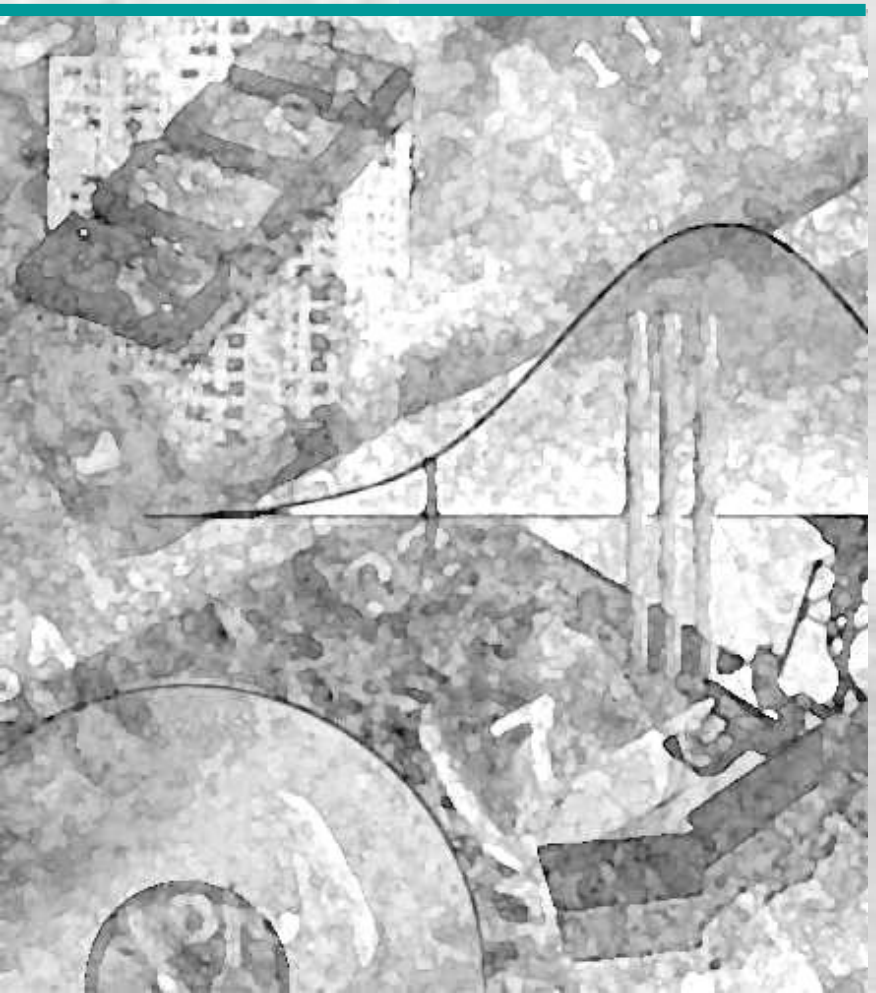
Chapter 11



Time Series And Business Forecasting

11.0 Time Series And Business Forecasting

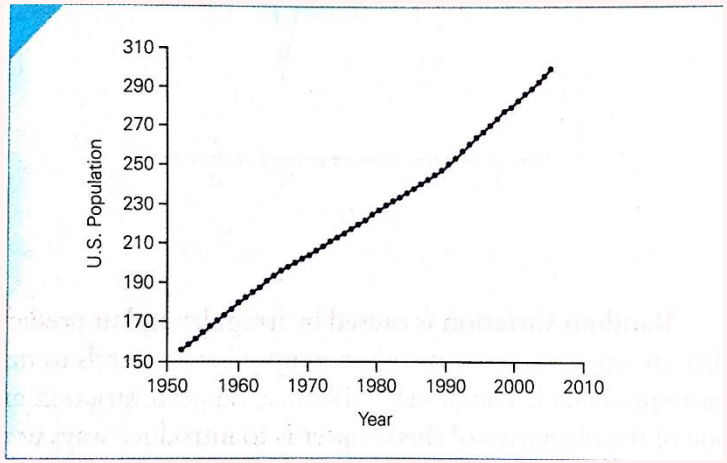
- Time series.
  - Any variable that is measured over time in sequential order.
- Forecasting.
  - To analyze time series in order to detect patterns that will enable us to forecast **future values** of the time series
- Application example
  - University often try to forecast the number of students who will be applying for their institution.
  - Many company attempt to predicts the demand for their products and their share of the market.

