

# | Chapter 10 |

## *Non-Parametric Statistics*

### **10.1 The Chi-Square Tests:**

**10.1.1 Goodness-of-fit Test**

**10.1.2 Independence Test**

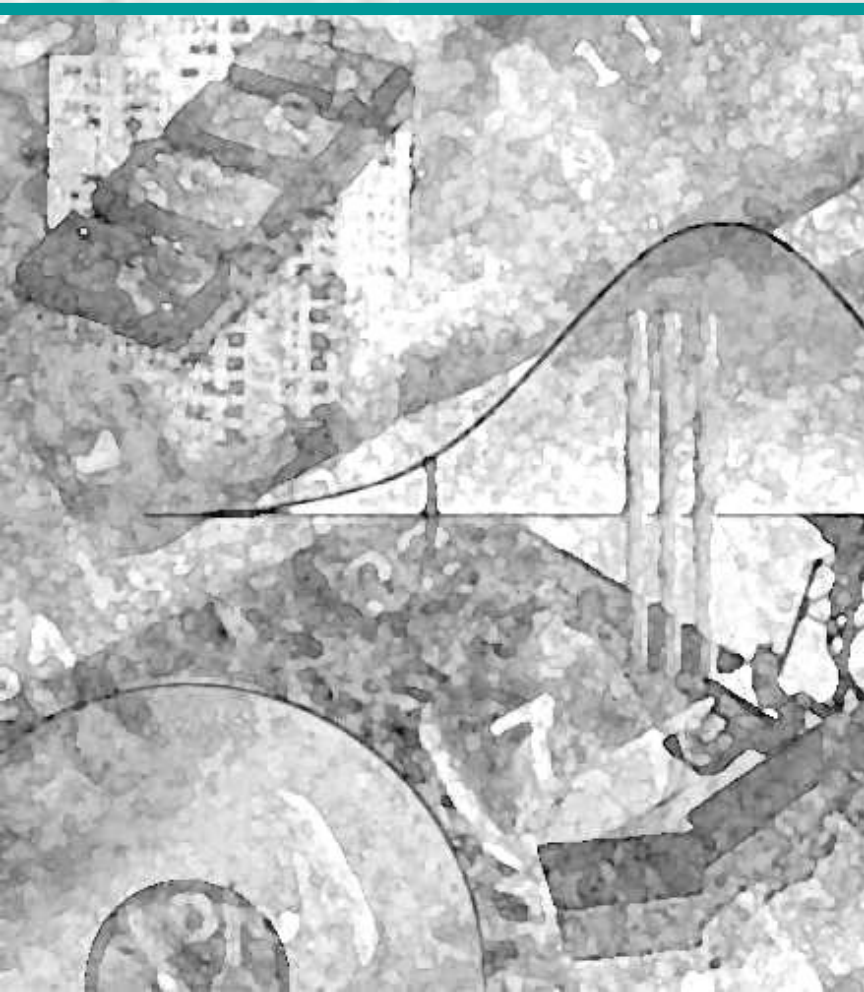
**10.1.3 Homogeneity Test**

### **10.2 Nonparametric Statistics:**

**10.2.1 Sign Test**

**10.2.2 Mann-Whitney Test**

**10.2.3 Kruskal Wallis Test**



# 10.0 Non-Parametric Statistics

- ◆ When assumptions of parametric tests were violated, nonparametric techniques can be used.
- ◆ These tests are less powerful than their parametric counterparts.

# 10.1 The Chi-Square Tests

- ◆ This subchapter develop two statistical techniques that involve nominal data.
- ◆ Chi-square distribution will be used in carrying out hypothesis to analyze whether:
  - i. A sample could have come from a given type of POPULATION DISTRIBUTION.
  - ii. Two nominal variable/categorical variable could be INDEPENDENT and HOMOGENEOUS of each other.
- ◆ There are two main types of chi-square test:
  - i. Goodness-of-fit Test (To analyze single categorical variable)
  - ii. The Chi-square Test For Homogeneity and Independence (To analyze the relationship between two categorical variables)

## 10.1 The Chi-Square Tests

- ◆ Assumption testing for nonparametric techniques is not as critical as for parametric methods. However, a number of generic assumptions apply.
- ◆ There are three assumptions you need to address before conducting chi-square tests:
  - Random sampling – observation should be randomly sampled from the population.
  - Independence of observations - observation should be generated by a different subject and no subject should be counted twice.
  - Size of expected frequencies – when the number of cells is less than 10 and has small sample size, the lowest expected frequency required is five. However, the observed frequency can be any value.

