Triphos: Isometric Coordinates and the Pythagorean Theorem

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Outline

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Motivation

- What if we used three axes to plot two-dimensional numbers?
- Additive coloring models work similarly: e.g., the RGB model of light.

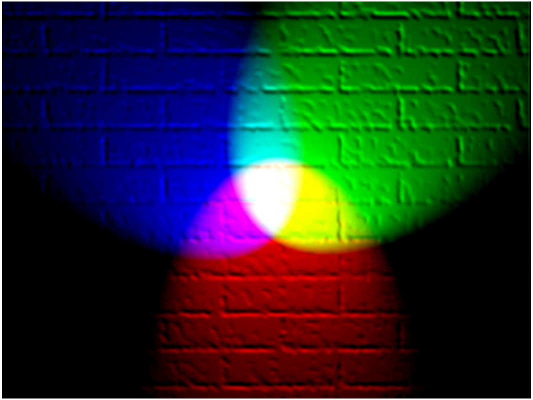

Figure: The RGB model of light is an additive coloring model which inspired the discovery of Triphos. [1] Source: https://en.wikipedia.org/wiki/File:RGB_illumination.jpg

Geometric Interpretation

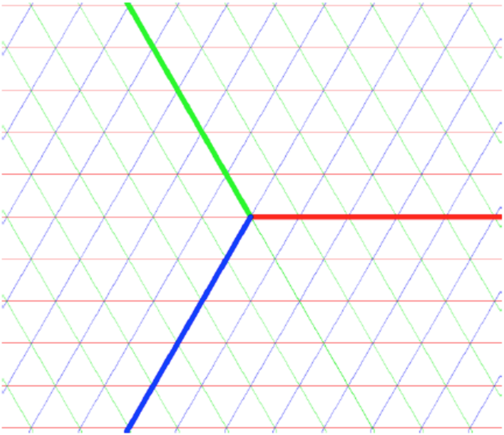

Figure: Triphos has three axes, equally spaced 120 degrees apart. We call these axes the red, green, and blue axes, due the motivation behind Triphos. Figure from [2].

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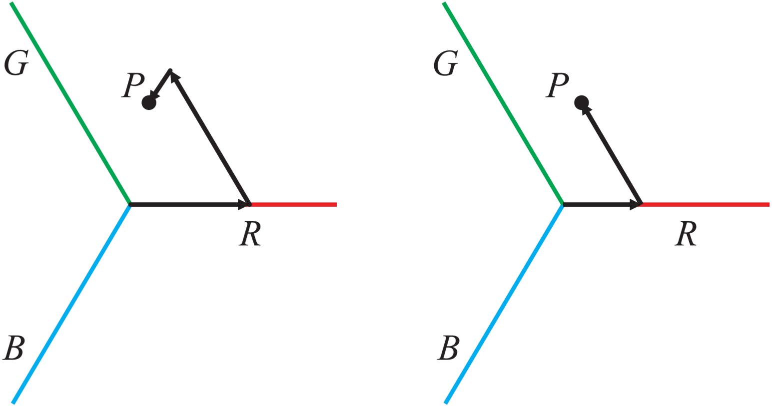

Figure: Each point in Triphosian coordinates is not uniquely determined by an ordered pair, unlike Cartesian coordinates. Left: $P = (3, 4, 1)$, Right: $P = (2, 3, 0)$. Figure from [1].

Formal Definition

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$$(r_2, g_2, b_2) = (r_1 + c, g_1 + c, b_1 + c).$$

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- The Triphosian reals, denoted $\mathbb{R}_T$, refers to the set of all such equivalence classes.
- All triples in the same equivalence class represent the same geometric point in Triphosian coordinates.

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- Every equivalence class of a Triphosian point has infinitely many members. We designate the standard form triple to represent all members of each equivalence class.

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Let $T = [(-2, 1, 2)]$. We convert to standard form by calculating $T^* = [(-2+2, 1+2, 2+2)] = [(0, 3, 4)]$.

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- If two Triphosian numbers are in standard form, they can only be equivalent if all three coordinates match. [2]

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Example

- $\mathbb{Z}$ or $\mathbb{R}$ with addition.
- $\mathbb{R} - \{0\}$ with multiplication.
- $\{0, 1\}$ with multiplication modulo 2.

