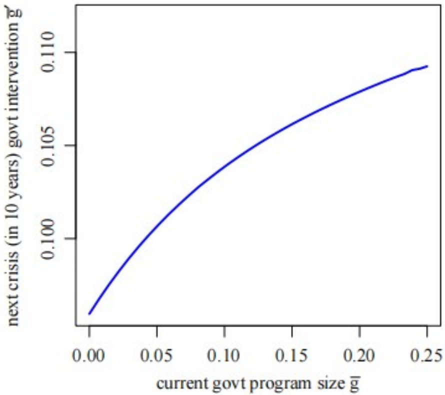
Next Crisis

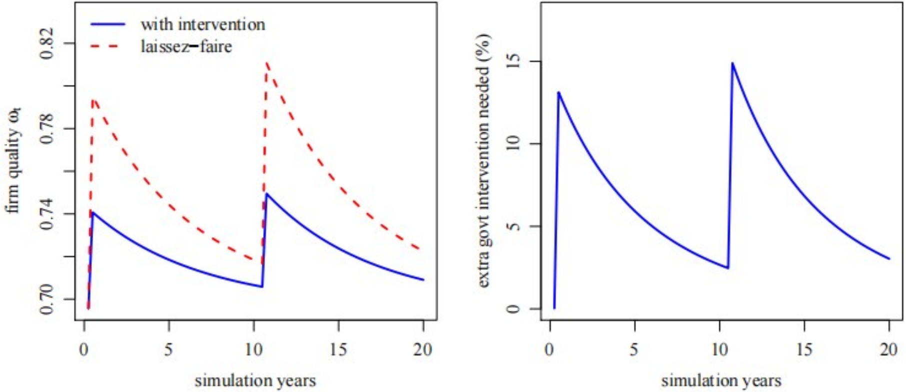


(a) Firm quality entering the next crisis



(b) Required intervention in the next crisis

Quality Dynamics



(a) Average capital quality (b) Additional intervention needed

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Conclusion

- Long-term consequences of credit intervention in crises.
- Cleansing effect of crises: High-productivity firms invest more in survival ($\uparrow$ capacity). Credit intervention weakens selection.
- Interventions reserves total capital K_t but lowers average productivity ω_t.
- Current intervention $\Rightarrow$ lower ω_t $\Rightarrow$ larger future interventions needed.
- Distortion feedback loop: Larger interventions $\rightarrow$ stronger distortions $\rightarrow$ demand for even larger interventions.
- Welfare implication: Optimal design improves outcomes vs. laissez-faire.