# One-Sample Tests of Hypothesis

Week 6

# Learning Objectives

LO10-1 Explain the process of testing a hypothesis

LO10-2 Apply the six-step procedure for testing a hypothesis

LO10-3 Distinguish between a one-tailed and a two-tailed test of hypothesis

LO10-4 Conduct a test of a hypothesis about a population mean

LO10-5 Compute and interpret a p-value

LO10-6 Use a t-statistic to test a hypothesis

# Hypothesis Testing

- Hypothesis testing begins with a hypothesis statement about a population parameter

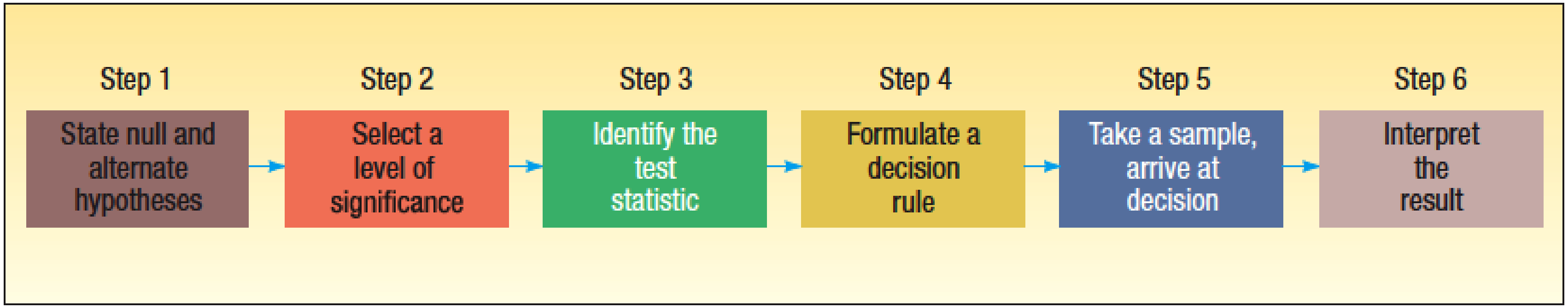
HYPOTHESIS A statement about a population parameter subject to verification

- Examples
- The mean speed of automobiles passing milepost 150 on the West Virginia Turnpike is 68 mph
- The mean cost to remodel a kitchen is $20,000

# Hypothesis Testing

- The objective of hypothesis testing is to verify the validity of a statement about a population parameter

> HYPOTHESIS TESTING A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.



# Step 1 of the Six-Step Process

- State the null hypothesis ($H_0$) and the alternate hypothesis ($H_1$)

> NULL HYPOTHESIS A statement about the value of a population parameter developed for the purpose of testing numerical evidence.

- The null hypothesis always includes the equal sign
  - For example; =, ≥, or ≤ will be used in $H_0$

> ALTERNATE HYPOTHESIS A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false.

- The alternate hypothesis never includes the equal sign
  - For example; ≠, <, or > is used in $H_1$

# Step 2 of the Process

- Next, you select the level of significance, $\alpha$

> LEVEL OF SIGNIFICANCE The probability of rejecting the null hypothesis when it is true.

- Sometimes called the level of risk
- Can be any value between 0 and 1
- Traditionally,
  - .05 is used for consumer research projects
  - .01 for quality assurance
  - .10 for political polling

# Possible Error in Hypothesis Testing

- Since the researcher cannot study every item or individual in the population, error is possible

> **TYPE I ERROR** Rejecting the null hypothesis, $H_0$, when it is true.

- Type I error is designated with the Greek letter alpha, $\alpha$

> **TYPE II ERROR** Not rejecting the null hypothesis when it is false.

- Type II error is designated with the Greek letter beta, $\beta$

| Null Hypothesis | Researcher: Does Not Reject $H_0$ | Researcher: Rejects $H_0$ |
|---|---|---|
| $H_0$ is true | Correct decision | Type I error |
| $H_0$ is false | Type II error | Correct decision |

# Step 3 of the Process

- Then, select the test statistic

> TEST STATISTIC A value, determined from sample information, used to determine whether to reject the null hypothesis.