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Example

- $\mathbb{Q}, \mathbb{R},$ and $\mathbb{C}$ with addition and multiplication.
- $\{0, 1\}$ with addition and multiplication both modulo 2.

Outline

Triphosian Addition

Addition definition

$$[(r_1, g_1, b_1)] + [(r_2, g_2, b_2)] = [(r_1 + r_2, g_1 + g_2, b_1 + b_2)]$$

Properties [2]

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- Well-defined on the Triphosian reals.

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- The Triphosian reals form an Abelian group under addition (closed, associative, and commutative).
- Additive Identity: $[(0, 0, 0)]$
- Additive Inverse: $[(-r, -g, -b)] = [(g + b, r + b, r + g)]$ [1]

Geometry of Addition

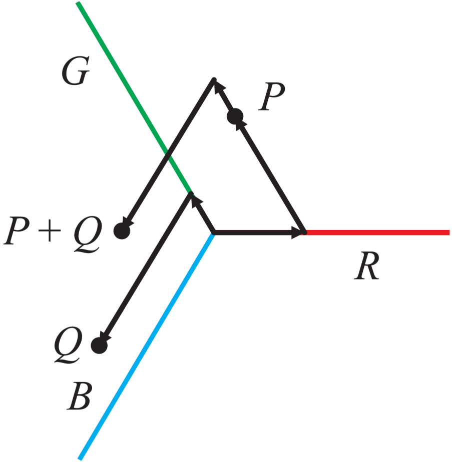

Figure: Adding in Triphos corresponds to Euclidean vector addition. The above figure from [1] shows the sum of $P = (2, 3, 0)$ and $Q = (0, 1, 4$, which gives $P + Q = (2, 4, 4)$.

Triphosian Multiplication

Multiplication definition

$$[(r_1, g_1, b_1)] \cdot [(r_2, g_2, b_2)] =$$

$$[(r_1 r_2 + g_1 b_2 + b_1 g_2, r_1 g_2 + g_1 r_2 + b_1 b_2, r_1 b_2 + g_1 g_2 + b_1 r_2)]$$

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- Multiplicative inverse: $1/n \cdot [(r, g, b)]$ where $n = r^2 + g^2 + b^2 - rb - rg - gb$ and $[(r, g, b)] \neq [(0, 0, 0)]$.

Geometry of Multiplication

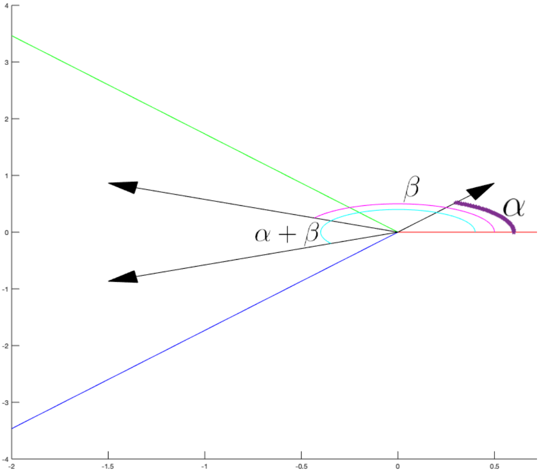

Figure: Triphosian multiplication looks messy, but geometrically, *angles add and magnitudes multiply*.

The Triphosian reals form a field

Distributive Property [2]

Given three Triphosian numbers, $[(r_1, g_1, b_1)], [(r_2, g_2, b_2)]$ and $[(r_3, g_3, b_3)]$, the distributive property holds, i.e.

$$[(r_1, g_1, b_1)] \cdot ([(r_2, g_2, b_2)] + [(r_3, g_3, b_3)]) = \\ [(r_1, g_1, b_1)] \cdot [(r_2, g_2, b_2)] + [(r_1, g_1, b_1)] \cdot [(r_3, g_3, b_3)].$$

Triphos is a field

Since $\mathbb{R}_T$ is an Abelian group under addition, $\mathbb{R}_T - \{[(0,0,0)]\}$ is an Abelian group under multiplication, and addition distributes over multiplication, $\mathbb{R}_T$ is a field.

