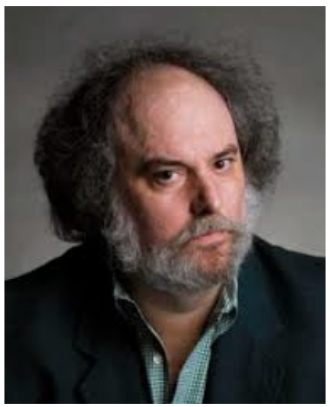
Modeling Module 1

August 13, 2020

Outline

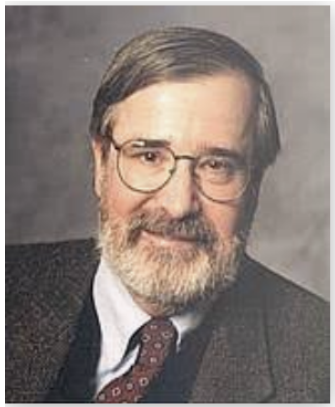
- What's a Model?
- State-Transition Systems (TS)
- Abstraction and Non-determinism in Modeling
- Modeling Examples
- Kripke Structures, Behaviors of TS

Model Checking: Cited for Turing Award in 2007 (Invented: 1986)



ACM Turing Award Hon...
cs.utexas.edu

E. Allen Emerson



Introduction to Model C...
moves.rwth-aachen.de

Edmund Clarke

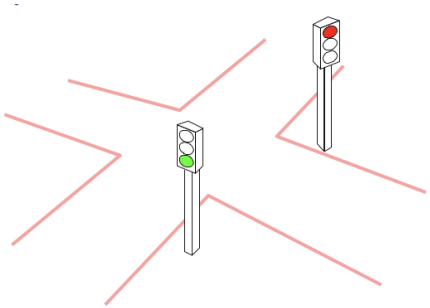
Joseph Sifakis | French com...
britannica.com

Joseph Sifakis

CITATION

Together with E. Allen Emerson and Joseph Sifakis, for their role in developing Model-Checking into a highly effective verification technology that is widely adopted in the hardware and software industries.

What Kind of Systems and Properties?



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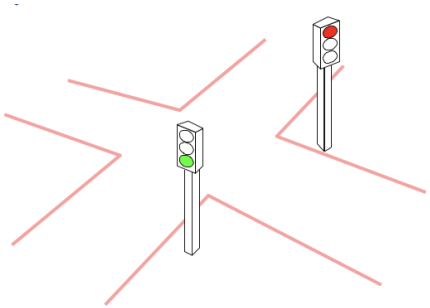

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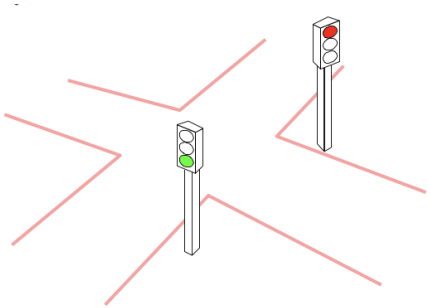

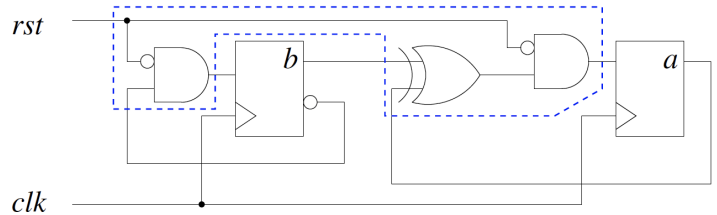

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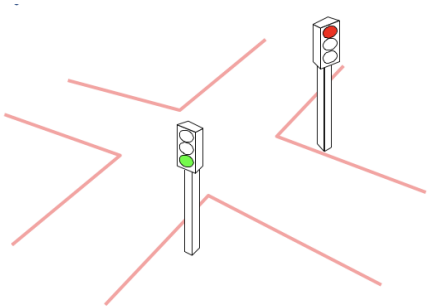