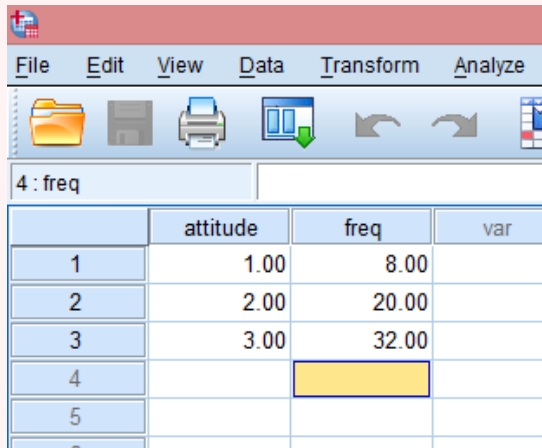
## 10.1.1 Goodness-of-fit Test

- ◆ The following table outlines the attitudes of 60 people towards US military bases in Australia. Test whether there are no differences between the attitude of Australian towards US military bases in Australia.

Attitude towards US military bases in Australia	Frequency of response
In favour	8
Against	20
Undecided	32

# 10.1.1 Goodness-of-fit Test

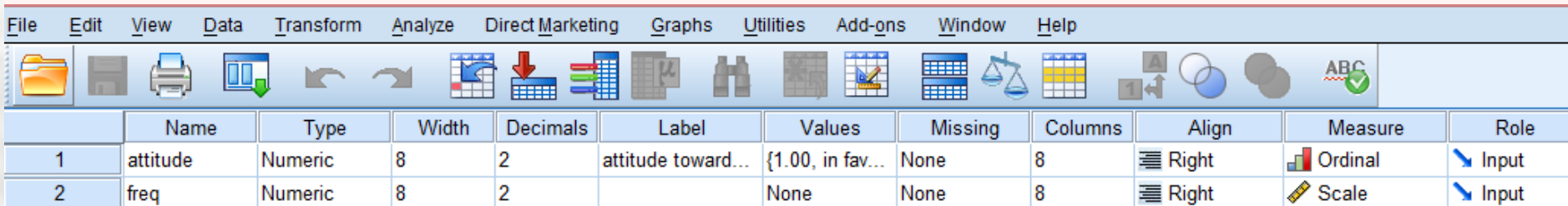
## ◆ In SPSS (Data view)