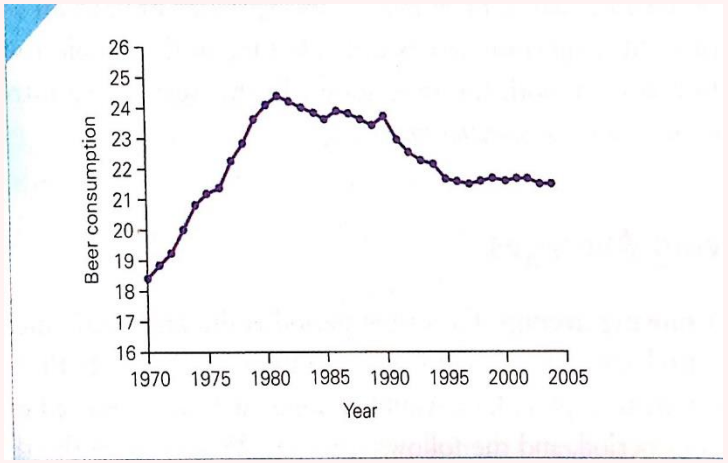
11.1 Time Series Data

- As mentioned previously, time series involved data over a period of time.
- Forecasting using time series data is a common practice among managers and government decision makers.
- Time series component:
  - Trend
  - Cyclical
  - Seasonal
  - Irregular

11.1 Time Series Data

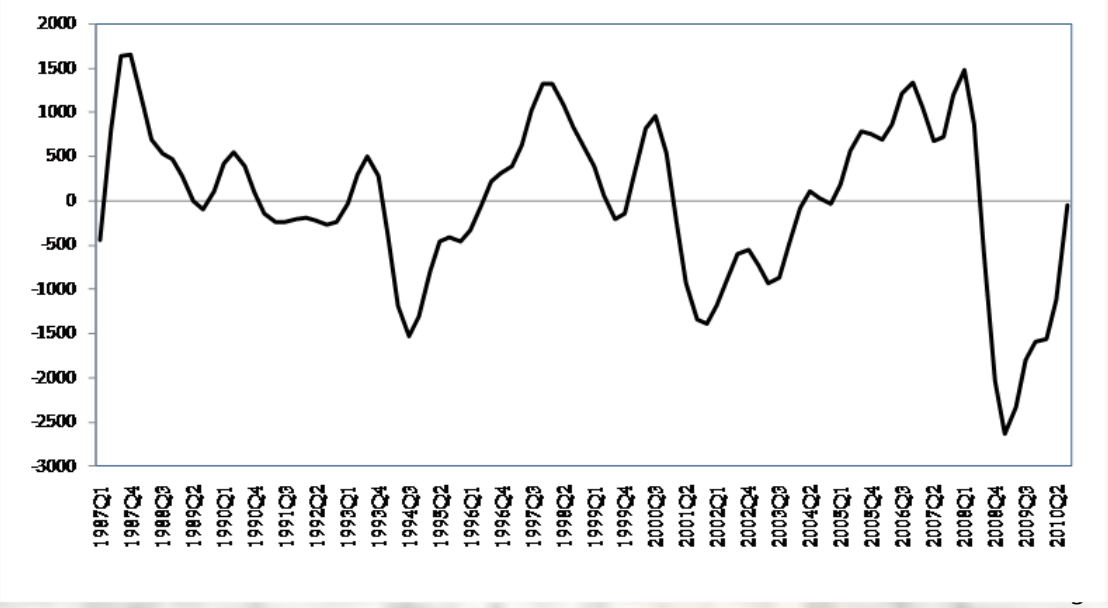
- Trend
  - A.k.a secular trend.
  - Long term smooth pattern/direction exhibited by a series.
  - Have a duration of more than one year.
  - Not always linear





