

Modeling Module 1

August 14, 2020

Outline

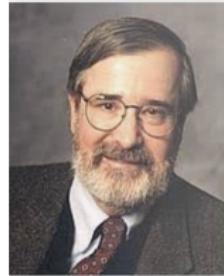
- ▶ What's a Model?
- ▶ State-Transition Systems (TS)
- ▶ Abstraction and Non-determinism in Modeling
- ▶ Modeling Examples
- ▶ Kripke Structures, Behaviors of TS

Model Checking: Cited for Turing Award in 2007 (Invented: 1986)



ACM Turing Award Hon...
cs.utexas.edu

E. Allen Emerson



Introduction to Model C...
moves.rwth-aachen.de

Edmund Clarke



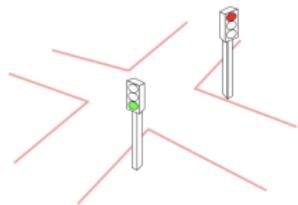
[Joseph Sifakis | French com...](#)
britannica.com

Joseph Sifakis

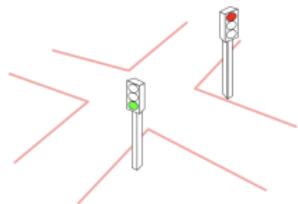
CITATION

Together with E. Allen Emerson and Joseph Sifakis, for their role in developing Model-Checking into a highly effective verification technology that is widely adopted in the hardware and software industries.

What Kind of Systems and Properties?

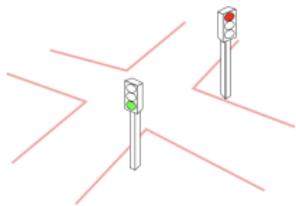


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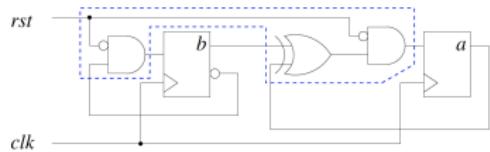


"traffic lights should never be green at the same time"

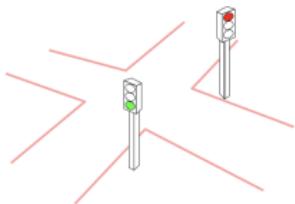
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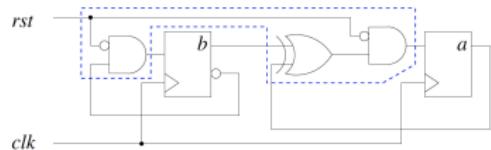
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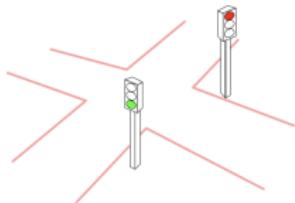


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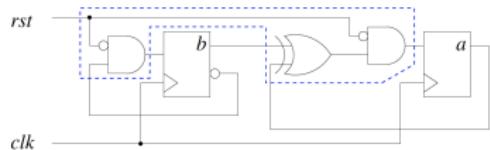


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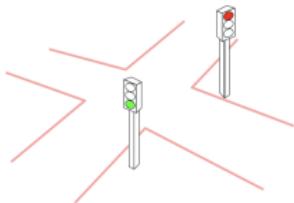
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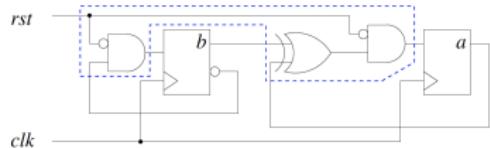
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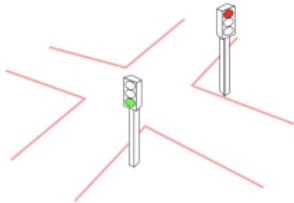
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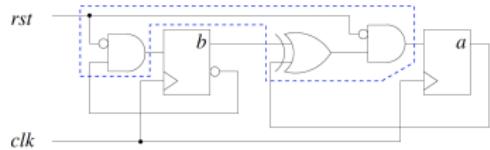


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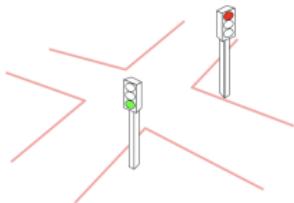
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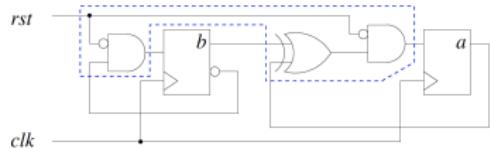
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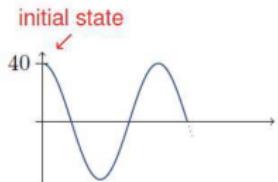
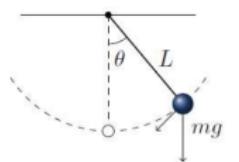
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Thread 1	Thread 2	Thread 3
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"the program has no data races"