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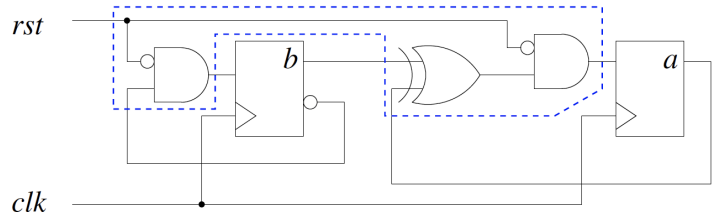

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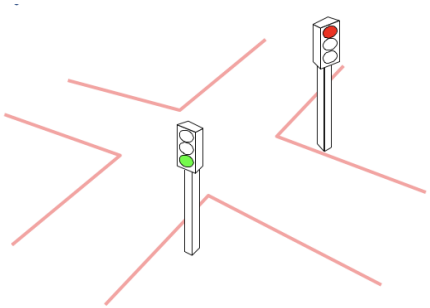

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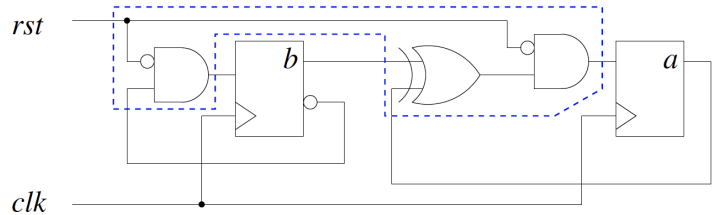

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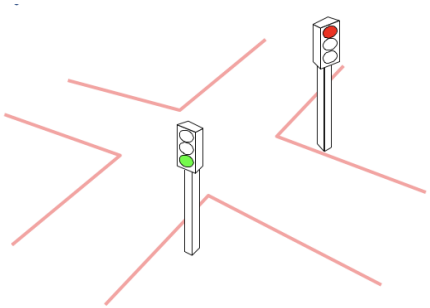

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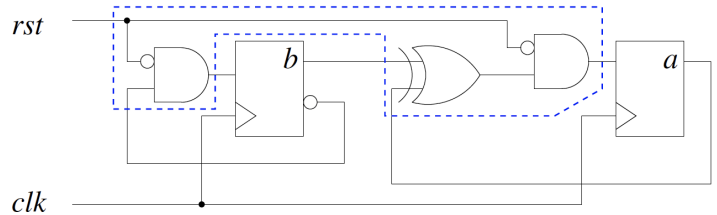

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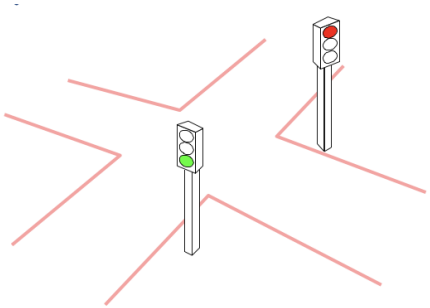

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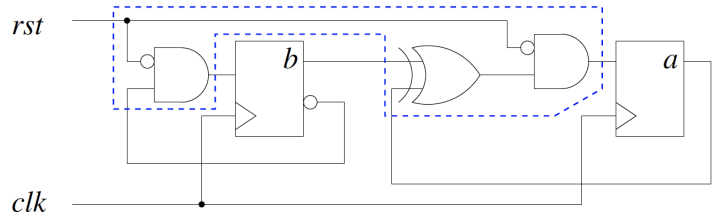

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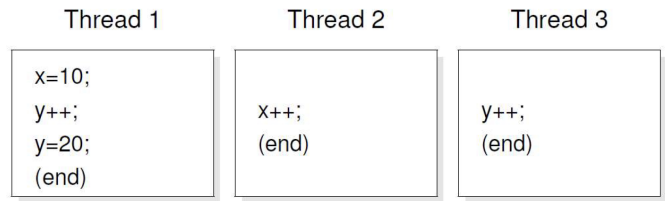

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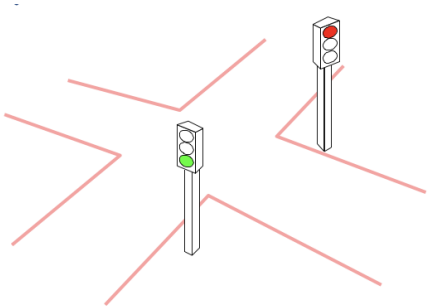