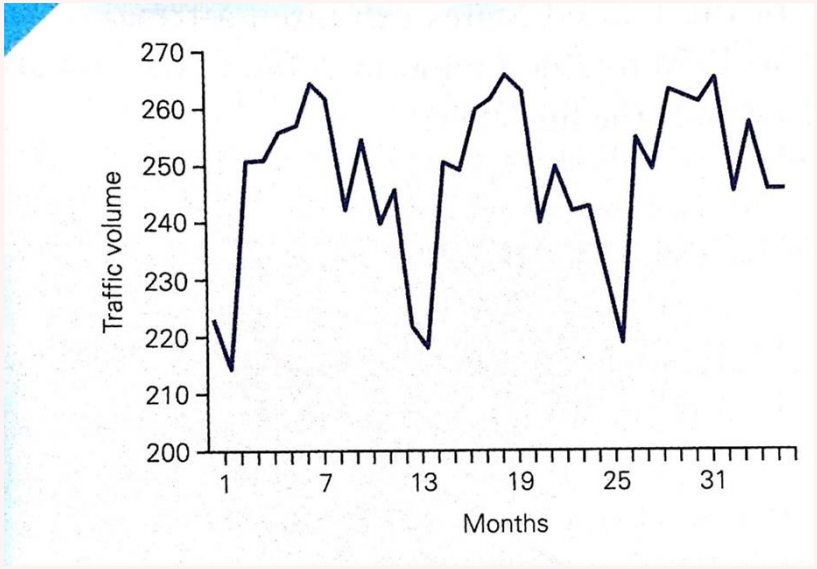
11.1 Time Series Data

- Cyclical
  - Wavelike pattern describing long term-trend.
  - Have a duration over a number of years.
  - Resulting in a cyclical effect.
  - Hard to make prediction.



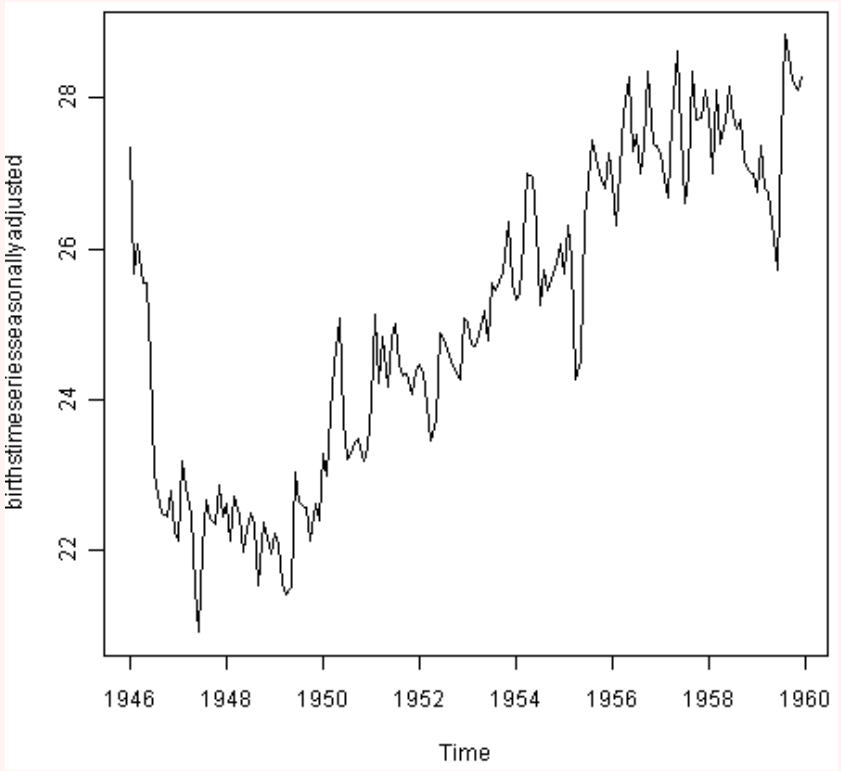
11.1 Time Series Data

- Seasonal.
  - Cycles that occur over short repetitive periods, which is less than one year.
  - E.g: Systematic pattern that occur during a month.



11.1 Time Series Data

- Irregular.
  - Causes by unpredictable changes in a time series that are not cause by any other components.
  - Exist in almost all time series.
  - This component need to be reduce in order to describe and measure other components – to make accurate predictions.