4 : freq

	attitude	freq	var
1	1.00	8.00	
2	2.00	20.00	
3	3.00	32.00	
4			
5			
6			

## ◆ In SPSS (Variable view)

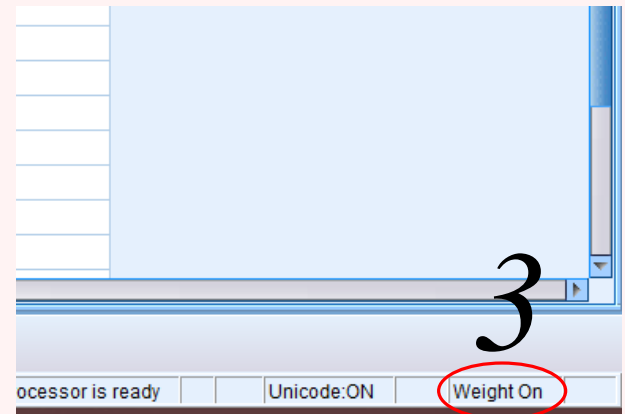
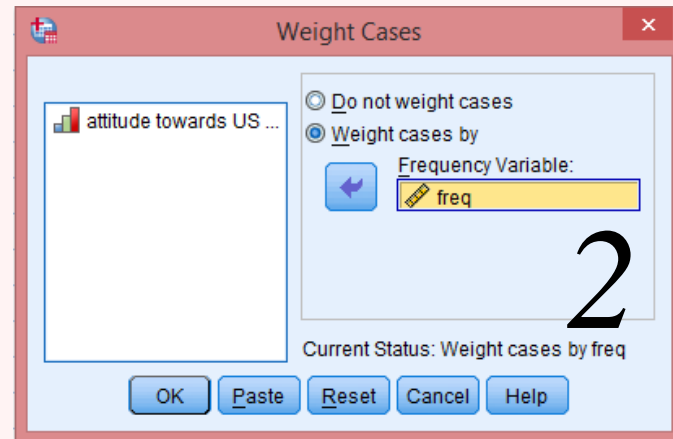
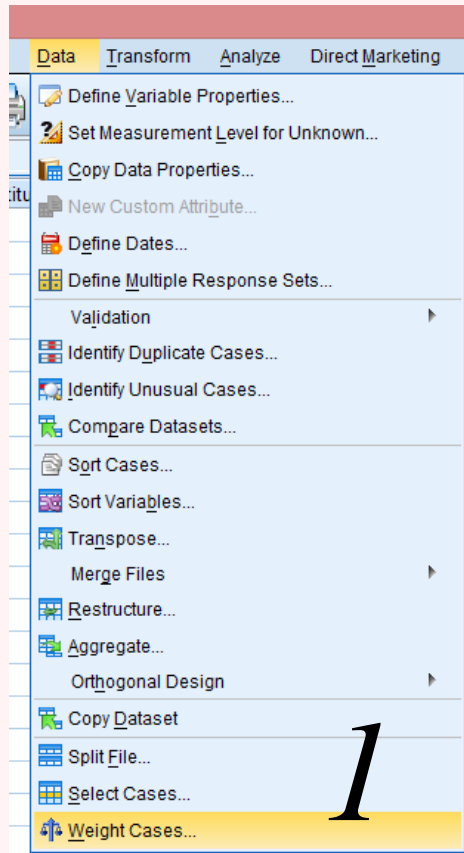


	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure	Role
