

Estimation of Nonhomogeneous Prestrain in a Continuum  
Ligament Model

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# Chapter 1

## Introduction

### 1.1 Background

Computational joint models are important tools that can be used to better understand joint mechanics and the conditions that can alter joint behavior. Knee models have been used to evaluate joint behavior that is difficult or impossible to measure *in-vitro* or *in-vivo*. Previous studies have used knee models to evaluate the joint's mechanics for healthy (Blankevoort and Huiskes, 1996; Pea et al., 2006), injured (Ali et al., 2016), and surgically repaired (Thompson et al., 2011; Amiri et al., 2012; Salehghaffari and Dhaher, 2015; Smith et al., 2016b) joint conditions. Specimen-specific knee models have been shown to more accurately predict the joint's behavior.

### Specimen-Specific Knee Models

Specimen-specific knee models are composed of specimen-specific geometry and material properties. The geometry is normally generated from MR imaging, and material properties for hard and soft tissues are determined with either experimentation (Gardiner and Weiss, 2003) or inverse modeling (Blankevoort and Huiskes, 1996; Baldwin et al., 2009; Ewing et al., 2015; Harris et al., 2016). The joint's kinematics and contact mechanics have been shown to be sensitive to variations in ligament properties under passive (Baldwin et al., 2009; Dhaher et al., 2010; Ewing et al., 2015) and dynamic (Smith et al., 2016a) joint loading, therefore specimen-specific ligament properties are needed to model the joint's behavior (Ewing et al., 2015). In addition to constitutive properties, ligament prestrain must also be defined because each ligament's geometry is defined from MR images, where the ligaments may be under an unknown amount of load (Weiss and Gardiner, 2001; Maas et al., 2016). Specimen-specific prestrain is important to define because joint

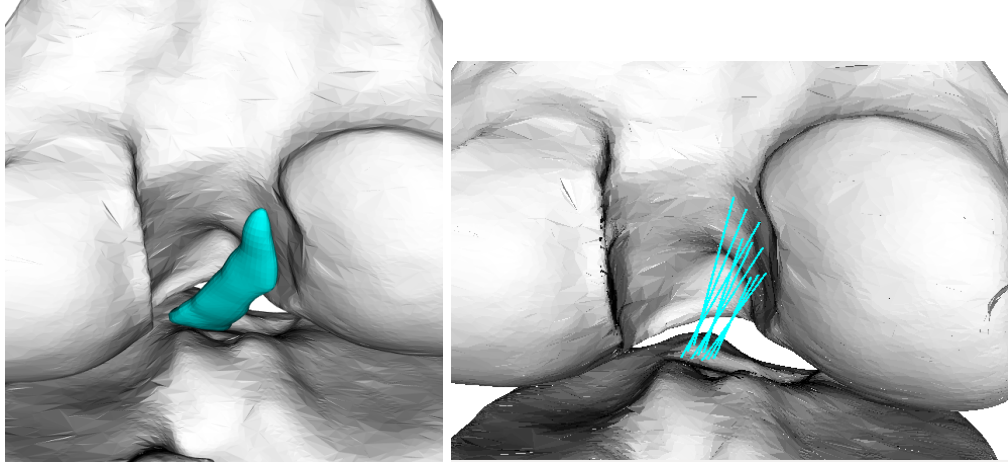


Figure 1.1: A comparison of the anterior cruciate ligament (ACL) modeled as a (left) continuum and (right) a bundle of nine individual springs arranged across the ligament's insertion site.

kinematics have been shown to be more sensitive to prestrain than stiffness (Baldwin et al., 2009). It has been shown that prestrain is not uniform throughout a ligament (Hull et al., 1996; Gardiner et al., 2001), and the way that prestrain is applied to a ligament varies depending on the ligament representation.

## Ligament Representation and Prestrain Definition

Ligaments are commonly represented as either bundles of nonlinear uniaxial springs (Blankevoort and Huiskes, 1991; Baldwin et al., 2009; Amiri et al., 2012; Kia et al., 2016), or as a solid continuum (Gardiner and Weiss, 2003; Pea et al., 2006; Dhaher et al., 2010) (Figure 1.1). Studies that utilize spring ligament representations can simulate nonhomogeneous prestrain in a ligament by varying the prestrain in each spring that composes a specific ligament. With the exception of Gardiner and Weiss (2003), specimen-specific nonhomogeneous prestrain is not utilized in continuum ligament models. Studies that utilize continuum ligament representations either apply uniform prestrain (Limbert et al., 2004; Song et al., 2004; Beidokhti et al., 2017), or nonhomogeneous prestrain based on values reported in the literature (Pea et al., 2006; Dhaher et al., 2010), where prestrains are derived from a combination of experimental studies and results of inverse modeling studies that utilize spring ligament models.

There are few studies that evaluate the effects of ligament representation on the model's performance, and improvements can be made to the studies that have addressed this topic. Beidokhti et al. (2017) used inverse modeling to generate comparable spring and continuum ligament models. The two types of models were considered equivalent if they yielded joint forces that were similar to experimentally measured values, however it was not shown that the spring and continuum ligament models yielded similar ligament forces at a given joint position. Additionally, the prestrain definition may not be equivalent between the spring

and continuum ligament representations. Conversely, Orozco et al. (2018) applied similar prestrain to their spring and continuum ligament models, however prestrain was uniformly applied throughout each ligament.