Outline

The Hexa-Metric Function

Definition: The Hexa-Metric function [2]

Let $T_1 = [(r_1, g_1, b_1)]$, $T_2 = [(r_2, g_2, b_2)]$ be Triphosian real numbers. The Hexa-Metric function, H, is defined as

$$\begin{aligned} H(T_1, T_2) = \min \{ &|(r_1 - b_1) - (r_2 - b_2)| + |(g_1 - b_1) - (g_2 - b_2)|, \\ &|(r_1 - g_1) - (r_2 - g_2)| + |(b_1 - g_1) - (b_2 - g_2)|, \\ &|(g_1 - r_1) - (g_2 - r_2)| + |(b_1 - r_1) - (b_2 - r_2)| \}. \end{aligned}$$

Hexa-Metric Properties

- Distance is non-negative: $H(T_1, T_2) \geq 0$ for all $T_1, T_2 \in \mathbb{R}_T$.
- The distance between a point and itself is zero, and no two distinct points have distance zero: $H(T_1, T_2) = 0 \iff T_1 = T_2$.
- Distance is symmetric: $H(T_1, T_2) = H(T_2, T_1)$.
- The Triangle Inequality holds: $H(T_1, T_2) \leq H(T_1, T_3) + H(T_3, T_2)$.

The Hexa-Metric Function

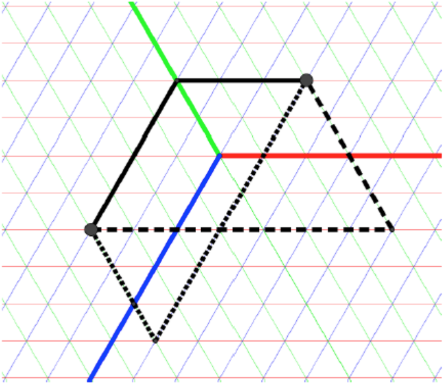

Figure: The Hexa-Metric only allows us to use paths parallel to the axes. Interestingly, there is no unique geodesic like we are used to in Euclidean space. In this figure from [2], all three paths result in a hexa-metric distance of 7 between the two points shown..

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The Triphosian Unit Circle and Pi

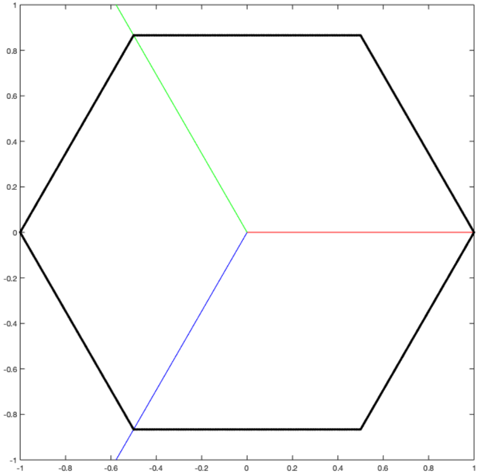

Figure: A plot of the Triphosian unit circle, generated with `MATLAB`.

The Triphosian Unit Circle and Pi

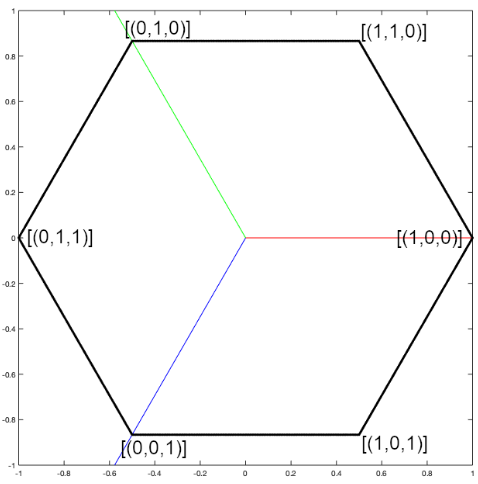

Figure: These points (in Triphosian coordinates) are the vertices of the unit circle.

The Triphosian Unit Circle and Pi

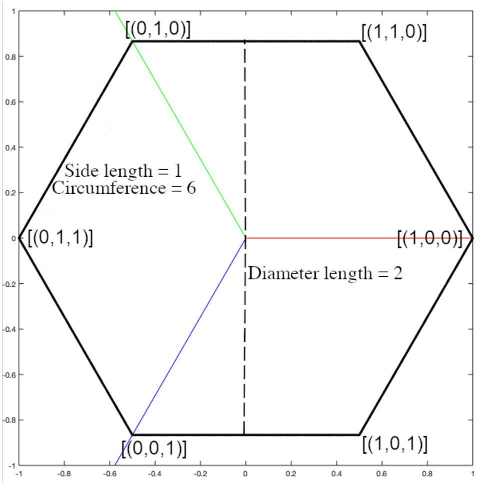

Figure: We compute the diameter of the unit circle, as well as the length of each side. Taking the ratio of the circumference to the diameter, we get $\pi_T = 3$.