1	attitude	Numeric	8	2	attitude toward...	{1.00, in fav...	None	8	Right	Ordinal	Input
2	freq	Numeric	8	2		None	None	8	Right	Scale	Input



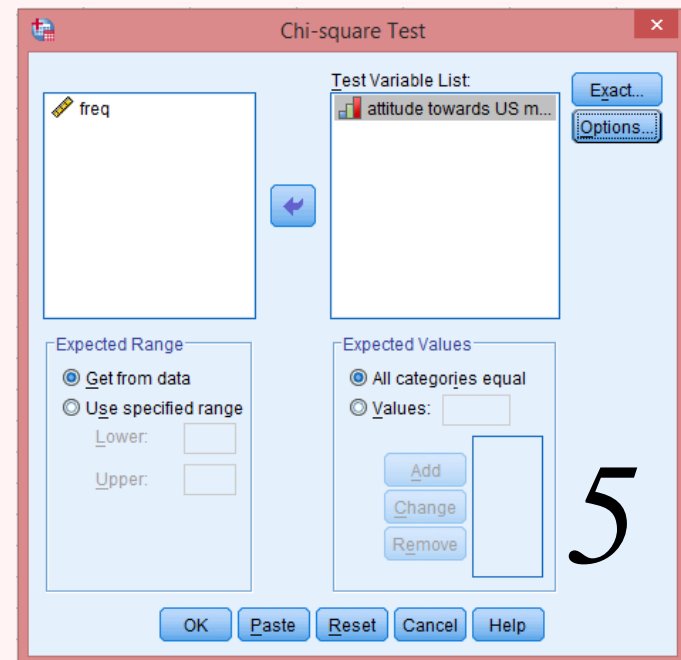
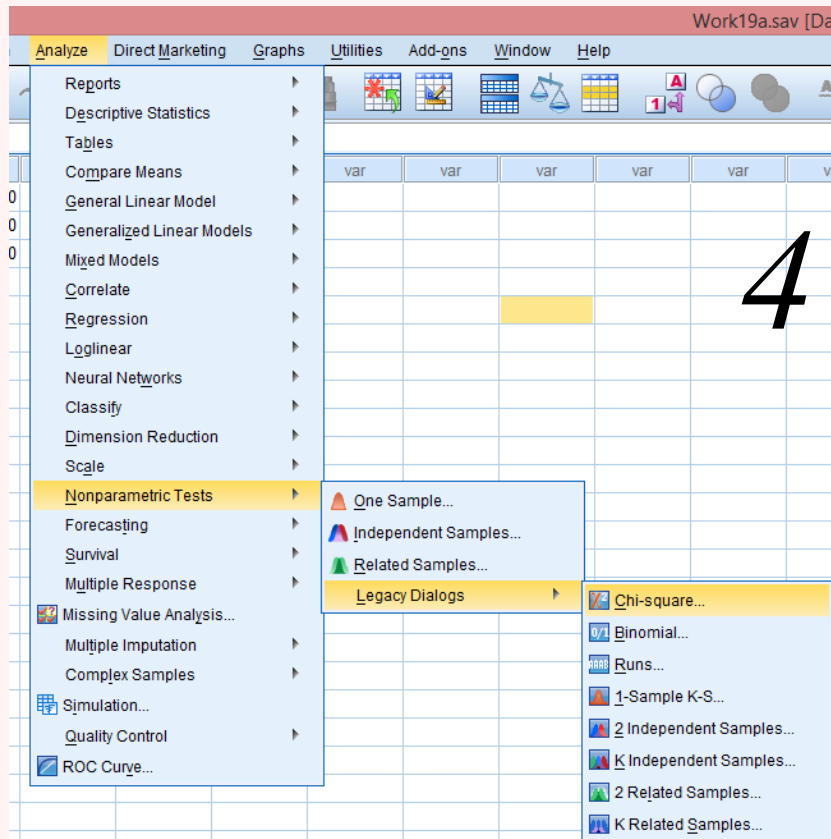
# 10.1.1 Goodness-of-fit Test

