

CV Test, Part 2

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Is lambda related to the value of r in the original population?

While I previously found that the amount of smoothing needed to recover the value of r from dimensionless data was much higher than I had expected, now I want to find out how r , the population intrinsic growth rate, relates to λ^* , the optimal smoothing value. I have three main hypothesis:

1. The value of λ^* is constant for all r ,
2. The value of λ^* is proportional to r , or
3. The value of λ^* is chaotic with respect to the value of r .

Right now, I am leaning towards hypothesis 2: I think there will be a linear relationship between the value of r and the value of λ^* .

Eventually I will wrap this up into a function, but right now I just want to do the experiment, so I will be copy-pasting most of my code from the previous experiment and wrapping it inside another loop. First I will source the necessary scripts and set a random seed, as well as setup variables I need.

Note here that I added the functions from the previous test to the `Helpers.R` script, so I didn't have to redefine them here.

```
source(here::here("Scripts", "Dimensionless_exploration.R"))
```

```
## Loading required package: MASS
```

```
source(here::here("Scripts", "Helpers.R"))
```

```
source(here::here("Scripts", "Least_squares_methods.R"))
```

```
rs <- seq(0.001, 1, 0.001) # Values of  $r$  to test
```

```
lambdas <- seq(10, 10000, 10) # Values of lambda for each  $r$ 
```

```
k <- 3 #  $k$ -fold cross validation constant
```

```
lstar <- numeric(length(rs))
```

```
for (r in rs) {
```

```
  # Generate and partition data
```

```
  df <- generate_dimensionless_logistic_data(.1, r, 50, 0.1)
```

```
  parts <- random_partition(df, k)
```

```
  combos <- combn(1:k, k-1, simplify = F)
```

```
  # Empty vector to contain cross-validation error values
```

```
  cves <- numeric(length(lambdas))
```

```
  # Do all of this stuff for each lambda using the current data.
```

```
  for (a in 1:length(lambdas)) {
```

```
    l <- lambdas[[a]]
```

```
    errors <- numeric(length(parts))
```

```

# Need to do this k number of times (# of parts)
for (i in 1:k) {
  # Grab the first combo of indices to use
  to_use <- combos[[i]]
  # Make a blank dataframe
  train <- data.frame()
  # This part has to be done k-1 times to build training data.
  for (j in to_use) {
    # Get the next data frame.
    new_dat <- parts[[j]]
    # Add this data frame to the bottom of the rest.
    train <- rbind(train, new_dat)
  }

  # Now train has k-1 of the parts in it.
  # Set test to be the part that wasn't selected.
  not_used <- (1:k)[!(1:k %in% to_use)]
  test <- parts[[not_used]]
  # Prep and model the data, specify time step manually.
  prepped <- prep_data(train, 0.1)
  mod <- model_logistic_data_dimensionless_smoothing(prepped, 1)
  # Next need to calculate predicted values using the estimated growth rate from the model.
  #  $P = (P_0 e^{rt}) / (1 + P_0(1 - e^{rt}))$ 
  t <- test$t
  pred <- (0.1 * exp(mod*t)) / (1 + 0.1 * (exp(mod*t) - 1))
  errors[i] <- calculate_SSE(test$P, pred)
}

cves[a] <- mean(errors)
}
lstar[match(r,rs)] <- lambdas[which(cves == min(cves))]
}