- In hypothesis testing for the mean, μ, when σ is known, the test statistic z is computed with the following formula

**TESTING A MEAN, σ KNOWN**

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad (10\text{–}1)$$

- We can determine whether the distance between $\bar{x}$ and μ is statistically significant by finding the number of standard deviations $\bar{x}$ is from μ

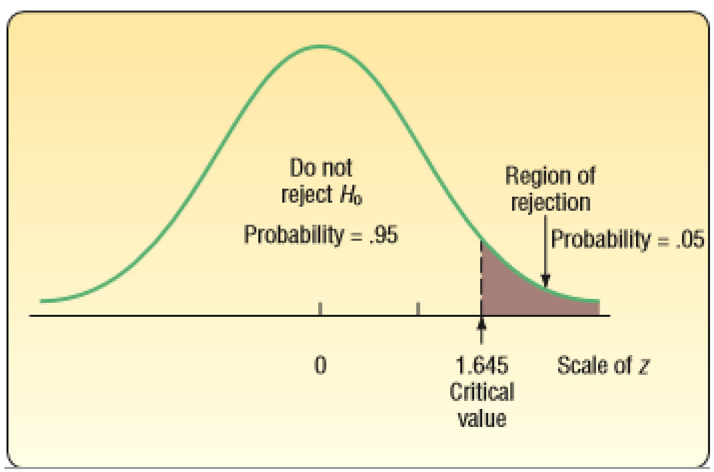
# Step 4 of the Process

- Formulate the decision rule
- The decision rule is a statement of specific conditions under which the null hypothesis is rejected and the conditions under which it is not rejected
- The region or area of rejection defines the location of all the values that are either so large or so small that their probability of occurrence under a true null hypothesis is remote

CRITICAL VALUE The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

# Critical Value

- The sampling distribution of the statistic z follows the normal distribution
- Here, an $\alpha$ of .05 is used in a one-tailed test
- The value 1.645 separates the regions where the null hypothesis is rejected and where it is not rejected
- The value 1.645 is the critical value



# Steps 5 & 6 of the Six-Step Process

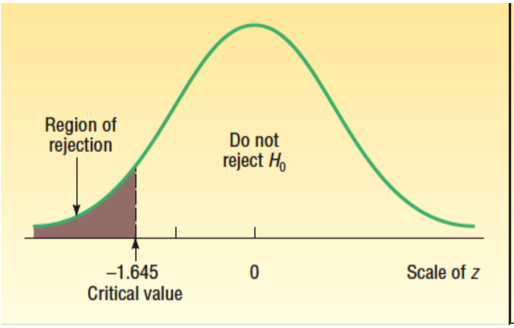
- Step 5 Make a decision
  - First, select a sample and compute the value of the test statistic
  - Compare the value of the test statistic to the critical value
  - Then, make the decision regarding the null hypothesis
- Step 6 Interpret the results
  - What can we say or report based on the results of the statistical test?

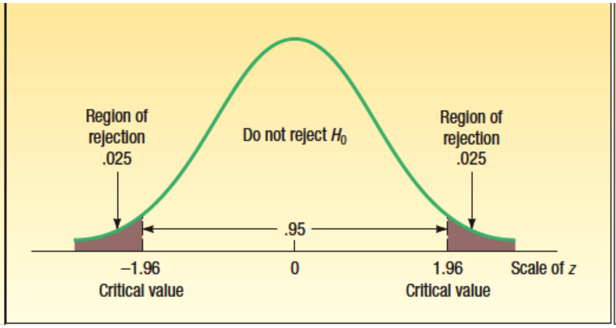
# One-Tailed and Two-Tailed Tests

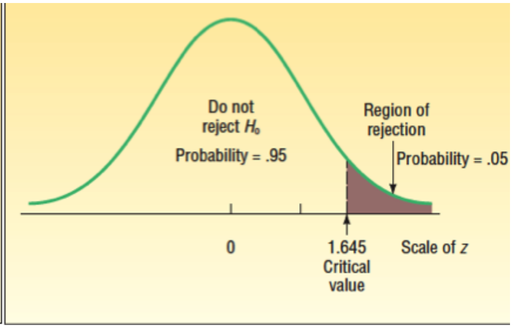
$H_0$: ≥ 60,000 miles
$H_1$: < 60,000 miles
with an $\alpha = .05$
Left-tailed test