## ◆ Analysis (for the case of even expected frequency)



# 10.1.1 Goodness-of-fit Test

## ◆ Analysis (for the case of even expected frequency)





# 10.1.1 Goodness-of-fit Test

- ◆ Output (for the case of even expected frequency)

## Chi-Square Test

### Frequencies

attitude towards US military bases

	Observed N	Expected N	Residual
in favour	8	20.0	-12.0
against	20	20.0	.0
undecided	32	20.0	12.0
Total	60		

### Test Statistics

	attitude towards US military bases
Chi-Square	14.400 <sup>a</sup>
df	2
Asymp. Sig.	.001

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 20.0.

Assumption  
no. 3

$H_0$  : There are no differences in the frequency of attitudes towards military bases in Australia.

$H_1$  : There are significant differences in the frequency of attitudes towards military bases in Australia.

$$P\text{-value} = 0.001 < \alpha = 0.05$$

Reject  $H_0$

There are significant differences in the frequency of attitudes towards military bases in Australia.

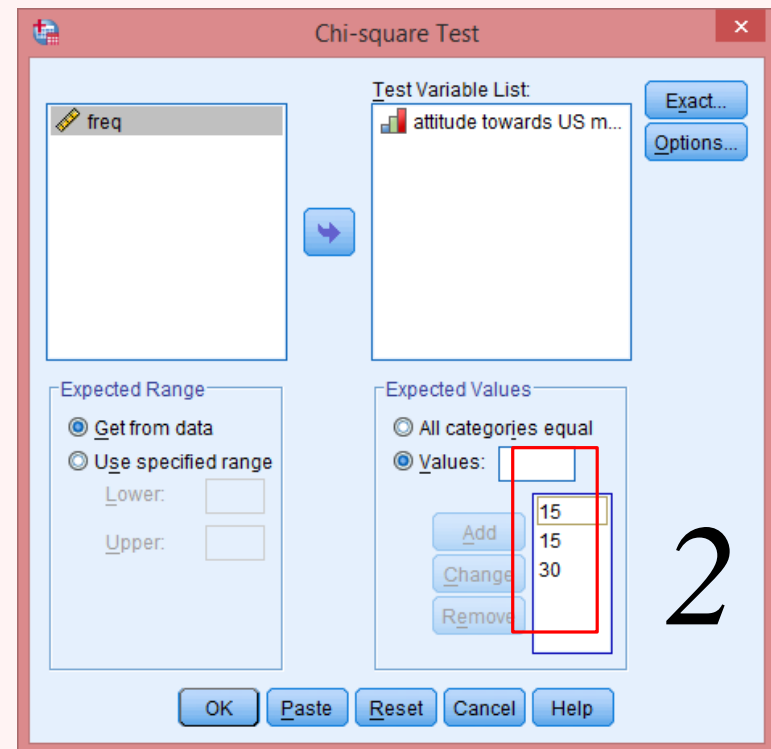
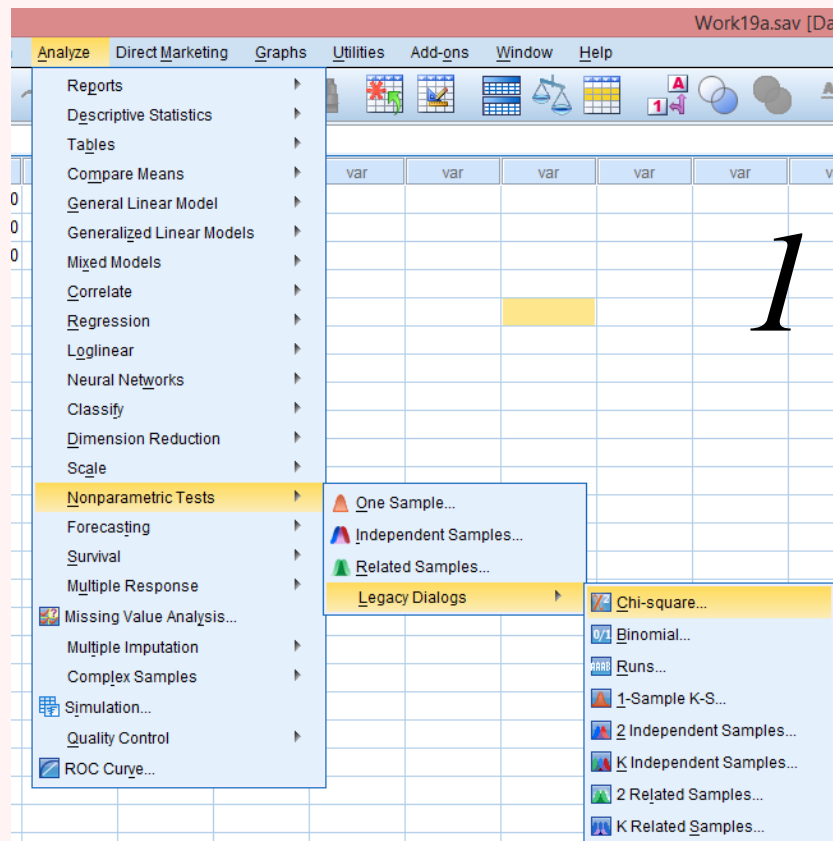
## 10.1.1 Goodness-of-fit Test

- ◆ Sometimes the expected frequencies are not evenly balanced. Across categories.
- ◆ For example, the following table outlines the attitudes of 60 people towards US military bases in Australia with their respected expected frequency of response. Test whether there is a different in the expected frequency and observed frequency

Attitude towards US military bases in Australia	Frequency of response	Expected frequency of response
In favour	8	15
Against	20	15
Undecided	32	30

# 10.1.1 Goodness-of-fit Test

## ◆ Analysis (for the case of uneven expected frequency)



# 10.1.1 Goodness-of-fit Test

## ◆ Output (for the case of uneven expected frequency)

### Chi-Square Test

#### Frequencies

##### attitude towards US military bases

	Observed N	Expected N	Residual
in favour	8	15.0	-7.0
against	20	15.0	5.0
undecided	32	30.0	2.0
Total	60		

#### Test Statistics

	attitude towards US military bases
Chi-Square	5.067 <sup>a</sup>
df	2
Asymp. Sig.	.079

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 15.0.

$H_0$  : There are no differences between the observed and expected frequency of attitudes towards military bases in Australia.

$H_1$  : There are differences between the observed and expected frequency of attitudes towards military bases in Australia for at least one attitude.

$P\text{-value} = 0.079 > \alpha = 0.05$

Fail to reject  $H_0$

There are no differences between the observed and expected frequency of attitudes towards military bases in Australia.