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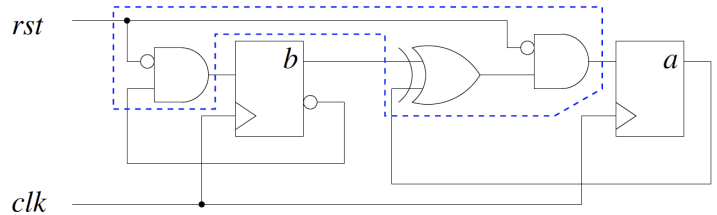

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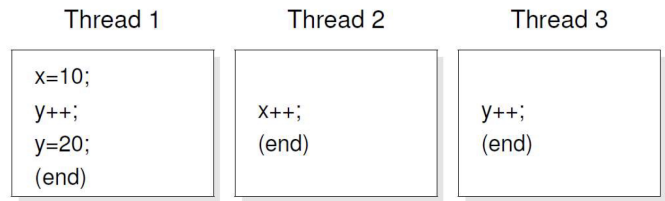
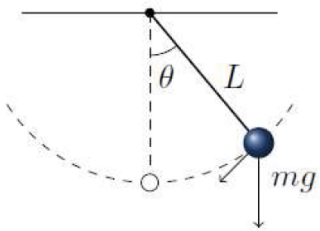
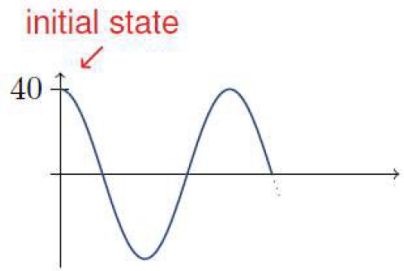


"the program has no data races"

What Kind of Systems Not

An Example





Our model:

$$\frac{\delta^2\theta}{\delta t^2} = -\frac{g}{L}\sin\theta$$

Plot of θ over time

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- Need a common formalism to model all systems of interest: **Transition Systems (M)**

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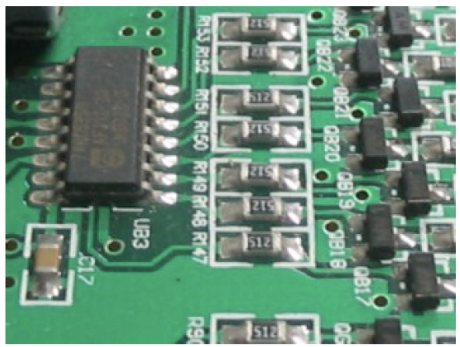
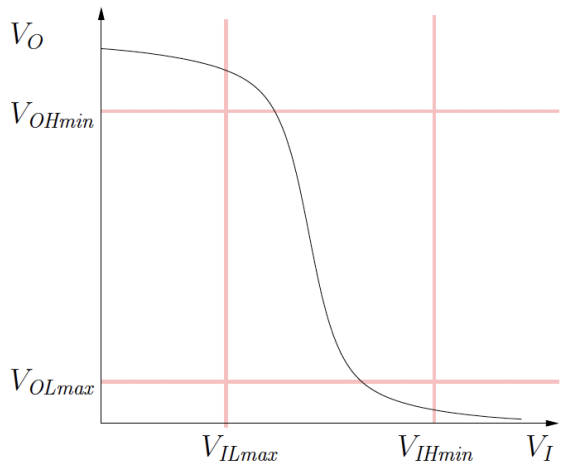
- Need a common formalism to model all systems of interest: **Transition Systems (M)**
- Need an expressive logic/language to specify desired properties **"Special-purpose Temporal Logics (Prop)"**
- Need efficient algorithms to automatically check properties **M** $\models$ **Prop**

A Hardware Abstraction Example

We always omit some detail.