What Kind of Systems Not

An Example



Our model:

$$\frac{\delta^2 \theta}{\delta t^2} = -\frac{g}{L} \sin \theta$$

Plot of θ over time

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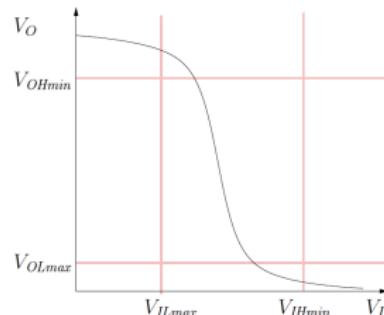
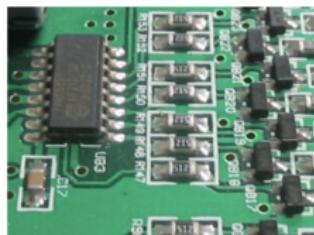
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- ▶ Need a common formalism to model all systems of interest:
Transition Systems (M)
- ▶ Need an expressive logic/language to specify desired properties
“Special-purpose Temporal Logics ($Prop$)”
- ▶ Need efficient algorithms to automatically check properties
 $M \models Prop$

A Hardware Abstraction Example

We always omit some detail.

Example:



x	$\neg x$
0	1
1	0

Abstraction

Modeling: States

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Definition Transition System

Definition (Transition System)

A *Transition System* is a 3-tuple $M = (S, S_0, T)$ consisting of

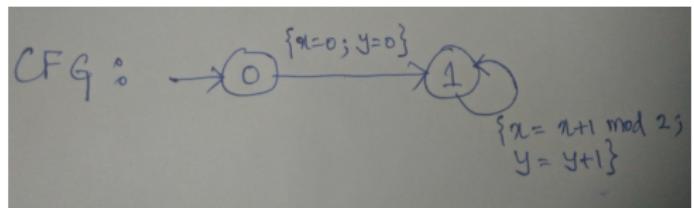
- ▶ a set of states S ,
- ▶ a set of initial states $S_0 \subseteq S$
- ▶ a transition relation $T \subseteq (S \times S)$

Modeling Sequential Code: An Example

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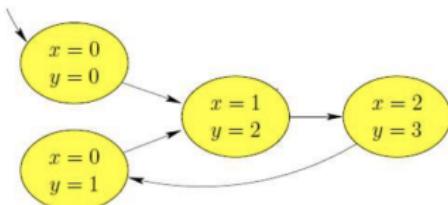
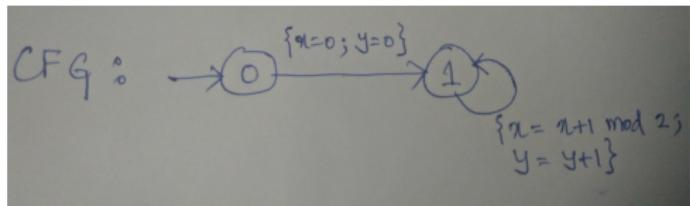
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TS : $M = (S, S_0, R)$

- ▶ $S = (D_1 \times D_2)$, where $D_1 = \{0, 1, 2\}$, $D_2 = \{0, 1, 2, 3\}$
- ▶ $S_0 = \{(0, 0)\}$
- ▶ $R = \{(0, 0), (1, 2)), ((0, 1), (1, 2)), ((1, 2), (2, 3)), ((2, 3), (0, 1))\}$

Towards Symbolic Representation for TS: Notation for States

- ▶ S - set of states
- ▶ $s \in S$ - one particular state
- ▶ s, x - the value of some variable x in state s .
These are called state variables

Towards Symbolic Representation for TS: The Transition Relation

The **transition relation** T relates a pre-state to a post-state:

$$T \subseteq S \times S$$

A diagram illustrating the transition relation T . At the top center is the expression $T \subseteq S \times S$, where T is written in red. Two arrows point downwards from this expression to the words "pre-state" and "post-state".

The transition relation captures how the state can be transformed by a **single transition**.

Towards Symbolic Representation for TS: Characteristic Functions

Notation: We typically write the transition relation using a characteristic function.