```

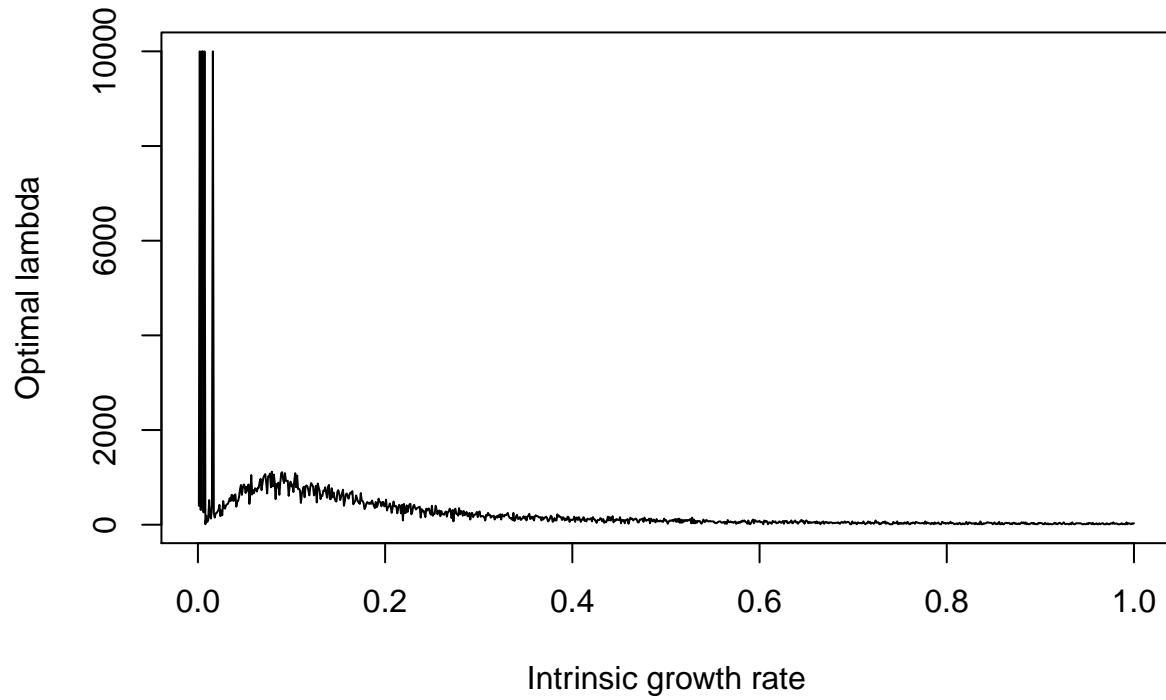
Now we need to collect the values of r and λ^* together and we can visualize the results.

```

res <- data.frame("r" = rs,
                  "l" = lstar)
with(res, plot(r,l, type = "l",
               xlab = "Intrinsic growth rate",
               ylab = "Optimal lambda",
               main = "Optimal lambda vs. R"))

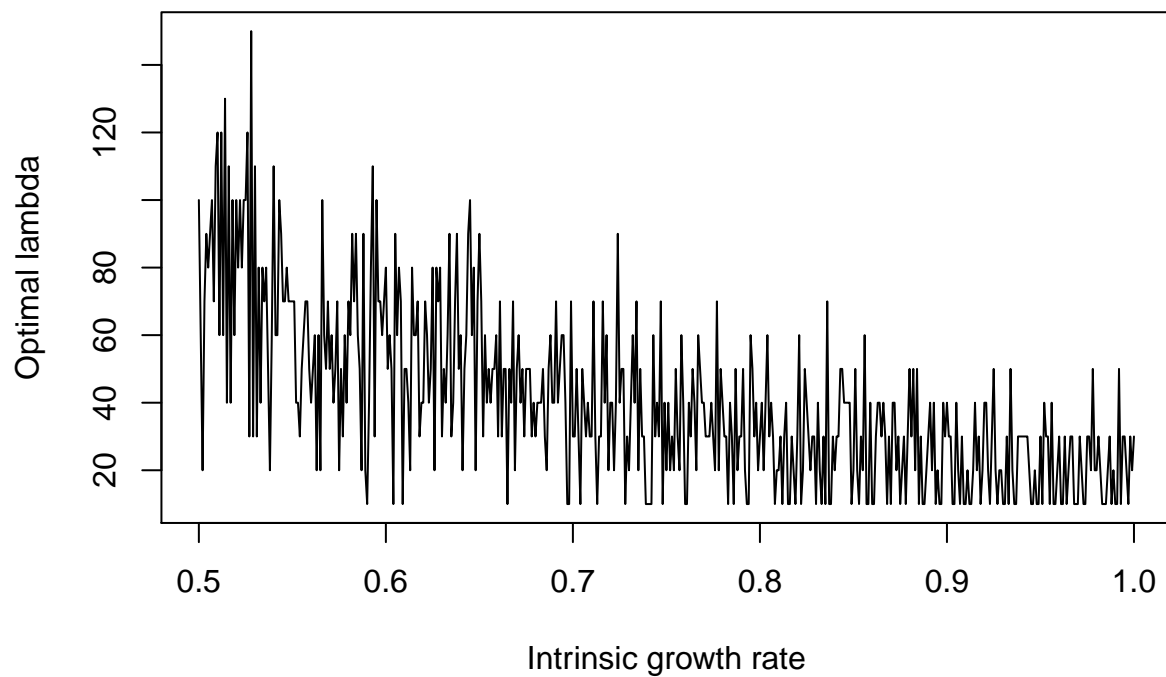
```