Example:





x	$\neg x$
0	1
1	0

Abstraction

Modeling: States

- Our systems have a state,
  the model has a set of states

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Definition Transition System

Definition (Transition System)
A *Transition System* is a 3-tuple $M = (S, S_0, T)$ consisting of

- a set of states S,
- a set of initial states $S_0 \subseteq S$
- a transition relation $T \subseteq (SXS)$

Modeling Sequential Code: An Example

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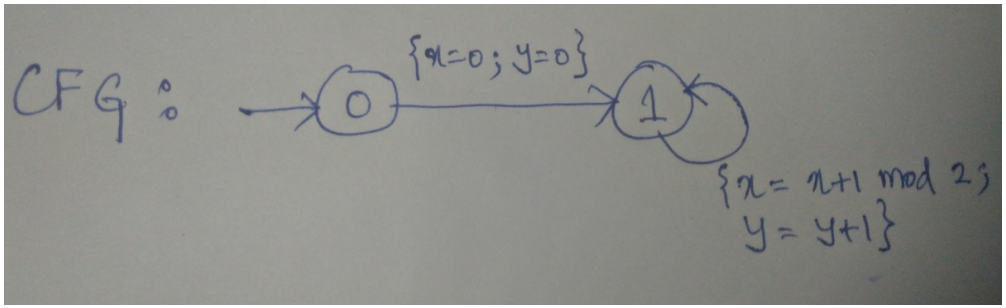

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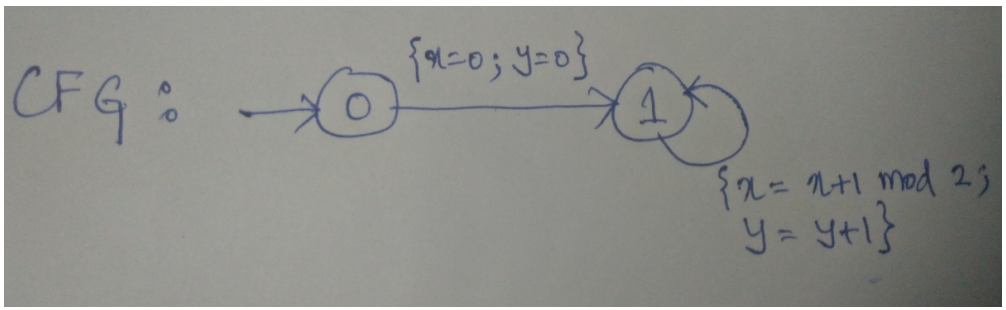
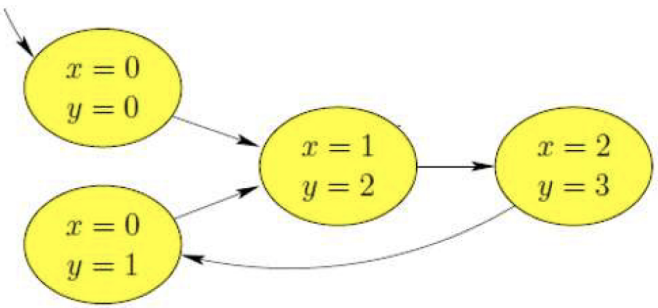




TS : $M = (S, S_0, R)$

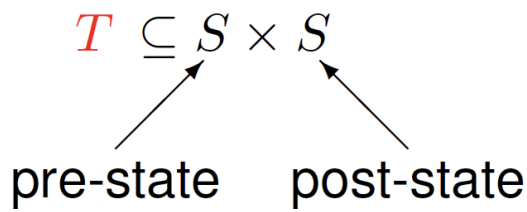
- $S = (D_1 X D_2)$, where $D_1 = \{0, 1, 2\}$, $D_2 = \{0, 1, 2, 3\}$
- $S_0 = \{(0, 0)\}$
- $R = \{((0, 0), (1, 2)), ((0, 1), (1, 2)), ((1, 2), (2, 3)), ((2, 3), (0, 1))\}$

Towards Symbolic Representation for TS: Notation for States

- S - set of states
- $s \in S$ - one particular state
- s, x - the value of some variable x in state s.
  These are called state variables

Towards Symbolic Representation for TS: The Transition Relation

The transition relation T relates a pre-state to a post-state:



The transition relation captures how the state can be transformed by a single transition.

Towards Symbolic Representation for TS: Characteristic Functions

Notation: We typically write the transition relation using a characteristic function.