## 1.2 Overview

A calibrated knee model that represents ligaments as springs could be used to estimate specimen-specific nonhomogeneous prestrain in continuum representations of ligaments. Lu et al. (2007) estimated the prestrain in blood vessels using an inverse elastostatics approach, where the deformed state of the geometry and forces were known, and the stress-free shape of the geometry was estimated. The overall purpose of this work is to (1) develop an approach to estimating specimen-specific nonhomogeneous prestrain in ligaments that are represented as a continuum and (2) compare the performance of the continuum ligament representation to an equivalent spring ligament representation.

There are three specific aims that are used to work towards the purpose of this work. The first aim is to collect specimen-specific force-displacement data under novel loading conditions. These data will be used in an inverse modeling scheme to estimate each specimen’s ligament slack lengths. Experimental data will be reduced to identify loading conditions that specifically load individual ligaments, and a subset of loading conditions that load all of the modeled ligaments. The results of the other specific aims will be used to support the estimation of subject-specific nonhomogeneous ligament prestrain in ligaments modeled as a continuum. Force results from the specimen-specific forward kinematics model will be used to define boundary conditions for an inverse elastostatics analysis, and the reduced set of loading conditions will be used to evaluate the calibrated continuum ligament models.

This work will develop and evaluate an approach to defining specimen-specific nonhomogeneous prestrain in ligaments modeled as a continuum. This approach could be used improve knee models that are used to evaluate ligament injury, and offer a better comparison between knee models that represent ligaments as springs and a continuum.

### 1.2.1 Specific Aims

#### Experimental Testing

Experimental testing was conducted to collect specimen-specific force-displacement data and MR images for two specimens. These tests applied novel loading that was designed to remove articular contact which focuses the joint’s force-displacement behavior on the soft tissue restraints. The applied joint forces and the corresponding joint kinematics were measured throughout testing. The MR imaging provides the data

necessary to create specimen-specific geometry, and the measurements taken during preparation and testing will provide the data necessary to simulate the experimental conditions.

*Status:* Complete

*Contribution:* Data necessary to create specimen-specific models and simulate the experimental tests.

### **Inverse Modeling**

Inverse modeling was used to estimate specimen-specific ligament slack-lengths. Use of an experimentally novel joint state and joint loads allows for joint contact to be neglected in the computational model, and may focus the joint's force-displacement behavior on the ligaments. Neglecting contact enables the use of a computationally efficient forward kinematics model, which is used in an inverse modeling scheme to estimate ligament slack-length. This will provide a calibrated model that recreates the experimental conditions.

*Status:* Complete

*Contribution:* A novel approach to using inverse modeling to estimate ligament properties.

### **Dataset Reduction**

The purpose of this aim is to determine (1) experimental loading conditions that target a small set of ligaments and (2) define a subset of the experimental tests that target all of the modeled ligaments. The experiments applied five different loading conditions at four different flexion angles for a total of twenty tests. Ligament recruitment varies with the loading condition and flexion angle. The specimen-specific geometry and kinematics can be used to estimate ligament length throughout each test, and these length measurements can be used to estimate which ligaments are recruited during specific tests without defining ligament slack lengths.

*Status:* Incomplete

*Contribution:* A set of experimental tests that can be used to evaluate the continuum ligament model's performance.

### **Continuum Ligament Prestrain**

This aim will use an inverse elastostatic analysis to estimate the undeformed geometry of individual ligaments. To evaluate the performance of this method, The calibrated continuum ligament model's forces will be compared to the corresponding spring model's forces under different loading conditions.

*Status:* Incomplete

*Contribution:* An approach to defining specimen-specific nonhomogeneous prestrain in a continuum ligament model.

## Chapter 2

# Distraction Experiment

### 2.1 Introduction

Laxity tests are used by physicians to assess the integrity of specific ligaments. These tests can involve the physician moving the joint to a specific position and manually applying loads to the joint. Under these passive loads, the joint is restrained by articular contact and ligaments. Different tests are used for specific ligaments, and the physician assess the amount of joint motion and restraint that is provided (or not provided) by the targeted ligament. In a research setting, similar laxity-style tests are conducted on cadaveric specimens with custom fixtures (Roth et al., 2015; Walker et al., 2014; Rachmat et al., 2016), or six degree-of-freedom robots (Song et al., 2004; Nawabi et al., 2016; Imhauser et al., 2017).

Experimental laxity tests are used to develop and validate computational knee models (Gardiner and Weiss, 2003; Kia et al., 2016), and these tests can provide data that are used to estimate ligament properties (Blankevoort and Huiskes, 1996; Baldwin et al., 2012; Ewing et al., 2015; Harris et al., 2016). These studies use inverse modeling to estimate ligament properties, where a knee model is used to simulate the experimental conditions, and ligament properties are iteratively changed until the model's results match the experimental values. This assumes that for a given joint position the externally applied forces balance with the internal articular contact and ligament forces (Blankevoort and Huiskes, 1996). An adjustment to the experimental protocol could remove articular contact and reduce the number of internal joint forces.