## 10.1.1 Goodness-of-fit Test

### ◆ Exercise

A coffee company is about to launch a range of speciality tea and would like to know whether some varieties are likely to sell better than others. Four of its new varieties are placed on a supermarket shelf for one week. The number of boxes sold for each variety is as follows:

Speciality tea	Frequency
Black tea	40
Green tea	20
Oolong tea	30
White tea	10

Determine whether the distribution of sales suggests that some varieties of tea are more popular than others.



# 10.1.1 Goodness-of-fit Test

**Frequency**

	Observed N	Expected N	Residual
10.00	10	36.7	-26.7
20.00	20	36.7	-16.7
40.00	80	36.7	43.3
Total	110		

**Test Statistics**

	Frequency
Chi-Square	78.182 <sup>a</sup>
df	2
Asymp. Sig.	.000

a. 0 cells (0.0%)  
have expected  
frequencies less  
than 5. The  
minimum  
expected cell  
frequency is 36.7.



## 10.1.2 Independence Test

- ◆ The arrangement of  $H_0$  and  $H_1$  are fixed and should not be arranged upside down.
- ◆ Hypothesis statement for Independence test:  
 $H_0$  : Row and column variable are independent  
 $H_1$  : Row and column variable are not independent



*Need to be changed according to question*

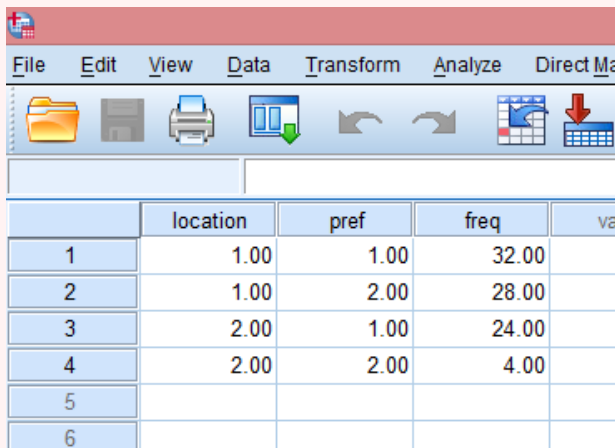
## 10.1.2 Independence Test

- ◆ A magazine publisher wishes to determine whether preference for certain **publications** **DEPENDS** on the **geographic location** of the reader. In a survey, the distribution of those preferring *The Australian Financial review* and *Newsweek* are as follows:

	The Australian Financial Review	Newsweek
Urban	32	28
Rural	24	4

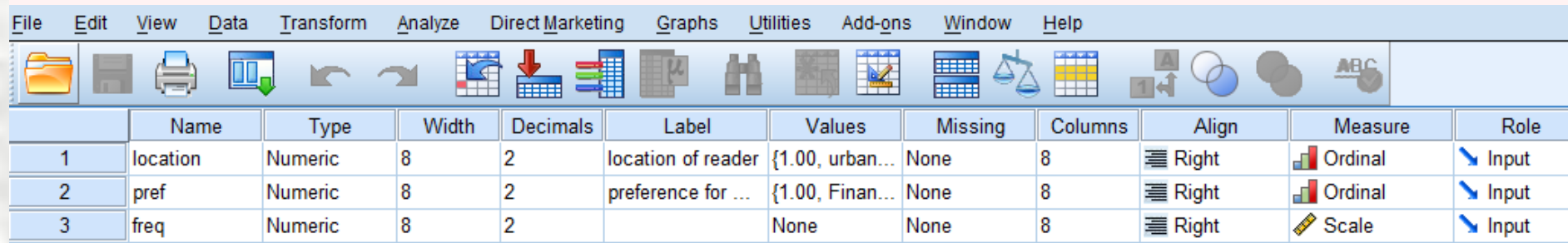
## 10.1.2 Independence Test

### ◆ In SPSS (Data view)



	location	pref	freq	va
1	1.00	1.00	32.00	
2	1.00	2.00	28.00	
3	2.00	1.00	24.00	
4	2.00	2.00	4.00	
5				
6				