This is useful for any kind of set:

$$S = \{0, 2, 4, 6, 8, \ldots\} \qquad x \in S \iff x \bmod 2 = 0$$

$$S = \{1, 2, \ldots, 9, 10\} \qquad x \in S \iff 1 \leq x \wedge x \leq 10$$

$$S = \{1, 2, 4, 8, 16, 32, \ldots\} \qquad x \in S \iff \exists y.\, x = 2^y$$

Towards Symbolic Representation for TS: The Transition Relation Again

Example:

$$T \subseteq \mathbb{N}_0 \times \mathbb{N}_0$$

$$T(x, x') \iff x' = x + 1$$

This corresponds to the set

$$\{(0, 1), (1, 2), (2, 3), \ldots\}$$

By convention, primed variables (x') denote the value of a state variable in the next state.

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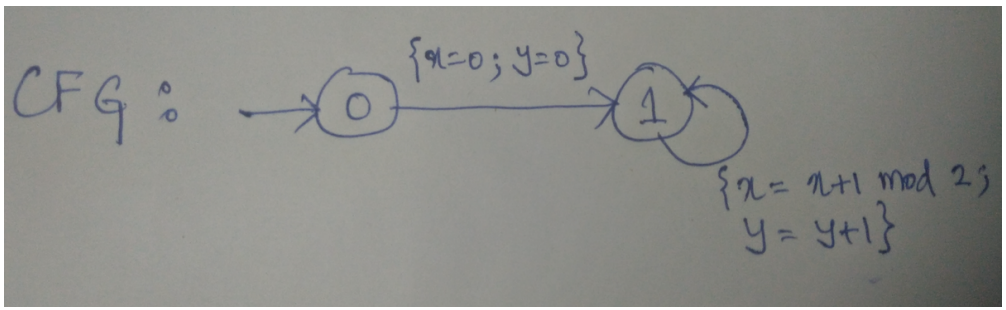

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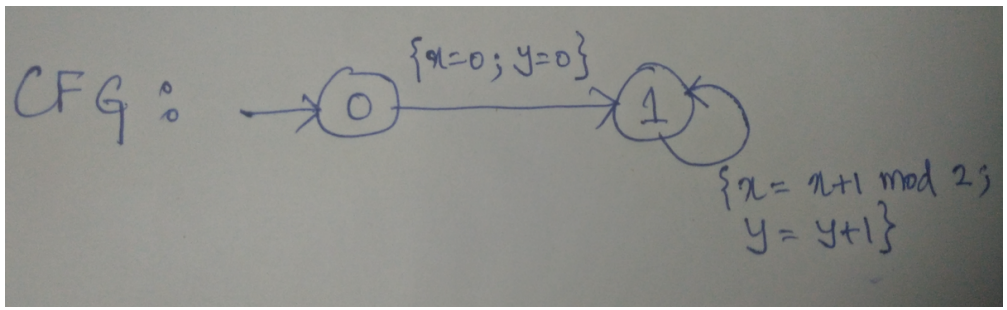

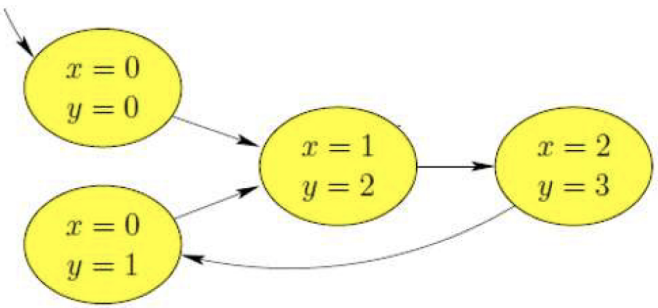

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  - Initial nondeterminism: Initial state need not be unique
  - Inputs (environment) nondeterminism: Inputs can change arbitrarily
  - Scheduler nondeterminism (concurrent systems): Multiple successor states possible for a transition