11.1 Time Series Data

- The common usage of time series data is to conduct forecasting.
- There are several forecasting method available. The simplest method known existed for forecasting is naïve method.
- Naïve method provide a benchmark against which more sophisticated models by omitting time series component.
- The assumption of this method is what happen yesterday will also happen today.
- It means that this forecast produce forecast that are equal to the last observed value.
- $F_{t+1} = D_t$; Where $F_{t+1} =$ Forecast value and $D_t =$ Observed value

11.1 Time Series Data

- Example:

An operator of five independent gas stations recorded the quarterly fuel sale (in thousand litre) for the past 4 years. Forecast the fuel sale for period 2 through period 16 using naïve method.

Time period	Year	Quarter	Fuel sale (in thousand litre)
1	1	1	39
2		2	37
3		3	61
4		4	58
5	2	1	18
6		2	56
7		3	82
8		4	27
9	3	1	41
10		2	69
11		3	49
12		4	66
13	4	1	54

11.1 Time Series Data

- Solution

Time period	Fuel sale (D)	Naïve method (F)
1	39	-
2	37	39
3	61	37
4	58	61
5	18	58
6	56	18
7	82	56
8	27	82
9	41	27
10	69	41
11	49	69
12	66	49
13	54	66
14	42	54
15	90	42
16	66	90

$$F_2 = D_1 = 39$$

11.1 Time Series Data

- However, if the existence of time series component can be determined/considered, better forecast can be made.
- Unfortunately, the existence of irregular component often make the task of identifying other components difficult.
- To ease the task, one of the simplest way is by smoothing time series data.
- There are two main methods of smoothing techniques.
  - Moving averages
    - Simple moving Average Model
    - Weighted Moving Average
  - Exponential smoothing

11.2 Simple moving Average

- A moving average for a time period is the mean of the values in that time period and those close to it.
- Common practice of simple moving average are 3 and 5 moving averages.
- 3-moving average

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2}}{3}$$

- 5-moving average

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + D_{t-3} + D_{t-4}}{5}$$

- Example
  - An operator of five independent gas stations recorded the quarterly fuel sale (in thousand litre) for the past 4 years. Calculate the three-moving averages and five-moving averages.

Time period	Year	Quarter	Fuel sale (in thousand litre)
1	1	1	39
2		2	37
3		3	61
4		4	58
5	2	1	18
6		2	56
7		3	82
8		4	27
9	3	1	41
10		2	69
11		3	49
12		4	66
13	4	1	54
14		2	42
15		3	90
16		4	66

11.2 Simple moving Average

- Solution

Time period	Fuel sale (D)	Three-Moving averages (F)	Five-Moving averages (F)
1	39	-	-
2	37		-
3	61		-
4	58	45.7	-
5	18	52.0	-
6	56	45.7	42.6
7	82	44.0	46
8	27	52.0	55
9	41	55.0	48.2
10	69	50.0	44.8
11	49	45.7	55
12	66	53.0	53.6
13	54	61.3	50.4
14	42	56.3	55.8
15	90	54.0	56
16	66	62.0	60.2