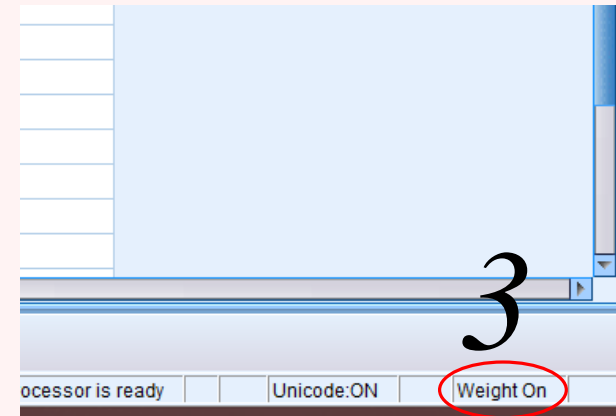
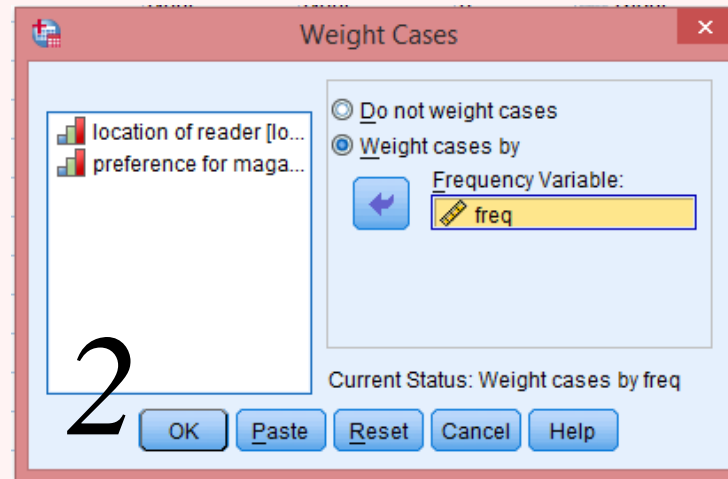
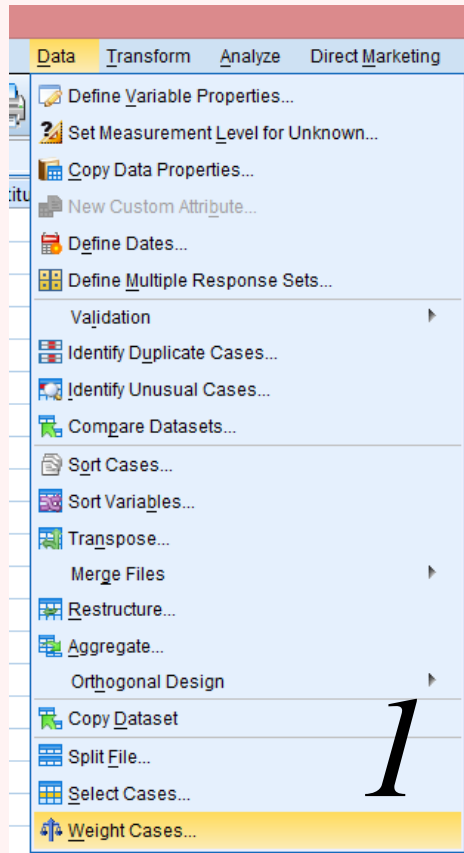
### ◆ In SPSS (Variable view)



	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure	Role
1	location	Numeric	8	2	location of reader	{1.00, urban...	None	8	Right	Ordinal	Input
2	pref	Numeric	8	2	preference for ...	{1.00, Finan...	None	8	Right	Ordinal	Input
3	freq	Numeric	8	2		None	None	8	Right	Scale	Input