This is useful for any kind of set:

$$S = \{0, 2, 4, 6, 8, \dots\} \quad x \in S \iff x \bmod 2 = 0$$

$$S = \{1, 2, \dots, 9, 10\} \quad x \in S \iff 1 \leq x \wedge x \leq 10$$

$$S = \{1, 2, 4, 8, 16, 32, \dots\} \quad x \in S \iff \exists y. x = 2^y$$

Towards Symbolic Representation for TS: The Transition Relation Again

Example:

$$T \subseteq \mathbb{N}_0 \times \mathbb{N}_0$$

$$T(x, x') \iff x' = x + 1$$

This corresponds to the set

$$\{(0, 1), (1, 2), (2, 3), \dots\}$$

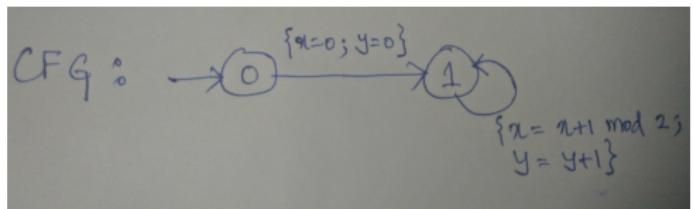
By convention, **primed variables (x')** denote the value of a state variable in the next state.

Symbolic TS for the Code Example

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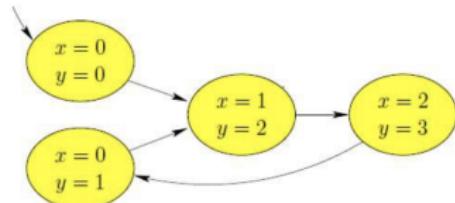
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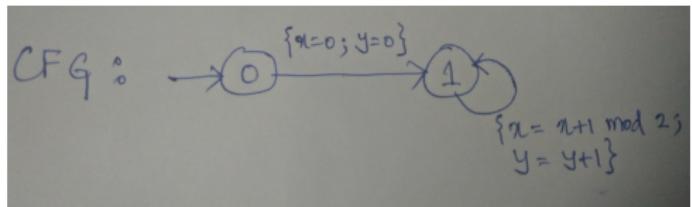
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TS : $M = (S, S_0, R)$

- ▶ $S = (D_1 \times D_2)$, where $D_1 = \{0, 1, 2\}$, $D_2 = \{0, 1, 2, 3\}$
- ▶ $S_0(s) : (s.x = 0 \wedge s.y = 0)$
- ▶ $R(s, s') : (s'.x = s.x + 1 \text{ mod } 3) \wedge (s'.y = s'.x + 1)$



Role of Nondeterminism in Modeling

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 - ▶ Scheduler nondeterminism (concurrent systems): Multiple successor states possible for a transition

Initial State Nondeterminism

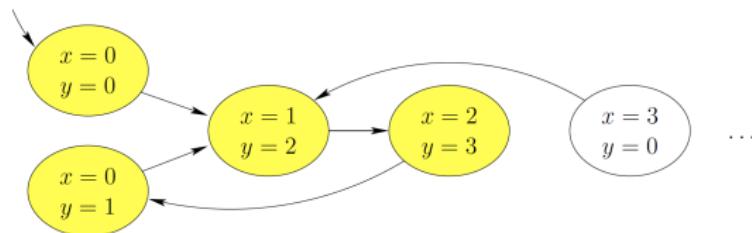
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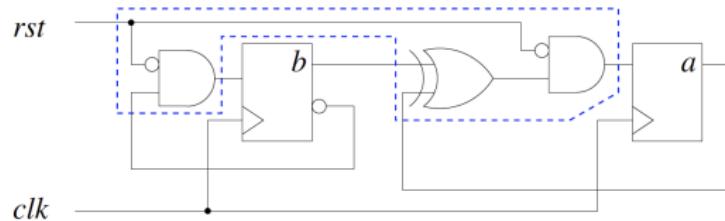
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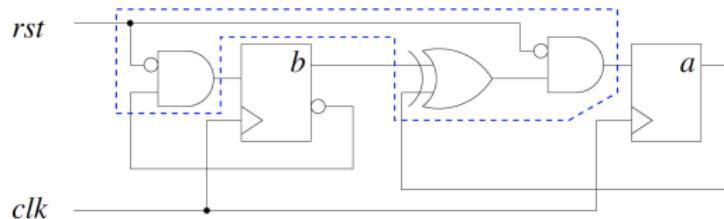
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Modeling Clocked Hardware: Modulo 4 Counter



- ▶ One global clock: *clk*
- ▶ One input: *rst*
- ▶ Next-state logic (dashed box):
a, b (flip-flops) change *synchronously* with every tick of *clk*
 - ▶ $\delta(a, b, rst).a \equiv \overline{rst} \cdot (a \oplus b)$
 - ▶ $\delta(a, b, rst).b \equiv \overline{rst} \cdot b$