$H_0$: = \$65,000 per year
$H_1$: ≠ \$65,000 per year
with an $\alpha = .05$
Two-tailed test

$H_0$: ≤ 453 grams
$H_1$: > 453 grams
with an $\alpha = .05$
Right-tailed test




Note that the total area in the normal distribution is 1.0000.

# Two-Tailed Test Example, σ Known

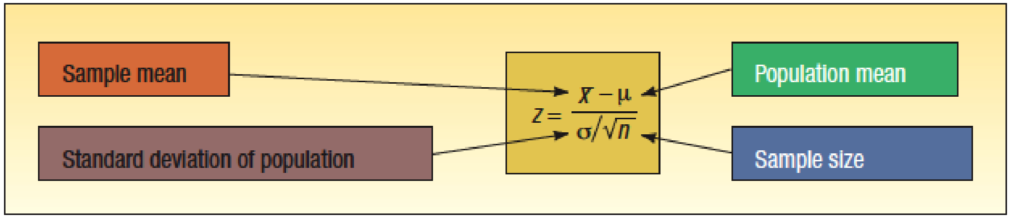
Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in New York State. At the Fredonia plant, the weekly production of the Model A325 desk follows a normal distribution with a mean of 200 and a standard deviation of 16. New production methods have been introduced and the vice president of manufacturing would like to investigate whether there has been a change in weekly production of the Model A325. Is the mean number of desks produced different from 200 at the 0.01 significance level?

Step 1: State the null hypothesis and alternate hypothesis.

$H_0$: $\mu = 200$ desks
$H_1$: $\mu \neq 200$ desks

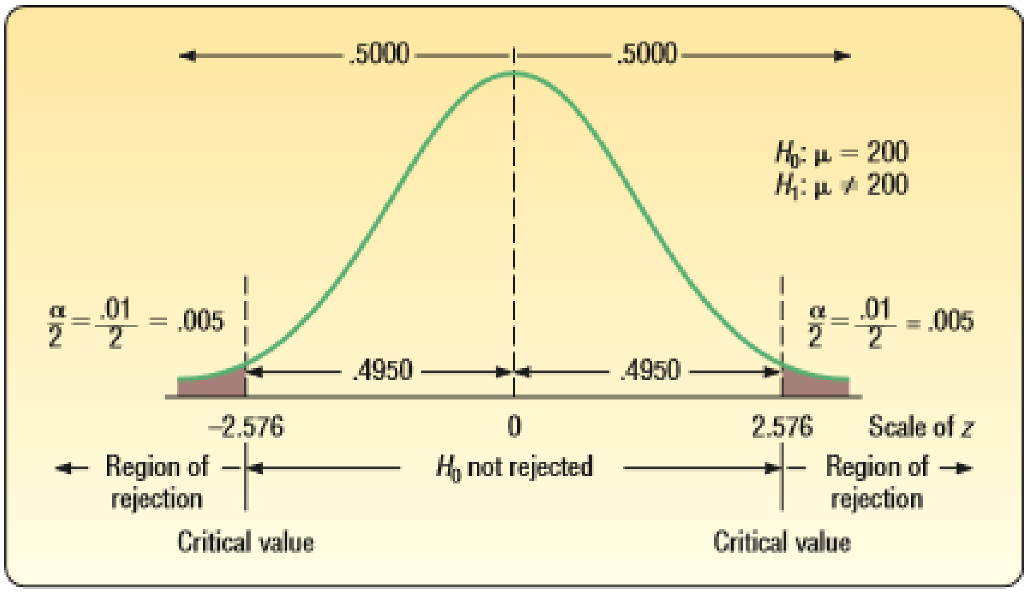
Step 2: Select the level of significance. Here $\alpha = .01$

Step 3: Select the test statistic. In this example, we'll use z



# Two-Tailed Test Example, σ Known

Step 4: Formulate the decision rule by first determining the critical values of z.