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The Euclidean Pythagorean Theorem

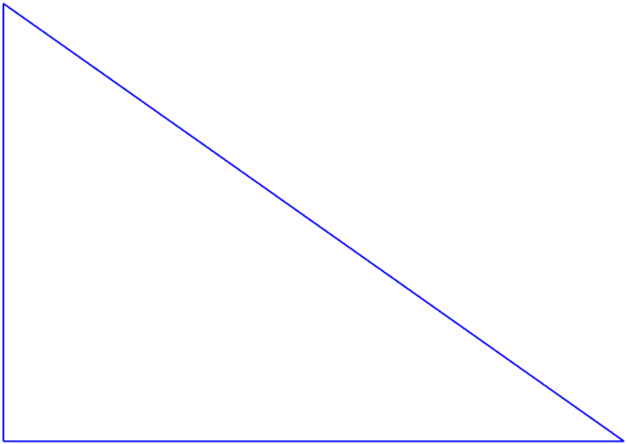

Figure: Suppose we have an arbitrary right triangle. I have rotated it this way for convenience, but any right triangle can be rotated to this orientation.

The Euclidean Pythagorean Theorem

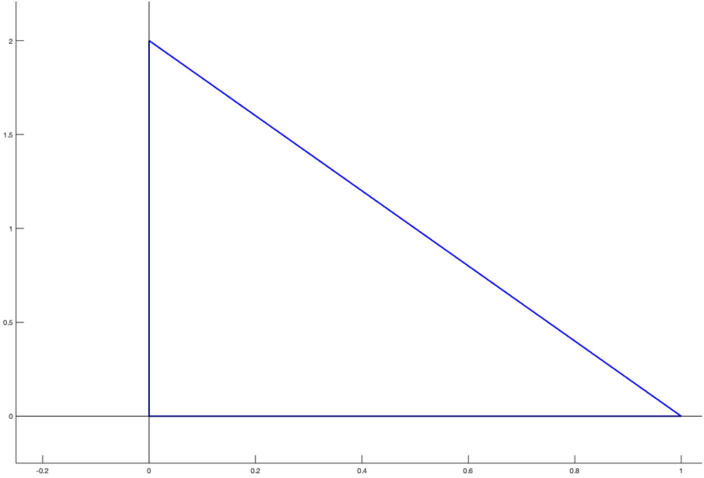

Figure: If we have any triangle with a right angle, we can impose the Euclidean axes over the right angle. We place the origin at the vertex of the right angle, so we can use the distance function to get the length of two legs.

The Euclidean Pythagorean Theorem

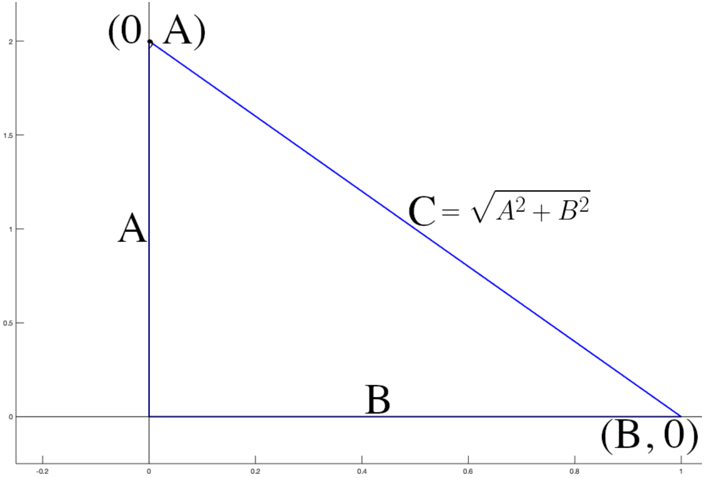

Figure: Then, we get the distance of the hypotenuse by taking the distance between the ends of both legs.