3-Moving Average

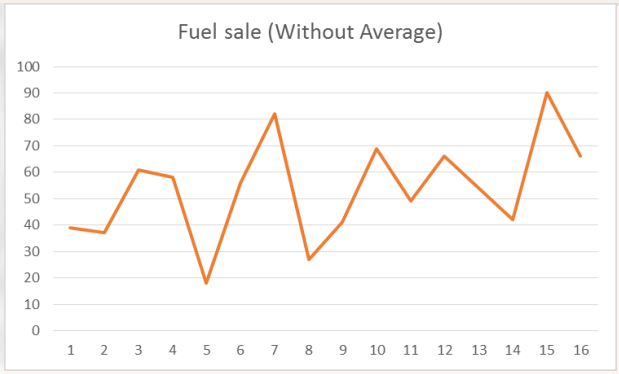
$$F_4 = \frac{61+37+39}{3} = 45.7$$

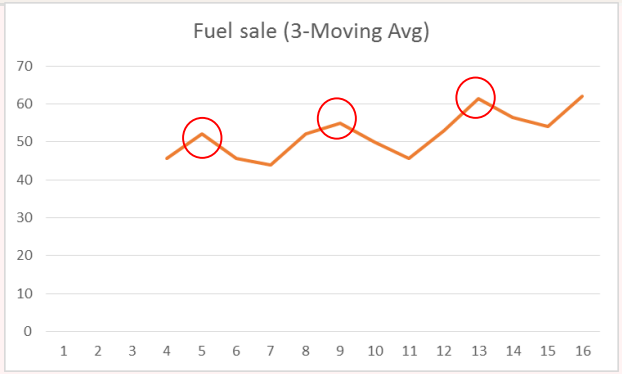
$$F_5 = \frac{58+61+37}{3} = 52.0$$

5-Moving Average

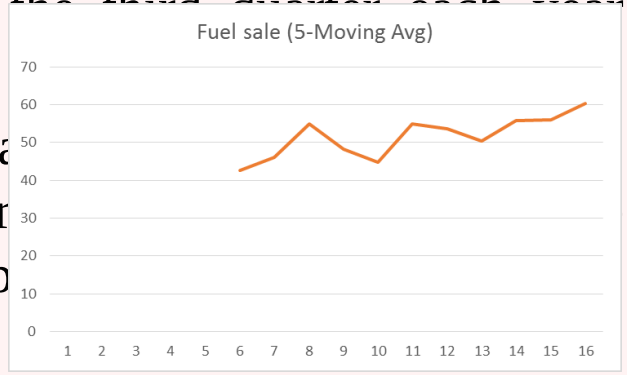
$$F_{16} = \frac{90+42+54+66+49}{5} = 60.2$$

11.1 Time Series Data





- Notice that the line of time series has been smooth out to reduce irregularity
- Without any smoothing techniques, it is hard to determine any component. (Fig. 1)
- Notice Fig. 2, seasonal pattern can be detected at the third quarter each year (periods 5, 9, 13). We can also see a long term upward
- As for Fig. 3 that use 5-moving averages, its line beca
  However, as we can see, for this case, 3-moving aver
  has become too smooth (seasonal pattern no longer app



11.3 Weighted Moving Average

- Similar to simple moving average with small difference.
- Simple moving averages place an equal weight on all of previous time period.
- A weighted moving average allows us to put more weight on the more recent data (total weight should equal to one).
- 3-moving average

$$F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2}$$

- 5-moving average

$$F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2} + w_4 D_{t-3} + w_5 D_{t-4}$$

- Example:
  - Compute the time series forecast for Period 4 through Period 16 using a 3-month weighted moving average. Use weights of 0.5, 0.33, and 0.17, with the heavier weights on the more recent months.

11.3 Weighted Moving Average

- Solution

Time period	Fuel sale	Three-Weighted moving averages
1	39	-
2	37	-
3	61	-
4	58	49.34
5	18	55.42
6	56	38.51
7	82	43.8
8	27	62.54
9	41	50.08
10	69	43.35
11	49	52.62
12	66	54.24
13	54	60.9
14	42	57.11
15	90	50.04
16	66	68.04

$F_4 = (0.15)39 + (0.33)37 + (0.5)61 = 49.34$

$F_5 = (0.15)37 + (0.33)61 + (0.5)58 = 55.42$

11.3 Weighted Moving Average

- Two drawbacks of moving average:
  - There will be no forecasting value for the early time period.
  - Moving average “forgets” most of the previous time series.
- Therefore, both of this problems are addressed by exponential smoothing.

11.4 Exponential Smoothing

- Exponential smoothing is a technique that can be applied to produce smoothed data. It able to provide forecasting value for each time period without loosing any previous data.
- The general formula for this method is:

$$F_{t+1} = F_t + \alpha(D_t - F_t)$$

- For example, if we want to forecast for time period 7 (F_{6+1}), the formula for exponential smoothing should be:

$$F_{6+1} = F_6 + \alpha(D_6 - F_6)$$

11.4 Exponential Smoothing

- Using our previous example of fuel sale, Use exponential smoothing with smoothing parameter of 0.3 and 0.5 to compute the forecast for each time period.