Decision Rule: If the computed value of z is not between -2.576 and 2.576, reject the null hypothesis. If z falls between -2.576 and 2.576, do not reject the null hypothesis.

# Two-Tailed Test Example, σ Known

Step 5: Take sample, compute the test statistic, make decision.

The mean number of desks produced last year (50 weeks because the plant was shut down 2 weeks for vacation) is 203.5. The standard deviation of the population is 16 desks per week. Compute z with formula 10-1.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{203.5 - 200}{16/\sqrt{50}} = 1.547$$

Decision: Because 1.547 does not fall in the rejection region, we decide not to reject $H_0$.

Step 6: Interpret the result.

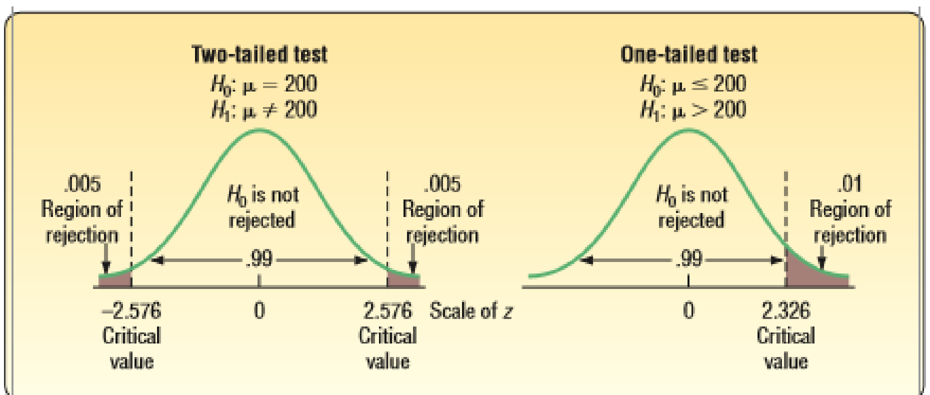
We did not reject the null hypothesis, so we have failed to show that the population mean has changed from 200 per week.

# One-Tailed Test

Suppose instead of wanting to know if there had been a change in the mean number of desks assembled, the vice president wanted to know if there had been an increase in the number of units assembled. Can we conclude, because of the improved production methods, that the mean number of desks assembled in the last 50 weeks was more than 200? Use $\alpha = .01$.

Before:
A two-tailed test
$H_0$: = 200 desks
$H_1$: ≠ 200 desks

Now:
A one-tailed test
$H_0$: ≤ 200 desks
$H_1$: > 200 desks



# The p-Value in Hypothesis Testing

p-VALUE The probability of observing a sample value as extreme as, or more extreme than, the value observed, given that the null hypothesis is true.

- Compare the p-value with the level of significance, $\alpha$
  - If the p-value is smaller than the significance level, reject $H_0$
  - If the p-value is larger than $\alpha$, $H_0$ is not rejected
- A p-value not only results in a decision about $H_0$, but gives additional insight about the strength of that decision

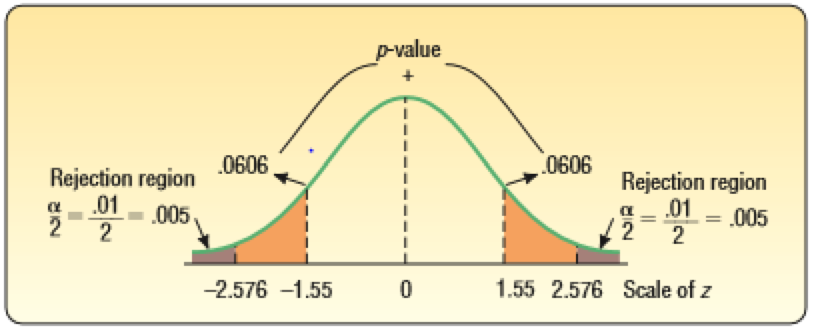
**INTERPRETING THE WEIGHT OF EVIDENCE AGAINST $H_0$**

If the *p*-value is less than

(a) .10, we have *some* evidence that $H_0$ is not true.
(b) .05, we have *strong* evidence that $H_0$ is not true.
(c) .01, we have *very strong* evidence that $H_0$ is not true.
(d) .001, we have *extremely strong* evidence that $H_0$ is not true.