Deriving a Pythagoren Theorem in Triphos

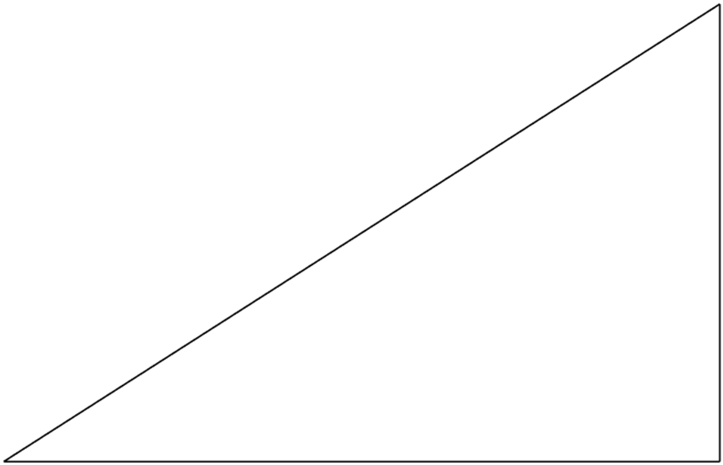

Figure: Suppose that we have an arbitrary triangle where one of the angles is 60°.

Deriving a Pythagoren Theorem in Triphos

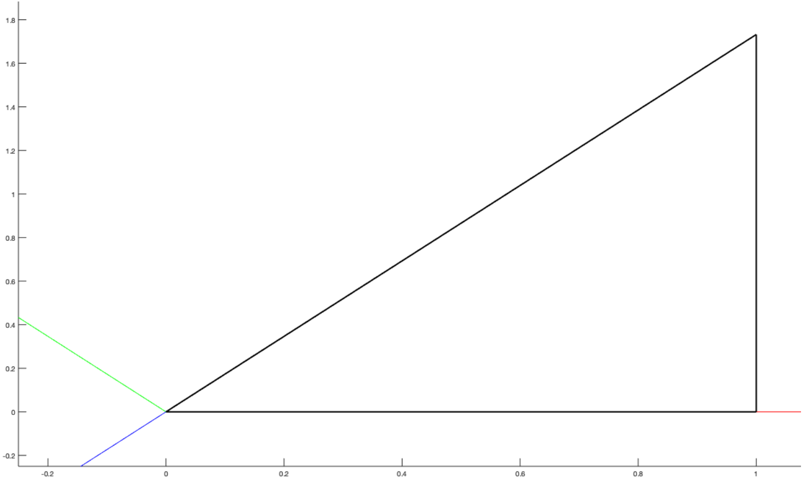

Figure: Due to the spacing of the three Triphosian axes, we can place the origin at the vertex of the 60° angle and impose Triphosian coordinates. We then know how long the two legs are.

Deriving a Pythagoren Theorem in Triphos

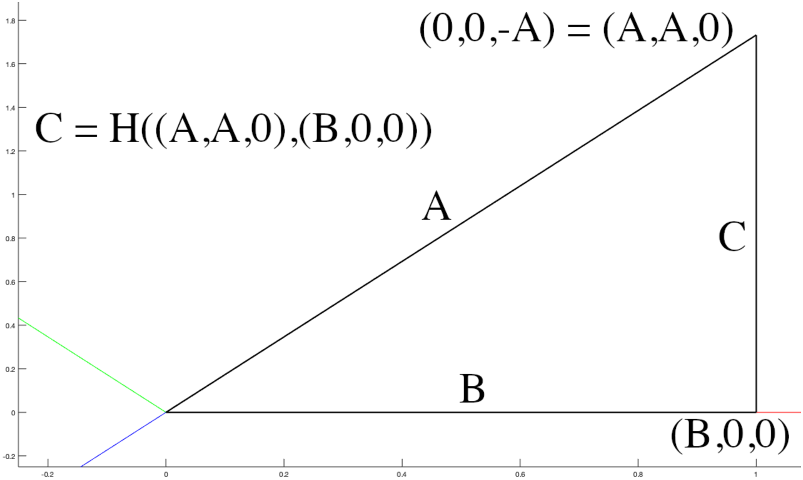

Figure: So, we know the coordinates of the points defining the “hypotenuse”, and we can evaluate the length of the other side.