Time period	Year	Quarter	Fuel sale (in thousand litre)
1	1	1	39
2		2	37
3		3	61
4		4	58
5	2	1	18
6		2	56
7		3	82
8		4	27
9	3	1	41
10		2	69
11		3	49
12		4	66
13	4	1	54
14		2	42
15		3	90
16		4	66

11.4 Exponential Smoothing

luti

Time period	Fuel sale (D)	α = 0.3 (F)	α = 0.5 (F)
1	39	39.0	39
2	37	39.0	39.0
3	61	38.4	38.0
4	58	45.2	49.5
5	18	49.0	53.8
6	56	39.7	35.9
7	82	44.6	45.9
8	27	55.8	64.0
9	41	47.2	45.5
10	69	45.3	43.2
11	49	52.4	56.1
12	66	51.4	52.6
13	54	55.8	59.3
14	42	55.2	56.6
15	90	51.3	49.3
16	66	62.9	69.7

$$F_2 = F_1 + 0.3(D_1 - F_1)$$
$$= 39 + 0.3(39 - 39) = 39$$

$$F_3 = F_2 + 0.3(D_2 - F_2)$$
$$= 39 + 0.3(37 - 39) = 38.4$$

997 and 2000

11.4 Exponential Smoothing

- Now that we have several forecasting methods. It is best to compare which one represent the best method to be used in forecasting our data.
- To do that, one simple method commonly used is Mean Absolute Deviation (MAD).
- The formula of MAD is:

$$MAD = \frac{\sum |E_t|}{n}$$

$$\text{Where } E_t = D_t - F_t$$

- Method that obtain the smallest MAD can be considered as the best forecasting model for our data.

11.4 Exponential Smoothing

- Example:

By using previous forecast of moving average and exponential smoothing, determine the best forecasting model using MAD (Compare period 6 through 16).

- Solution

Time period	Fuel sale	Three-Moving averages	Three-Weighted moving averages	Exponential smoothing	
				$\alpha = 0.3$	$\alpha = 0.5$
6	56	|56-45.7|=10.3	|56-38.5|=17.5	|56-39.7|=16.3	|56-35.9|=20.1
7	82	|82-44|=38.0	|82-43.8|=38.2	|82-44.6|=37.4	|82-45.9|=36.1
8	27	|27-52|=25.0	|27-62.5|=35.5	|27-55.8|=28.8	|27.64|=37.0
9	41	|41-55|=14.0	|41-50.1|=9.1	|41-47.2|=6.2	|41-45.5|=4.5
10	69	|69-50|=19.0	|69-43.4|=25.7	|69-45.3|=23.7	|69-43.2|=25.8
11	49	|49-45.7|=3.3	|49-52.6|=3.6	|49-52.4|=3.4	|49-56.1|=7.1
12	66	|66-53|=13.0	|66-54.2|=11.8	|66-51.4|=14.6	|66-52.6|=13.4
13	54	|54-61.3|=7.3	|54-60.9|=6.9	|54-55.8|=1.8	|54-59.3|=5.3
14	42	|42-56.3|=14.3	|42-57.1|=15.1	|42-55.2|=13.2	|42-56-6|=14.6
15	90	|90-54|=36.0	|90-50|=40.0	|90-51.3|=38.7	|90-49.3|=40.7
16	66	|66-62|=4.0	|66-68|=2.0	|66-62.9|=3.1	|66-69.7|=3.7
	MAD	**16.8**	**16.8**	**18.7**	**17**

- Hence, the best forecasting model for our example is Three-Moving averages / Three-Weighted moving averages.

11.4 Exponential Smoothing

- Exercise:

The following quarterly sales (in RM thousand) of a department store chain were recorded for the years 1997-2000:

	Year			
Quarter	2001	2002	2003	2004
1	18	33	25	41
2	22	20	36	33
3	27	38	44	52
4	31	26	29	45

1. Compute the sales forecast using the naive method.
2. Compute the sales forecast using a 3- moving average.

u Compute the sales forecast using a 5- moving average.

u Compute the sales forecast using a 3- weighted moving average. Use weights of 0.5, 0.3, and 0.2, with the heavier weights on the more recent periods.

u Use exponential smoothing with smoothing parameter of 0.3 and 0.5 to compute the sales forecast.

1. Compute the mean absolute deviation for the third and fourth quarter for each of the methods used. Which method would you use to forecast the sales?