# Finding a p-Value

- In the previous example about desk production, the computed z was 1.547 and $H_0$ was not rejected
- Round the computed z-value to two decimal places, 1.55
- Using the z-table, find the probability of finding a z-value of 1.55 or more by .5000 - .4394 = .0606
- Since this is a two-tailed test 2(.0606) = .1212
- In this chart, we can easily compare the p-value with the level of significance



# Testing for the Population Mean: Population Variance Known

- *Assumption*: A sample of size $n$ from a normal population with unknown mean but known variance.
- $H_0$: $\mu = \mu_0$ versus $H_1$: $\mu > \mu_0$, $\mu < \mu_0$, or $\mu \neq \mu_0$.
- Test Statistic: $z = \dfrac{(\bar{x} - \mu_0)}{\sigma / \sqrt{n}}$ .
- Rejection Region: $z > z_\alpha$, $z < -z_\alpha$, or $|z| > z_{\alpha/2}$.

# Example

- A sample of 36 observation is selected from a normal population. The sample mean is 49, and the population standard deviation is 5. conduct the following test of hypothesis using the 0.05 significant level.
  - $H_0: \mu = 50$
  - $H_1: \mu \neq 50$
- *(a) Is this a one- or two-tailed test ?*
- *(b) What is the decision rule ?*
- *(c) What is the value of the test statistics ?*
- *(d) What is your decision regarding $H_0$ ?*
- *(e) What is the p-value ? Interpret it.*

# Solution

1. a. Two-tailed, because the alternate hypothesis does not indicate a direction.
   b Reject $H_o$ when $z$ does not fall in the region from – 1.96 and 1.96
   c. –1.2, found by $z = \dfrac{49 - 50}{(5/\sqrt{36})}$
   d. Fail to reject $H_o$
   e. $p$ = 0.2302, found by 2(0.5000 – 0.3849). There is a 23.02% chance of finding a $z$ value this large by "sampling error" when $H_o$ is true.

# Example

- A sample of 36 observation is selected from a normal population. The sample mean is 12, and the population standard deviation is 3. conduct the following test of hypothesis using the 0.01 significant level.
  - $H_0: \mu = 10$
  - $H_1: \mu > 10$
- *(a) Is this a one- or two-tailed test ?*
- *(b) What is the decision rule ?*
- *(c) What is the value of the test statistics ?*
- *(d) What is your decision regarding $H_0$ ?*
- *(e) What is the p-value ? Interpret it.*

# Answer

2. a. One-tailed, because the alternate hypothesis indicates a greater than direction.
   b. Reject $H_o$ when $z > 2.326$
   c. 4, found by $z = \frac{12 - 10}{(3/\sqrt{36})}$
   d. Reject $H_o$ and conclude that $\mu > 10$
   e. The $p$-value is close to 0. So there is very little chance $H_o$ is true.

# Hypothesis Testing, σ Unknown

- When testing a hypothesis about a population mean

| | | |
|---|---|---|
| **TESTING A MEAN, σ UNKNOWN** | $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ | **[10–2]** |

- The major characteristics of the t distribution are
  - It is a continuous distribution
  - It is bell-shaped and symmetrical
  - There is a family of t distributions, depending on the number of degrees of freedom
  - As the number of degrees of freedom increases, the shape of the t distribution approaches that of the standard normal distribution
  - It is flatter than the standard normal distribution

# Testing for the Population Mean: Small Sample, Population Variance Unknown

- *Assumption*: A sample of size $n$ (small) from a normal population with unknown mean and unknown variance.
- $H_0$: $\mu = \mu_0$ versus $H_1$: $\mu > \mu_0$, $\mu < \mu_0$, or $\mu \neq \mu_0$.
- Test Statistic: $t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ .
- Rejection Region: $t > t_{\alpha}$, $t < -t_{\alpha}$, or $|t| > t_{\alpha/2}$;

  Degrees of freedom = $n$ - 1.