Adjusting experimental loading to distract the joint would tension the ligaments while also removing contact loads (Figure 2.1). This adjusted loading could focus the joint's force-displacement behavior on the soft tissue restraints, and an inverse modeling scheme similar to other studies Blankevoort and Huiskes (1996); Baldwin et al. (2012); Ewing et al. (2015); Harris et al. (2016) could be used to estimate ligament properties.

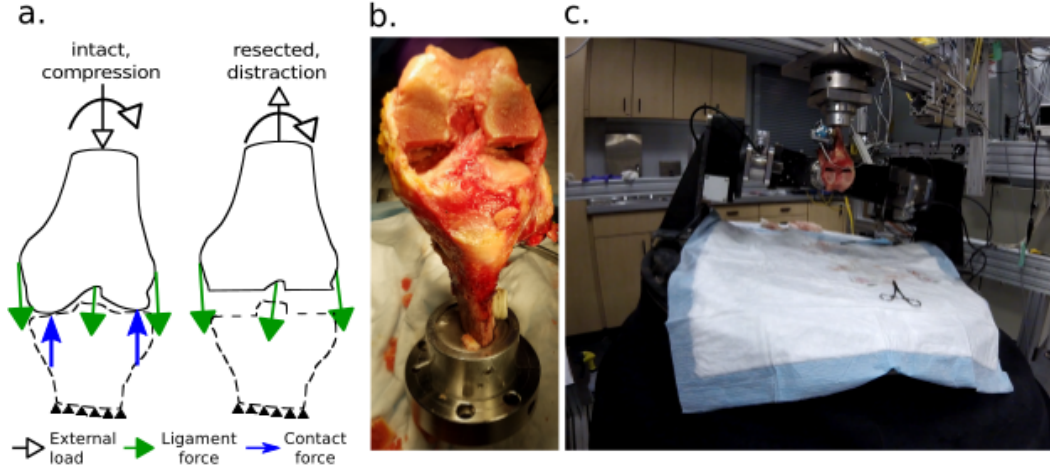


Figure 2.1: (a) A simplified representation of a traditional test with compression, and a distraction test. (b) A resected specimen with the femoral articulating surface removed. (c) The robot that applied the laxity-style distraction loading, with the specimen in 90° flexion.

The purpose of this work was to use novel joint loading to collect specimen-specific force-displacement data for use in an inverse modeling scheme to estimate ligament properties.

## 2.2 Methods

### Specimen Preparation

Experimental tests were conducted on two knee specimens. The specimens were initially prepared following the OpenKnee(s) protocol (Erdemir, 2016). In short, whole leg specimens from femoral head to foot were initially dissected by removing the soft tissue 8 cm proximal and distal to the knee. To facilitate model generation, three registration markers were fixed to the femur and tibia (six markers total) approximately 8 cm proximal and distal to the joint line, respectively. Optoelectronic sensors were fixed to the femur and tibia, and the position and orientation of the sensors was measured with an optoelectronic camera system (Optotrak, Northern Digital Inc., Waterloo, Ontario, Canada). Osseous landmarks (medial and lateral femoral epicondyles, femoral head, medial and lateral tibial plateau, and the medial and lateral malleolus) and ten points around each registration marker were digitized. After digitization, the tibia and femur were cut approximately 19 cm proximal and distal to the joint line, respectively. The specimen was then MR imaged, then the femur and tibia were potted into fixtures that are used to mount to the robot.

Each specimen was initially tested under traditional laxity-style tests (not described here) with the joint in two dissected states. The first state was intact, and the second state was cleaned, where an orthopedic surgeon dissected the skin, muscle, patella, and menisci, leaving the ligament structures intact. Following testing in

these two joint states, an orthopedic surgeon removed the articulating surface of the femur (Figure 2.1).

## Distraction Testing

Similar to the test in the intact and cleaned states, a six degree-of-freedom robot (Robotpod 2000, Mikrolar, Hampton NH, USA) with a custom alignment fixture (Figure 2.1) was used to apply laxity-style distraction loading to the specimen. A total of five laxity-style distraction tests were conducted at 0°, 30°, 60°, 90° flexion. The applied joint forces and resulting joint kinematics were measured throughout each test.

1. Distraction

- (a) Increased distraction load from 25 N to 100 N (step size: 25 N)

2. Anterior-Posterior drawer

- (a) Constant 75 N distraction
- (b) Increased anterior or posterior load from 20 N to 100 N (step size: 20 N)

3. Varus-Valgus distraction

- (a) Held knee at  $\pm 5^\circ$  Varus
- (b) Increased distraction load from 25 N to 100 N  
(step size: 25 N)

4. Internal-External rotation

- (a) Constant 75 N distraction
- (b) Increased internal or external rotation torque from 0 Nm to 5 Nm (step size: 1. Nm)

5. Kinetic Plane

- (a) Constant 100 N distraction
- (b) Constant 25 N load swept 360° around the coronal plane
  - i. Begin in anterior tibial drawer, then transition to lateral, posterior, then medial tibial drawer, and end in anterior tibial drawer (24 steps total)

All off axis loads were minimized throughout each test. The applied loads were designed to test the envelope of tissue recruitment. These loads were determined with pilot tests on a different specimen where successively larger distraction forces were applied until the joint remained out of contact during the laxity tests. The maximum resultant load was less than 130 N, which was nearly half the applied load in clinical tests (Nagai et al., 2014).