Optimal lambda vs. R

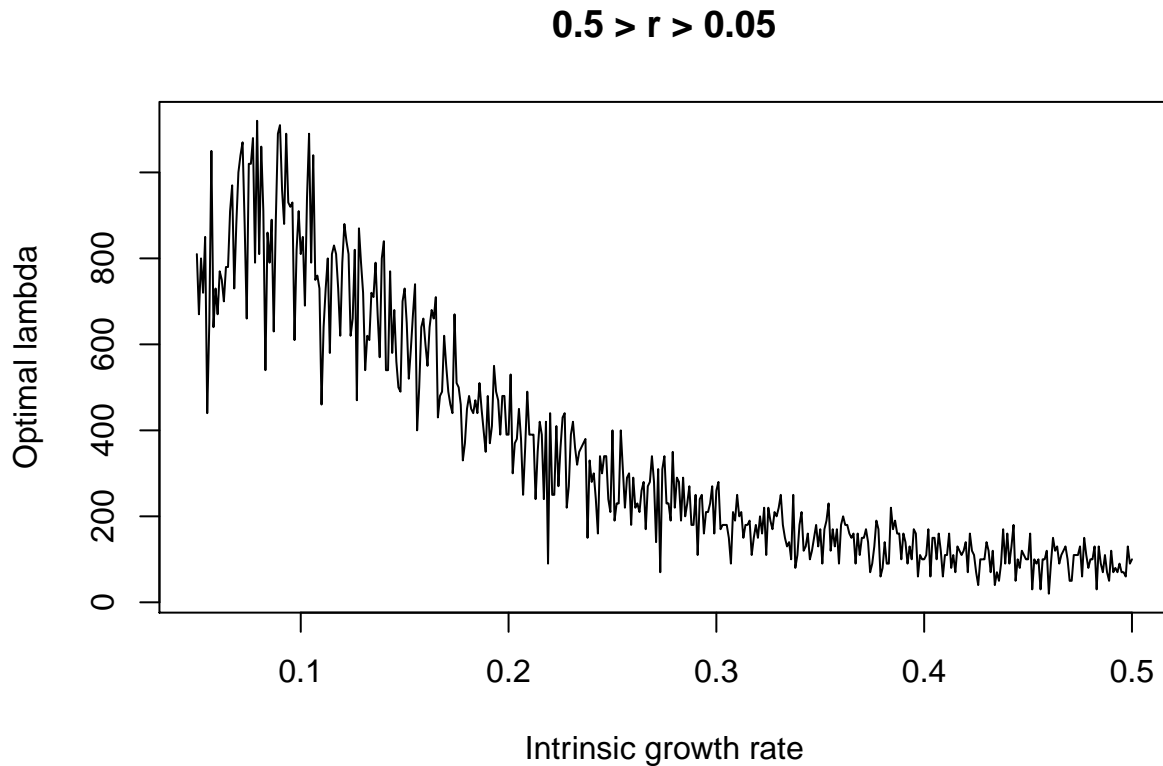


```
with(res[res$r >= 0.5 & res$r <= 1, ], plot(r,l, type = "l",  
      xlab = "Intrinsic growth rate",  
      ylab = "Optimal lambda",  
      main = "1 > r > 0.5"))
```