# Hypothesis Testing, σ Unknown Example

The McFarland Insurance Company Claims Department reports the mean cost to process a claim is $60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the cost-cutting measures, a random sample was taken of 26 claims processed last month and the cost to process each claim was recorded (see below).

At the .01 significance level, is it reasonable to conclude the mean cost to process a claim is now less than $60?

| | | | | | |
|---|---|---|---|---|---|
| $45 | $49 | $62 | $40 | $43 | $61 |
| 48 | 53 | 67 | 63 | 78 | 64 |
| 48 | 54 | 51 | 56 | 63 | 69 |
| 58 | 51 | 58 | 59 | 56 | 57 |
| 38 | 76 | | | | |

Step 1: State the null hypothesis and the alternate hypothesis

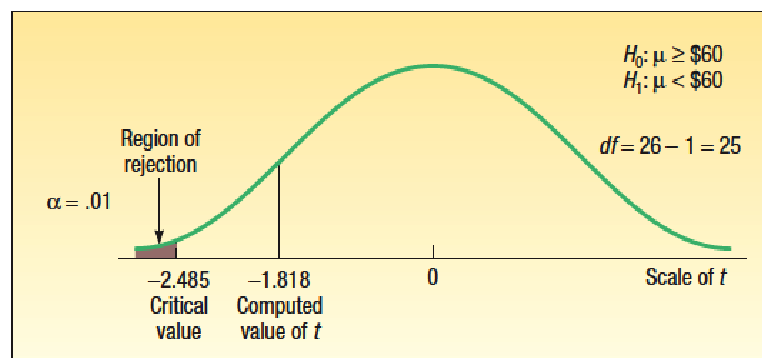
$$H_0: \mu = \$60$$
$$H_1: \mu < \$60$$

# Hypothesis Testing, σ Unknown Example

Step 2: Select the level of significance; we will use .01
Step 3: Select the test statistic; we will use t
Step 4: Formulate the decision rule; reject $H_0$ if t is less than -2.485
Step 5: Take sample, make decision; Do not reject $H_0$
Step 6: Interpret the result; The test results do not allow the claims manager to conclude the cost-cutting measures have been effective.

| | Confidence Intervals | | | | | |
|---|---|---|---|---|---|---|
| | 80% | 90% | 95% | 98% | 99% | 99.9% |
| | Level of Significance for One-Tailed Test, α | | | | | |
| df | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
| | Level of Significance for Two-Tailed Test, α | | | | | |
| | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{\$56.423 - \$60}{\$10.41/\sqrt{26}} = -1.82$$



# Exercise

- Given the following hypotheses :

$$H_0: \mu = 10$$
$$H_1: \mu > 10$$

A random sample of 10 observations is selected from a normal distribution. The sample mean was 12 and the sample standard deviation was 3. Using the 0.05 significance level :

a) State the decision rule
b) Compute the value of the test statistics
c) What is your decision regarding the null hypothesis ?

# Answer

a. Reject $H_o$ when $t > 1.833$

b. $$t = \frac{12 - 10}{(3/\sqrt{10})} = 2.108$$

c. Reject $H_o$. The mean is greater than 10.

# Exercise

- Given the following hypotheses :

$$H_0: \mu = 400$$
$$H_1: \mu \neq 400$$

A random sample of 12 observations is selected from a normal distribution. The sample mean was 407 and the sample standard deviation was 6. Using the 0.01 significance level :

a) State the decision rule
b) Compute the value of the test statistics
c) What is your decision regarding the null hypothesis ?

# Answer

a. Reject $H_o$ if $t < -3.106$ or $t > 3.106$

b. $$t = \frac{407 - 400}{(6/\sqrt{12})} = 4.041$$

c. Reject $H_o$, the mean does not equal 400