Initial State Nondeterminism

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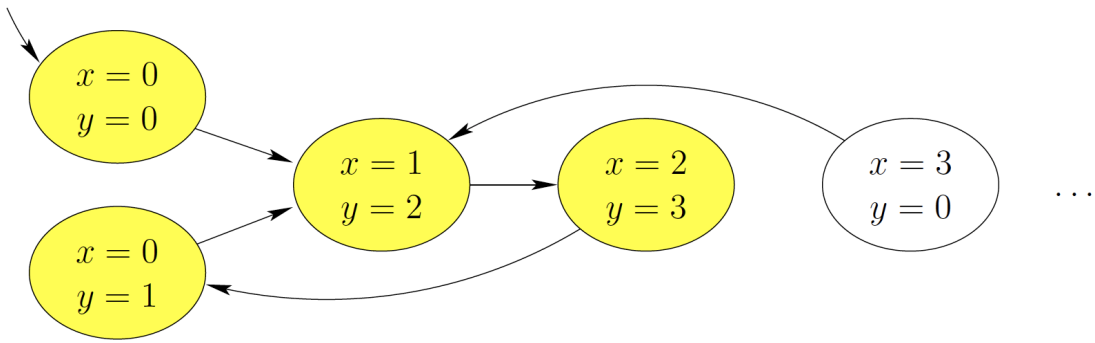

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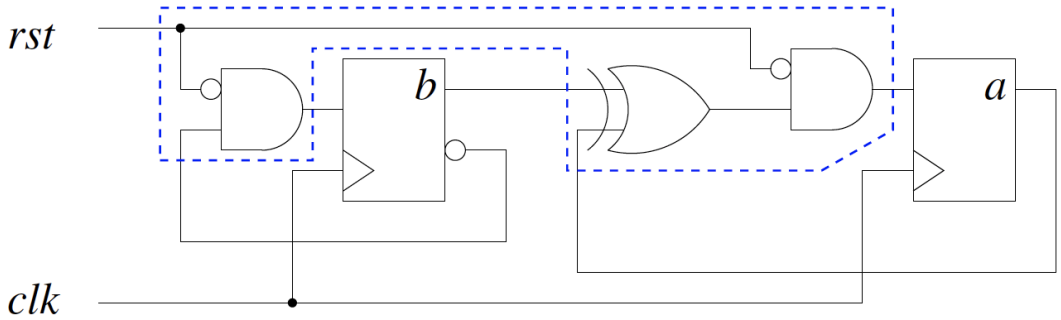
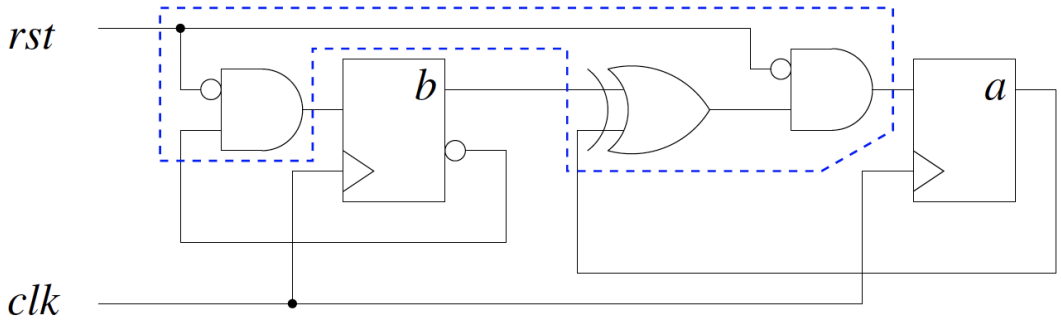
Modeling Clocked Hardware: Modulo 4 Counter



- One global clock: *clk*
- One input: *rst*
- Next-state logic (dashed box):
  a, b (flip-flops) change *synchronously* with every tick of *clk*
  - $\delta(a, b, rst).a \equiv \overline{rst} \ . \ (a \oplus b)$
  - $\delta(a, b, rst).b \equiv \overline{rst} \ . \ \overline{b}$