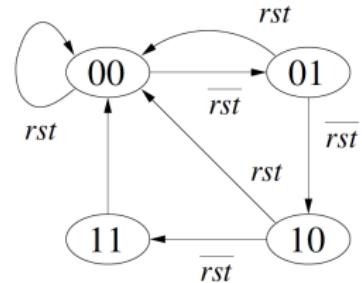
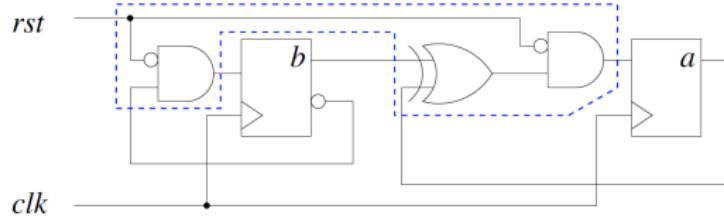
Modeling Inputs in TS



$$(\delta(s.a, s.b, s.rst) = s'.ab) \vee$$

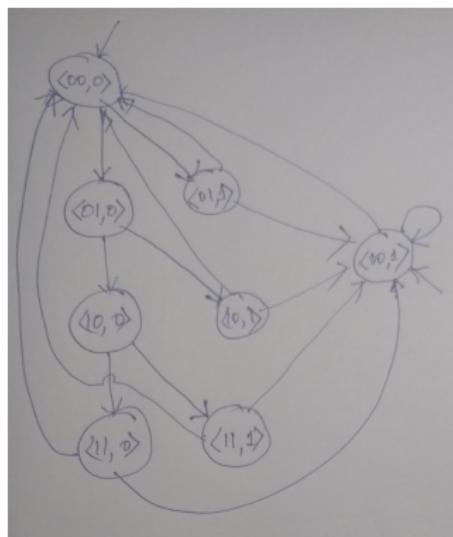
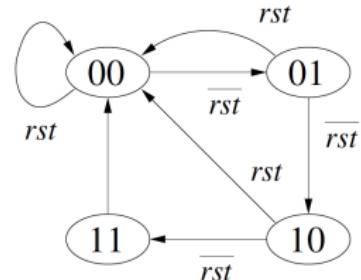
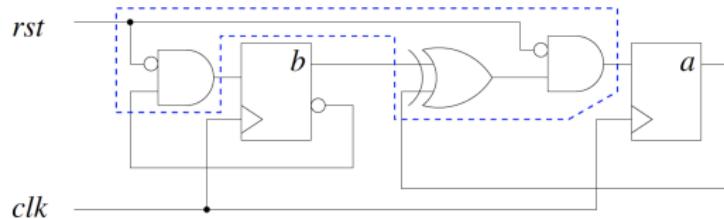
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Assumptions on Transition Systems

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1. Domains of state variables are finite and discrete
2. Transition relation is total (No loss of generality)
 - ▶ How about terminating systems, e.g., code?
 - ▶ If $T(s, s') \equiv \text{false}$, then redefine it as $T'(s, s') \equiv (s = s')$

Terminating TS

```
Void main () {
```

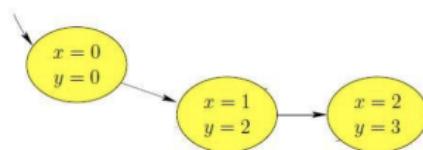
```
    int x = 0, y = 0;
```

```
    while (y<=2) {
```

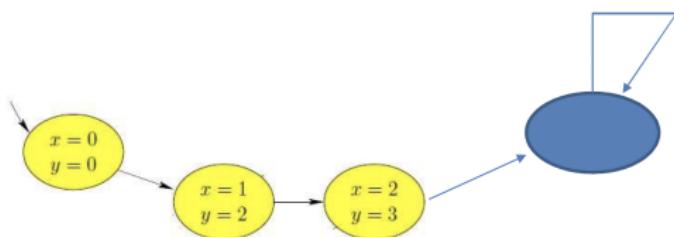
```
        x = x+1 mod 3;
```

```
        y = x+1; }
```

```
}
```



Without loss of generality we make
all our TS non-terminating

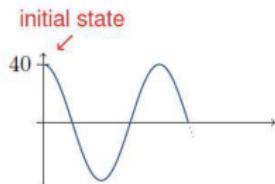
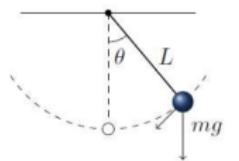


This means our TS relation is assumed to be TOTAL



An Continuous Domain Example

An Example



Our model:

$$\frac{\delta^2 \theta}{\delta t^2} = -\frac{g}{L} \sin \theta$$

Plot of θ over time