## 2.3 Contribution

The novel loading protocol used in the laxity-style distraction tests can be used to evaluate the limits of tissue recruitment while avoiding potentially confounding effects of articular contact. Additionally, these tests are similar to clinically relevant loading that is used during total knee arthroplasty. A surgeon may use a gap balanced surgical technique, where a distraction force across the joint, which tensions the ligaments, and cuts are made in order to achieve a desirable joint state D’Lima and Colwell (2017).

## Chapter 3

# Inverse Modeling

### 3.1 Introduction

Inverse modeling is normally used to estimate ligament properties using computational knee models and experimental laxity tests (Blankevoort and Huiskes, 1996; Baldwin et al., 2012; Ewing et al., 2015; Harris et al., 2016). These methods assume that for a given joint position the external joint forces balance with the internal ligament and contact forces (Blankevoort and Huiskes, 1996). This approach uses optimization to estimate the ligament properties that minimize the residual model and experimental values. Due to the need to model joint contact, these methods use forward dynamics models because joint contact forces have been shown to be sensitive to kinematic errors (Fregly et al., 2008; Yao et al., 2008).

Adjusting experimental conditions to remove articular contact would focus the joint's force-displacement behavior on the soft-tissue restraints. Additionally, this adjustment leaves the soft-tissue restraints as the only source for internal joint force (Figure 3.1), and this may lead to an improved estimation of ligament properties. This work uses novel experimental loading conditions in an inverse modeling scheme that is similar to previous studies (Blankevoort and Huiskes, 1996; Baldwin et al., 2012; Ewing et al., 2015; Harris et al., 2016) to estimate ligament slack lengths. This work focused on estimating slack length because Baldwin et al. (2009) showed that passive joint kinematics are more sensitive to ligament slack length than stiffness. The purpose of this work was to use novel experimental loading in an inverse modeling scheme to estimate ligament slack lengths.

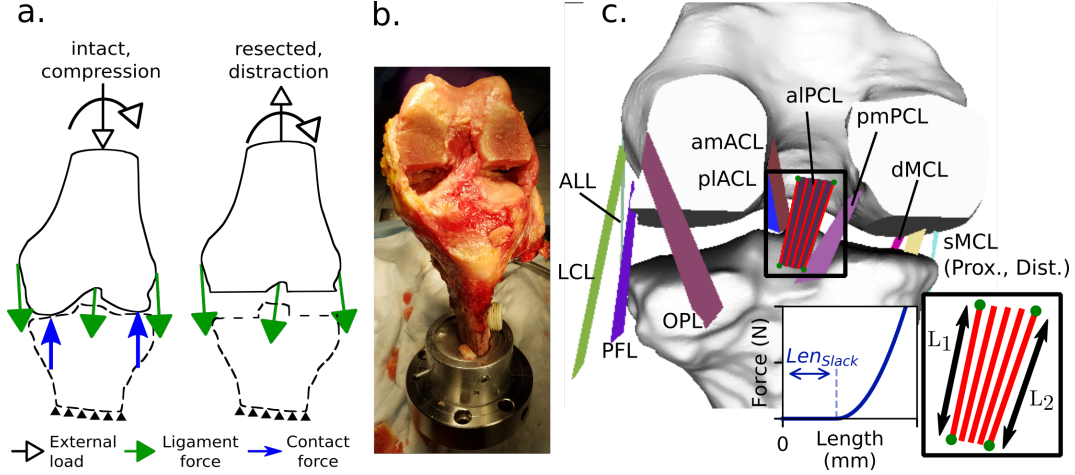


Figure 3.1: (a) A simplified representation of the joint forces during an intact laxity-style test compared to a distraction laxity-style test. (b) The specimen with resected femoral articulating surfaces. (c) The knee model with 11 ligament bundles represented. (inset) The non-linear spring model, and the definition of slack length for a ligament with five fibers.

## 3.2 Methods

Specimen-specific geometry was defined using the specimen's MR images. The femur and the tibia were segmented, and 11 ligament bundles were defined using the MR images and literature descriptions (Figure 3.1). The anterior and posterior cruciate ligaments were modeled as four bundles. These bundles were the anteromedial and posterolateral anterior cruciate ligament (amACL, plACL) (Duthon et al., 2006) and the anterolateral and posteromedial posterior cruciate ligament (aIPCL, pmPCL) (Anderson et al., 2012). The medial collateral ligament (MCL) was modeled as three bundles. Two of those bundles composed the proximal and distal superficial MCL (psMCL, dsMCL), and one bundle defined the deep MCL (dMCL) (LaPrade, 2007b). Additional ligament bundles included the lateral collateral ligament (LCL), the popliteofibular ligament (PFL) (LaPrade et al., 2003), the anterolateral ligament (ALL) (Claes et al., 2013), and the oblique popliteal ligament (OPL) (LaPrade, 2007a; Hedderwick et al., 2017).

The ligament bundles were defined by manually placing two points at the margins of femoral and tibial insertion sites (four points total). Each bundle consisted of 25 fibers, where two fibers connect the points that define the insertion area, and the remaining 23 fibers are evenly mapped between the margins. Each ligament fiber was modeled as a nonlinear spring (Blankevoort and Huiskes (1996)). Uniform stiffness was applied across each ligament, where the fiber stiffnesses summed to the ligament's equivalent stiffness (Table 3.1).