1 > r > 0.5

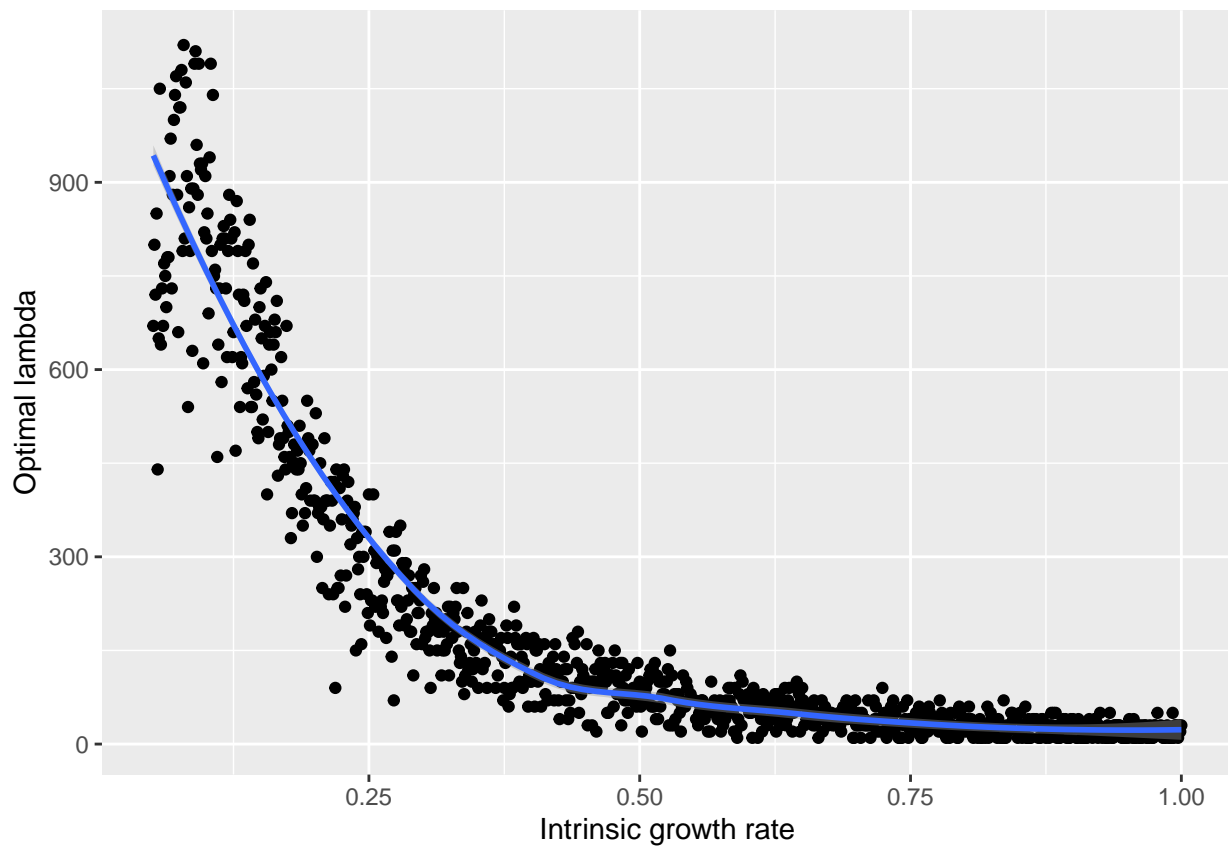


```
with(res[res$r <= 0.5 & res$r >= 0.05, ], plot(r,l, type = "l",
        xlab = "Intrinsic growth rate",
        ylab = "Optimal lambda",
        main = "0.5 > r > 0.05"))
```



This curve (starting at around $r = 0.05$ to exclude the major outliers) can probably be modeled well by an exponential decay curve. Here, I have just modeled it with a LOESS curve but an exponential fit might be worth examining.

```
library(tidyverse)
res %>%
  filter(res$r > 0.05) %>%
  ggplot(aes(x = r, y = l)) +
  geom_point() +
  labs(x = "Intrinsic growth rate", y = "Optimal lambda") +
  geom_smooth(method = "loess")
```



```
res %>%
  filter(res$r > 0.05) %>%
  ggplot(aes(x = r, y = l)) +
  geom_point() +
  scale_y_log10() +
  labs(x = "Intrinsic growth rate", y = "log10(optimal lambda)") +
  geom_smooth(method = "lm")
```

