MATH 430/530 Project 1 Spring 2019

Complete this project in your assigned group. Please upload your project to Blackboard as a LaTeX document, attaching the `.pdf` file, a working `.tex` file, and any necessary graphics files. These all may be submitted as a single compressed folder in `.zip`, `.tar`, or `.gz` format. Upload only **one copy** for the whole group. For *this* project, you may use MS Word with equation editor if you are unfamiliar with LaTeX. Please submit the document, a `.pdf` file, and all graphics.

Showing your work: For computations please provide a summary of the output, either in the text of the report or as an appendix. You may elect to include a copy of the code as an appendix (with liberal comments). You may also submit *Mathematica* `.nb` files or screenshots of direction fields as supporting evidence, but all relevant results must appear in your main document itself. Paper and pencil results should be documented appropriately. Graphics should be embedded in the file either in the text or, if there are many figures, in the appendix.

Rubric: (See Blackboard.) There are a total of 24 points. The categories are

- Clarity and style (4 pts.)
- Identification of a problem and setup of the model (including identifying variables and/or submodels) (4 pts)
- Solving and verifying the model, and analyzing its strengths and weaknesses (8 pts)
- Mathematical accuracy (8 pts),

WCU Academic Integrity Policy: You are authorized to use any print or electronic resources available to you if they are cited appropriately. The only person outside of your group with whom you may discuss your project is the instructor. See the course syllabus for more detail.

Integrated pest management

In sustainable agriculture, the practice of Integrated Pest Management (IPM) involves the introduction of a predator insect to control the population of a pest that threatens a crop. For example, aphids are a problem for organic apple growers, so an IPM method is to introduce ladybugs to prey on the aphids. The farmers measure the predator and prey populations weekly and report this to the crop scientists. The crop scientists in turn provide you, the mathematician, with a model in the form of a system of Lotka-Volterra differential equations,

$$\begin{aligned} \frac{dA}{dt} &= rA - \alpha AL \\ \frac{dL}{dt} &= -cL + \beta AL \end{aligned} \tag{1}$$

where time t is the number of weeks since the release of the ladybugs, $A(t)$ is the aphid population at time t measured in millions, and $L(t)$ is the ladybug population at time t in tens of thousands. All parameters are constant and positive. The growing season is roughly half a year.

Your report should address all of the following.

1. Interpret each term of each equation including the sign of the term.
2. Determine the equilibria of this system symbolically.
3. Let $r = 0.21$, $\alpha = 0.13$, $c = 0.10$, and $\beta = 0.08$. Analyze the stability of all equilibria using both graphical techniques, such as nullclines and phase plane plots, and linearization techniques.
4. What will happen if you add a carrying capacity term so that the aphids follow a logistic growth model? Refer to equation (3.47) from Beltrami Exercise 3.5 (pp. 52-53). You will need to make a reasonable choice of carrying capacity. Does it introduce new equilibria? Does it change the stability of the equilibria?
5. Write a recommendation for the introduction of ladybugs to manage the aphid population. Your recommendation must contain arguments that will convince the crop scientist, but also can be understood by the farmer. Remember that the goal of IPM is not necessarily eradication of the pest but rather management of the pest. (It is acceptable to organize your recommendation in two parts, a technical report and a nontechnical explanation.)
6. Return to the original system in (1). Suppose that in addition to IPM the farmer applies a pesticide that kills the same fraction f of each species of insect. (Do not apply the pesticide at a kill rate that exceeds either r or c.) Could a well-intended application of a pesticide be counterproductive?
7. Write a recommendation regarding the use of IPM and pesticides in tandem. Again, you must convince the crop scientists and the farmers.