Experimental kinetics were applied to the forward kinematics model, and the length of each ligament fiber was used to calculate the magnitude of force carried by each fiber. The line of action was defined using each fiber's femoral and tibial insertion point. Based on visualization of the kinematics, some ligaments

Table 3.1: The equivalent stiffness (N/ $\epsilon$ ) of each ligament bundle with units of force per unit strain.

Ligament	Stiffness	Reference
amACL	2120	Amiri et al. (2007), Kia et al. (2016)
plACL	2880	Amiri et al. (2007), Kia et al. (2016)
alPCL	5625	Amiri et al. (2007), Kia et al. (2016)
pmPCL	3375	Amiri et al. (2007), Kia et al. (2016)
psMCL	1375	Amiri et al. (2007)
dsMCL	1375	Amiri et al. (2007)
dMCL	1000	Amiri et al. (2007)
LCL	2000	Amiri et al. (2007)
ALL	750	Ewing et al. (2015)
PFL	1000	Ewing et al. (2015)
OPL	1000	Ewing et al. (2015)

simulated wrapping around either the femur, or a cylinder that approximated the tibia’s surface. Tibial reaction forces were calculated by summing the force carried by each ligament fiber along its line of action (Mommersteeg et al., 1996), and when applicable, the wrapping reaction forces. Tibial reaction moments were calculated with tibial insertion points, wrapping points, and each fiber’s force magnitude and line of action.

Ligament slack lengths were estimated using a constrained sequential quadratic programming algorithm. To simulate nonhomogeneous prestrain, two values were used to define ligament slack length for every fiber in a ligament bundle. The length of the two fibers at the margins of the insertion site was defined, and linear interpolation was used to define the slack length for the remaining fibers (Figure 3.1).

The optimization minimized the residual between the model and experimentally measured tibial reaction forces for the anterior-posterior drawer and fixed varus-valgus distraction tests at 0°, 30°, 60°, 90° flexion (Equation 3.1). Three experimental tests were excluded from the optimization to evaluate the inverse modeling scheme’s performance.

$$\begin{aligned}
 &\underset{\mathbf{x}}{\text{minimize}} && f(\mathbf{x}) = \sum_{j=1}^{72} \sum_{i=1}^6 [w_j(M_{ij}(\mathbf{x}) - E_{ij})]^2 \\
 &\text{subject to} && \\
 &&& h(x_m) \geq 0.1, \ m = 1, \dots, 22
 \end{aligned} \tag{3.1}$$

Where  $i$  is number of the step in the loading cycle, and  $j$  is the force degree of freedom. The weighting factor ( $w$ ) applied a weight of 1 to the forces, and a weight of 20 to the moments. Constraints ( $h(x_m)$ ) enforced that the fibers at the margin of every ligament experienced a resultant force greater than 0.1 N at

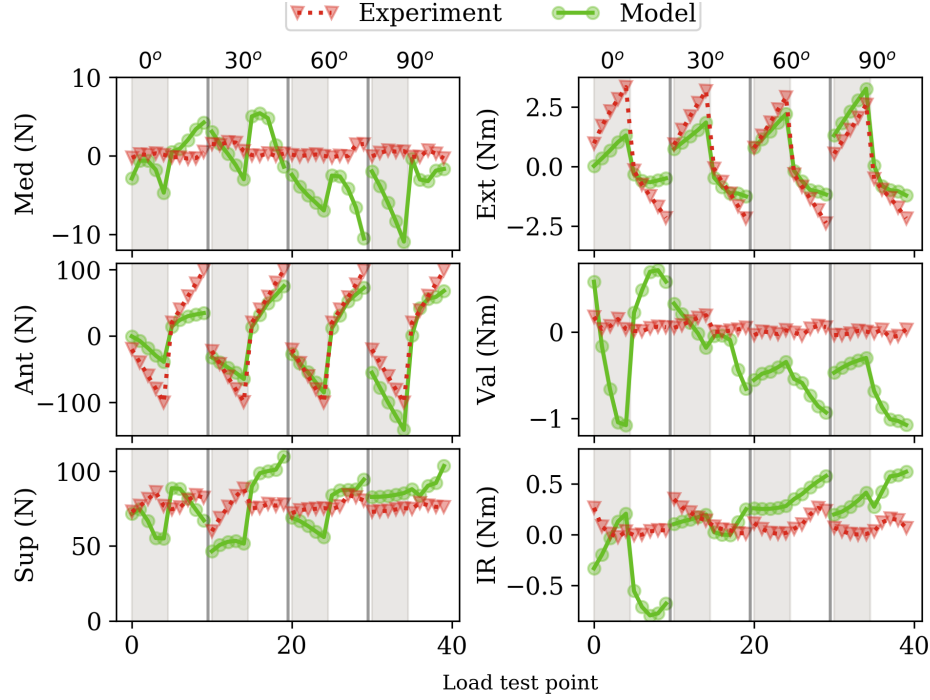


Figure 3.2: Tibial reaction forces and moments for the (grey) anterior and (white) posterior drawer tests for the experiment and calibrated model at  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$  flexion. Med, Ant, Sup are the medial, anterior, and superior tibial reaction forces, respectively. Ext, Val, IR are the extension, valgus, and internal tibial rotation reaction moments, respectively.

a minimum of one point throughout the loading cycle. This ensured that every control variable can have an impact on the objective function.