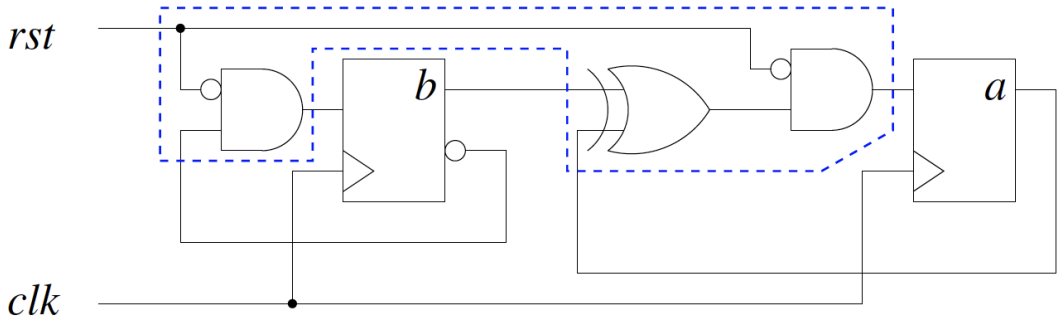
Modeling Inputs in TS

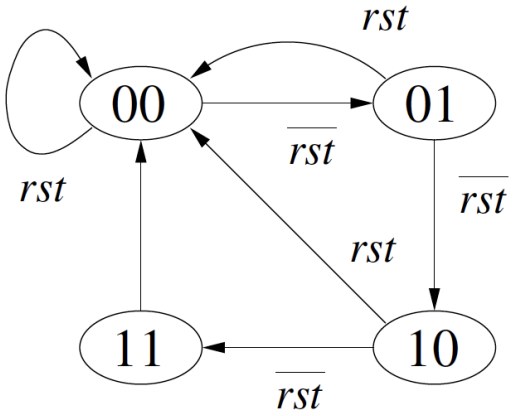


$$T(s, s') \equiv$$

$$(\delta(s.a, s.b, s.rst) = s'.ab \vee$$

Modeling Inputs in TS





$$T(s, s') \equiv$$

$$(\delta(s.a, s.b, s.rst) = s'.ab \vee$$

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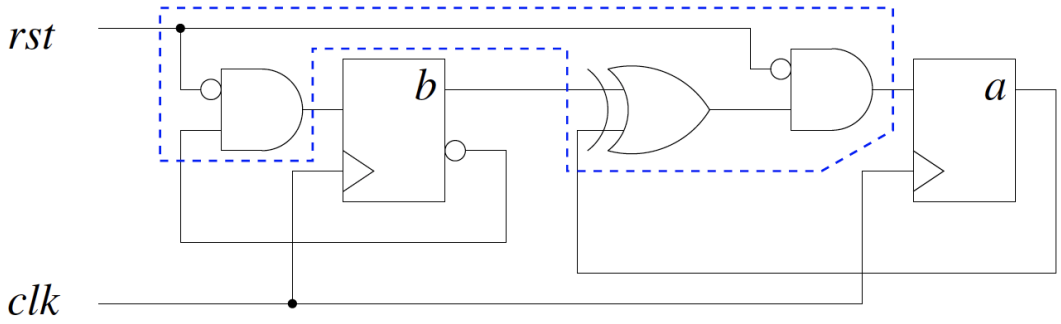