Triphosian Pythagorean Theorem

At this point, I enlisted Mathematica to assist in computations.

```
HM[r1_, g1_, b1_, r2_, g2_, b2_] :=
 Min[Abs[(r1 - b1) - (r2 - b2)] + Abs[(g1 - b1) - (g2 - b2)], Abs[(r1 - g1) - (r2 - g2)] + Abs[(b1 - g1) - (b2 - g2)],
  Abs[(g1 - r1) - (g2 - r2)] + Abs[(b1 - r1) - (b2 - r2)]]

d1 = HM[r1, g1, b1, 0, 0, 0]
d2 = HM[r2, g2, b2, 0, 0, 0]

Min[Abs[b1 - r1] + Abs[g1 - r1], Abs[-b1 + g1] + Abs[-b1 + r1], Abs[b1 - g1] + Abs[-g1 + r1]]

Min[Abs[b2 - r2] + Abs[g2 - r2], Abs[-b2 + g2] + Abs[-b2 + r2], Abs[b2 - g2] + Abs[-g2 + r2]]

PT = HM[d1, 0, 0, d2, d2, 0]

Min[Abs[Min[Abs[b1 - r1] + Abs[g1 - r1], Abs[-b1 + g1] + Abs[-b1 + r1], Abs[b1 - g1] + Abs[-g1 + r1]]] +
  Abs[Min[Abs[b2 - r2] + Abs[g2 - r2], Abs[-b2 + g2] + Abs[-b2 + r2], Abs[b2 - g2] + Abs[-g2 + r2]]],
 Abs[Min[Abs[b1 - r1] + Abs[g1 - r1], Abs[-b1 + g1] + Abs[-b1 + r1], Abs[b1 - g1] + Abs[-g1 + r1]] -
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 Abs[Min[Abs[b1 - r1] + Abs[g1 - r1], Abs[-b1 + g1] + Abs[-b1 + r1], Abs[b1 - g1] + Abs[-g1 + r1]]] +
  Abs[-Min[Abs[b1 - r1] + Abs[g1 - r1], Abs[-b1 + g1] + Abs[-b1 + r1], Abs[b1 - g1] + Abs[-g1 + r1]] +
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```

Triphosian Pythagorean Theorem

By simplification, we obtain an equation which is (slightly) less messy.

```
SPT = FullSimplify[PT]

Min[Abs[Min[Abs[b1 - g1] + Abs[b1 - r1], Abs[b1 - g1] + Abs[g1 - r1], Abs[b1 - r1] + Abs[g1 - r1]] -
    Min[Abs[b2 - g2] + Abs[b2 - r2], Abs[b2 - g2] + Abs[g2 - r2], Abs[b2 - r2] + Abs[g2 - r2]]] +
  Min[Abs[b1 - g1] + Abs[b1 - r1], Abs[b1 - g1] + Abs[g1 - r1], Abs[b1 - r1] + Abs[g1 - r1]],
 Abs[Min[Abs[b1 - g1] + Abs[b1 - r1], Abs[b1 - g1] + Abs[g1 - r1], Abs[b1 - r1] + Abs[g1 - r1]] -
    Min[Abs[b2 - g2] + Abs[b2 - r2], Abs[b2 - g2] + Abs[g2 - r2], Abs[b2 - r2] + Abs[g2 - r2]]] +
  Min[Abs[b2 - g2] + Abs[b2 - r2], Abs[b2 - g2] + Abs[g2 - r2], Abs[b2 - r2] + Abs[g2 - r2]],
 Min[Abs[b1 - g1] + Abs[b1 - r1], Abs[b1 - g1] + Abs[g1 - r1], Abs[b1 - r1] + Abs[g1 - r1]] +
  Min[Abs[b2 - g2] + Abs[b2 - r2], Abs[b2 - g2] + Abs[g2 - r2], Abs[b2 - r2] + Abs[g2 - r2]]]
```

So, given a triangle with a 60° angle, we can rotate it, impose coordinates, and use this formula to find the length of the third side.

Outline

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- Dr. Penland and Dr. Lawson have also posed questions to me regarding the topology and curvature of Triphos respectively.
- The authors of [2] and [1] propose interesting open problems regarding calculus, fractals, and functions in Triphos.

Conclusions

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- Finally, I derived a notion of a "Pythagorean theorem" in Triphos.
- There are a lot of open questions remaining about Triphos!

References

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