### 3.3 Results

For the tests that were included in the optimization (anterior-posterior drawer and varus-valgus), the RMS error of the joint kinetics across all flexion angles was 5.8 N, 22.5 N and 17.1 N for the medial, anterior, and superior loads respectively, and 0.80 Nm, 0.60 Nm, and 0.33 Nm for the extension, valgus, and internal rotation moments respectively (Figure 3.2).

For the distraction test, which was not part of the optimization, the RMS error of the joint kinetics across all flexion angles was 2.5 N, 13.4 N and 14.2 N for the medial, anterior, and superior loads respectively, and 0.54 Nm, 0.36 Nm, and 0.31 Nm for the extension, valgus, and internal rotation moments respectively (Figure 3.3).

For the kinetic plane and distraction tests, the highest errors in anterior force occurred at  $90^\circ$  flexion, and the highest distraction force errors occurred at  $0^\circ$  flexion (Table 3.2). The internal-external rotation model demonstrated the largest difference in distraction force (Table 3.2). For the internal-external rotation tests,

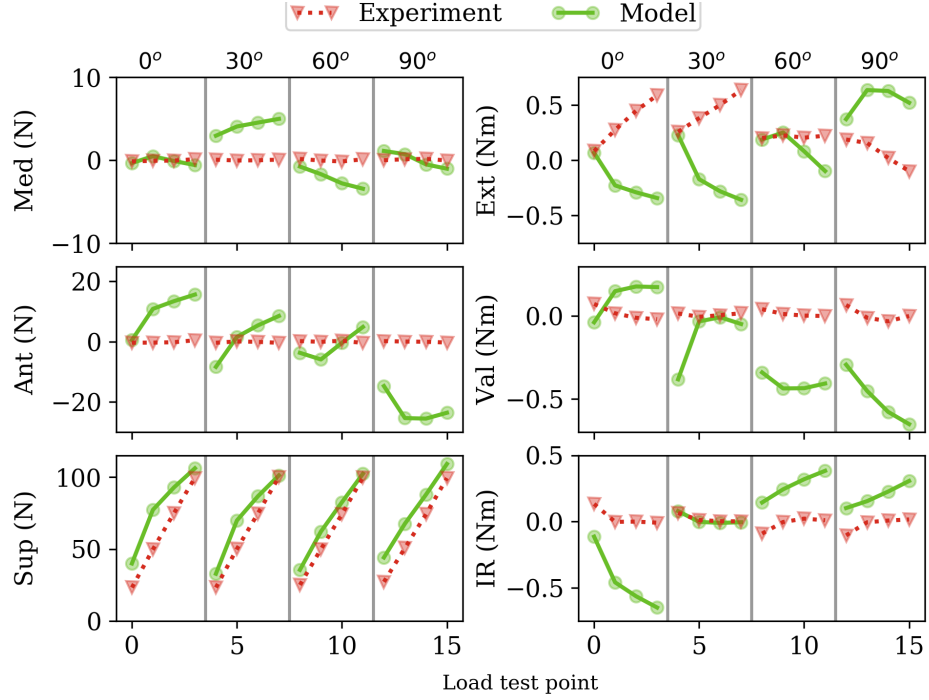


Figure 3.3: Tibial reaction forces and moments for the calibrated model compared to the experimentally measured values for the distraction test. Results are shown for 0°, 30°, 60°, 90° flexion. This test was not included in the optimization.

the model performed better when the applied torque was lower. Across flexion angles, the RMS distraction force error was 13.5 N at  $\pm 1$  Nm internal rotation torque, and 46.2 N at  $\pm 5$  Nm internal rotation torque.

### 3.4 Discussion and Contribution

This work demonstrated how novel experimental loads can be used to estimate ligament properties while avoiding the confounding effects of articular contact. This approach may yield a more repeatable estimate of ligament slack length because the joint's force-displacement behavior is dictated by the soft-tissue restraints, which are the subject of the inverse modeling scheme. Additionally, these methods use a constrained optimization, where the constraints ensure that every ligament can contribute to the objective function. Without these constraints, the optimization algorithm could effectively remove a ligament from the model. This effect is obvious with a forward kinematics, however the constraints used in this study are applicable to forward dynamics knee models as well.

Table 3.2: RMS errors between the experimental and calibrated model joint reaction forces (N) and moments (Nm) for the distraction, kinetic plane, and external-internal rotation tests. These tests were not part of the optimization.