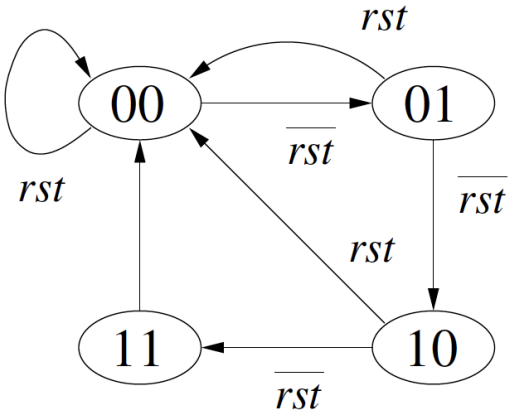

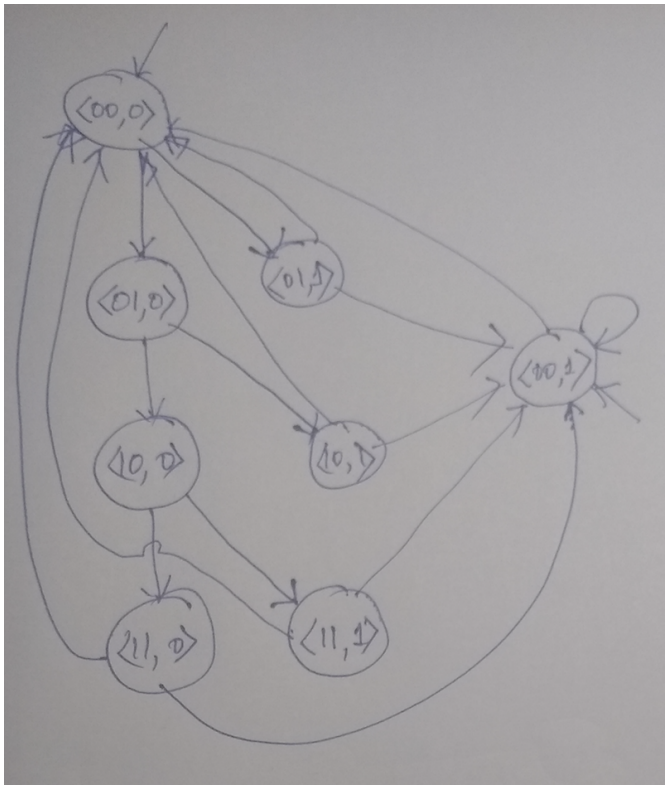

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Assumptions on Transition Systems

Definition (Transition System)
A *Transition System* is a 3-tuple $M = (S, S_0, T)$ consisting of

- a set of states S,
- a set of initial states $S_0 \subseteq S$
- a transition relation $T \subseteq (SXS)$

1. Domains of state variables are finite and discrete

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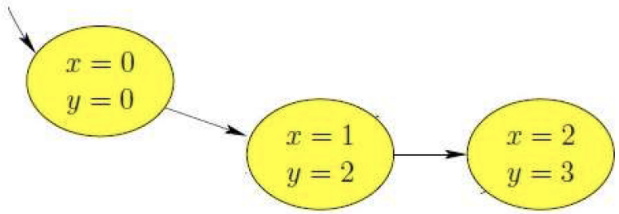
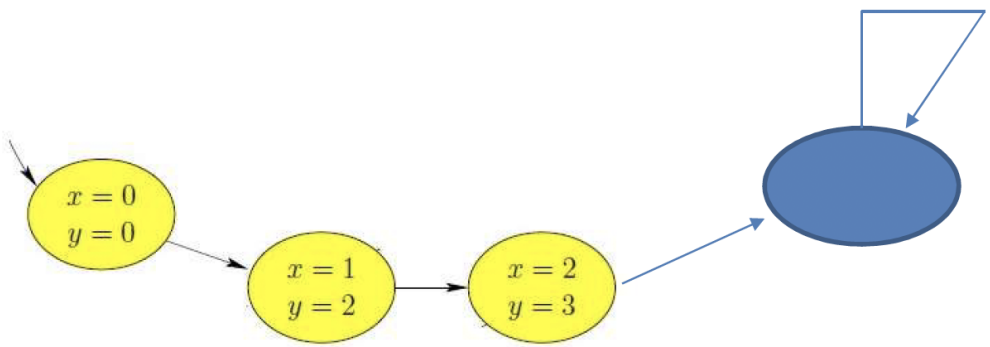
1. Domains of state variables are finite and discrete
2. Transition relation is total (No loss of generality)
   - How about terminating systems, e.g., code?
   - If $T(s, s') \equiv \mathit{false}$, then redfeine it as $T'(s, s') \equiv (s = s')$

Terminating TS

```
Void main () {

int x = 0, y = 0;

while (y<=2) {

x = x+1 mod 3;
y = x+1; }

}
```



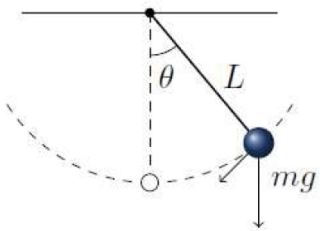
Without loss of generality we make
all our TS non-terminating

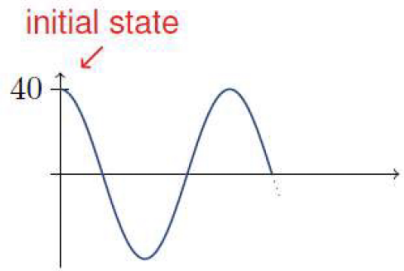


This means our TS relation is assumed to be TOTAL

An Continuous Domain Example

An Example





Our model:

$$\frac{\delta^2\theta}{\delta t^2} = -\frac{g}{L}\sin\theta$$

Plot of θ over time