<b>Flexion</b>	<b>Test</b>	$F_{med}$	$F_{ant}$	$F_{sup}$	$M_{ext}$	$M_{val}$	$M_{ir}$
0°	K. Pl.	9.87	19.29	10.47	1.06	0.18	0.66
	Dist.	0.46	11.64	18.68	0.64	0.16	0.50
	ER	2.73	11.94	13.41	1.00	0.21	2.72
	IR	3.67	20.99	28.32	2.00	0.93	3.32
30°	K. Pl.	11.84	11.07	9.85	1.01	0.17	0.07
	Dist.	4.19	6.63	12.48	0.69	0.20	0.01
	ER	3.60	9.03	32.09	0.94	0.99	2.86
	IR	6.07	36.09	45.38	1.25	0.34	3.24
60°	K. Pl.	8.65	6.45	9.09	0.42	0.44	0.37
	Dist.	2.41	4.42	9.13	0.17	0.42	0.29
	ER	0.81	9.70	53.11	0.61	0.60	3.02
	IR	2.30	13.58	59.35	0.82	0.29	3.24
90°	K. Pl.	7.58	25.15	7.99	0.72	0.65	0.30
	Dist.	0.87	22.71	14.62	0.50	0.51	0.22
	ER	5.58	6.43	43.39	0.93	0.45	2.72
	IR	3.66	18.70	33.68	0.33	0.36	2.71

K. Pl.: kinetic plane, Dist.: distraction, ER.: external tibial rotation, IR.: internal tibial rotation.

$F_{med}$ ,  $F_{ant}$ ,  $F_{sup}$ : medial, anterior and superior tibial reaction forces, respectively.

$M_{ext}$ ,  $M_{val}$ ,  $M_{ir}$ : extension, valgus, and internal tibial rotation tibial reaction moments, respectively.

## Chapter 4

# Continuum Prestrain

### 4.1 Introduction

Ligament geometry is defined from MR images where the ligament is under an unknown amount of load, therefore the unloaded reference geometry of the ligament is unknown. The amount of deformation need to achieve the reference geometry should be taken into account when evaluating joint mechanics (Maas et al., 2016). If ligaments are modeled as a bundle of springs, then the ligament's slack length can be adjusted to account for this deformation. However, ligaments that are represented as springs cannot model stress, or the ligament's elastic response to deformation. The three dimensional behavior of a ligament may have an impact on a knee model's performance. There are few studies that evaluate the effects of ligament representation on the model's performance, and improvements can be made to the studies that have addressed this topic. Beidokhti et al. (2017) used inverse modeling to generate comparable spring and continuum ligament models. The two types of models were considered equivalent if they yielded joint forces that were similar to experimentally measured values, however it was not shown that the spring and continuum ligament models yielded similar ligament forces at at a given joint position. Additionally, the spring ligament representation could account for uneven prestrain, however the continuum representation did not account for nonhomogeneous prestrain.

The approach used by Beidokhti et al. (2017) could be altered to create a continuum ligament model that can account for nonhomogeneous prestrain. This could provide a more equivalent comparison between the two types of ligament models because both will be able to account for nonhomogeneous prestrain. This approach would use two steps where (1) inverse modeling is used to calibrate a knee model with spring ligaments, and (2) the ligament force results from the calibrated model are used as boundary conditions



when estimating nonhomogeneous prestrain in individual continuum ligament models. Similar approaches have been used to evaluate the reference geometry in blood vessels where the pressure inside the vessel is assumed to be known (Fachinotti et al., 2008; Sellier, 2011; Lu and Li, 2016). Similarly, Maas et al. (2016) presented an approach to estimating the prestrain in ligaments where the ligament loads are known, however the source for the ligaments forces were not a focus of that study. The purpose of this work is to estimate nonhomogeneous ligament prestrain using data from a calibrated knee model, and evaluate the performance of the specimen-specific continuum ligament representation.

## 4.2 Status and Methods Overview

The methods for this specific aim have not been fully developed. There are two general approaches that are used to estimate the unloaded configuration of a solid body. One approach iteratively uses a traditional finite element analysis where an estimated reference geometry is loaded and adjusted until the deformed reference geometry matches the imaged geometry (Maas et al., 2016; Sellier, 2011). The other approach modifies the an existing finite element algorithm to use a Eulerian description of motion (Fachinotti et al., 2008; Lu et al., 2007; Lu and Li, 2016). By using an Eulerian description of motion, it is assumed that the forces and geometry in the deformed state is known, and the equations of elasticity are used to calculate the undeformed geometry. Fachinotti et al. (2008); Lu and Li (2016) have shown that existing software that implements the finite element method can be modified to use an Eulerian description of motion while still using established strain energy functions.

This work will use approaches similar to Fachinotti et al. (2008) and Lu and Li (2016) to modify existing finite element software to use an Eulerian description of motion. The individual ligament forces from the calibrated knee model that represents ligaments as springs (Figure 4.1) will be used to calculate the ligament's reference geometry. The calculated reference geometry will be used to simulate different experimental conditions to estimate the amount of force the ligament exerts across the joint. These forces will be compared to the corresponding spring ligament representation's forces to evaluate the performance of the continuum ligament representation.

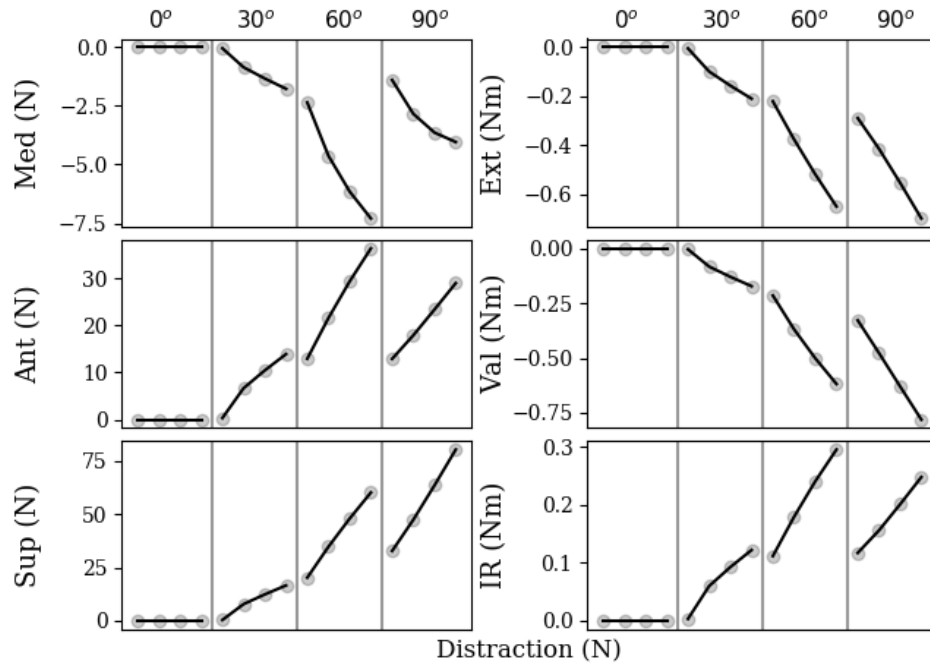


Figure 4.1: An example of the forces and moments that the aIPCL exerts on the tibia at four points throughout a distraction test at 0°, 30°, 60°, 90° flexion.

## Chapter 5

# Dataset Reduction

### 5.1 Introduction

Ligament lengths are used to infer ligament recruitment throughout joint motion (Blankevoort et al., 1991; Hosseini et al., 2014). These studies analyze a set of joint motions where ligament lengths are estimated and it is assumed that the ligaments carry load at some point throughout the test. Recognizing that ligaments only carry force in tension, it is inferred that ligaments are most likely to carry force when they are at their longest length. This analysis can indicate which points in the loading cycle that ligaments are loaded without the need to define any ligament properties.

Depending on the desired outcome, ligament lengths can be analyzed to indicate a small set of experimental tests that potentially recruit the most ligaments, or conversely what tests recruit the fewest ligaments. Identifying a subset of experiments that are likely to recruit all of the modeled ligaments would be applicable to future experimental design. For this work, identifying tests that recruit individual ligaments would be applicable to evaluating the performance of a calibrated continuum ligament model (presented in chapter 4). The purpose of this work is to (1) identify a subset of experimental loading conditions that recruit all of the modeled ligaments and (2) to identify individual experimental tests that isolate a small set of ligaments. The results from (1) will be applicable to future experiment design, and the results from (2) will help evaluate the performance of the calibrated continuum and spring ligament representations.

### 5.2 Status and Methods Overview

The methods for this specific aim have not been fully developed. Ligament recruitment has previously been estimated by analyzing ligament length throughout a set of laxity (Blankevoort and Huiskes, 1991) and

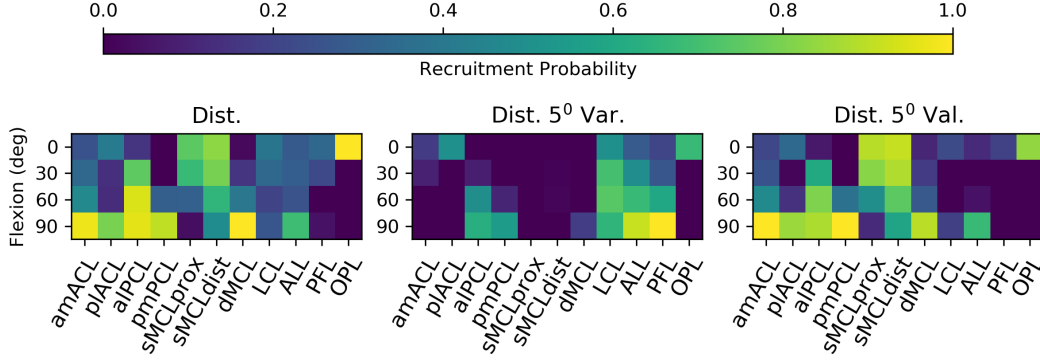


Figure 5.1: The recruitment probability for 11 ligament bundles for specimen throughout distraction and varus-valgus distraction tests.

dynamic. (Hosseini et al., 2014) loading conditions. If the strain at ligament failure is assumed, then the recruitment probability (Equation 5.1, Figure 5.1) (Blankevoort et al., 1991) can be calculated for every test.

$$P_i = (L_i - R_i) / (\epsilon_m R_i) \quad \text{for } L_i > R_i \quad (5.1)$$

$$P_i = 0 \quad \text{for } L_i \leq R_i \quad (5.2)$$

Where  $i$  indicates the ligament,  $e_m$  is the strain at failure for the ligament, and  $R_i$  is the minimal recruitment length defined by

$$R_i = \frac{L_{max,i}}{1 + e_m} \quad (5.3)$$

Where  $L_{max,i}$  is the maximum length of the ligament throughout all of the loading cycles. The value for  $e_m$  is normally assumed to be 0.1, and it is also assumed that the ligaments are elastically deformed and uninjured throughout the experimental tests.

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