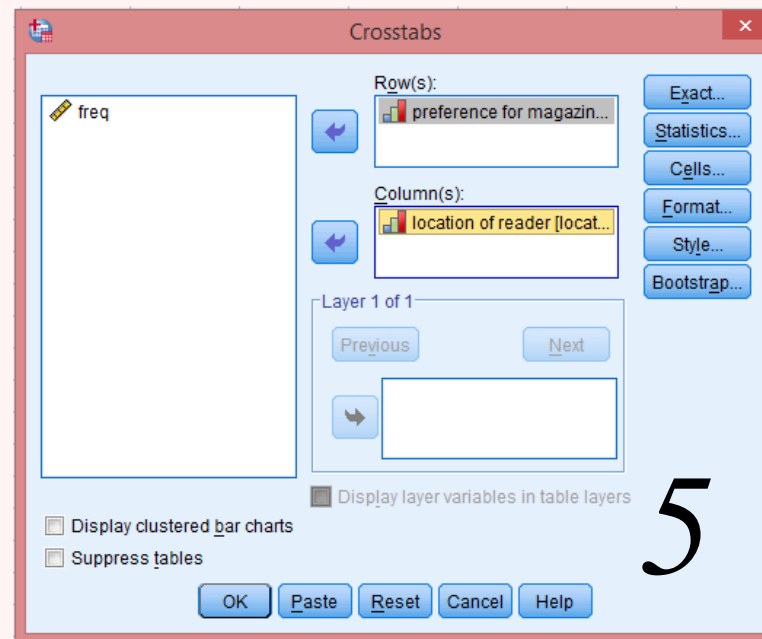
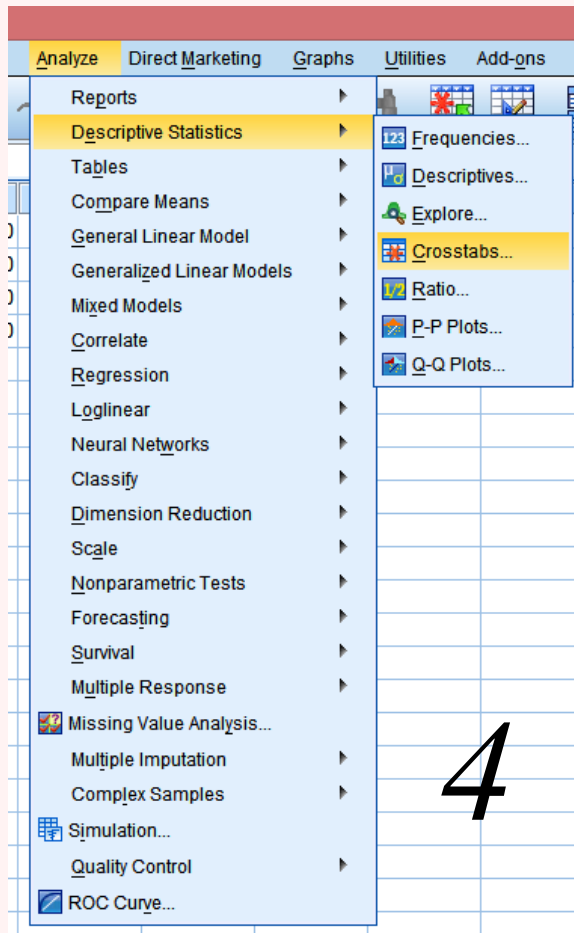
## 10.1.2 Independence Test

### ◆ Analysis



# 10.1.2 Independence Test

## ◆ Analysis



## 10.1.2 Independence Test

### ◆ Analysis

**Crosstabs: Statistics**

☒ Chi-square ☐ Correlations

**Nominal**

☐ Contingency coefficient  
☐ Phi and Cramer's V  
☐ Lambda  
☐ Uncertainty coefficient

**Ordinal**

☐ Gamma  
☐ Somers' d  
☐ Kendall's tau-b  
☐ Kendall's tau-c

**Nominal by Interval**

☐ Eta

☐ Kappa  
☐ Risk  
☐ McNemar

☐ Cochran's and Mantel-Haenszel statistics  
Test common odds ratio equals: 1

Continue Cancel Help

6

**Crosstabs: Cell Display**

**Counts**

☒ Observed  
☒ Expected  
☐ Hide small counts  
Less than 5

**z-test**

☐ Compare column proportions  
☐ Adjust p-values (Bonferroni method)

**Percentages**

☒ Row  
☒ Column  
☒ Total

**Residuals**

☐ Unstandardized  
☐ Standardized  
☐ Adjusted standardized

**Noninteger Weights**

☒ Round cell counts ☐ Round case weights  
☐ Truncate cell counts ☐ Truncate case weights  
☐ No adjustments

Continue Cancel Help

7



# 10.1.2 Independence Test

## ◆ Output

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
preference for magazine * location of reader	88	100.0%	0	0.0%	88	100.0%

preference for magazine \* location of reader Crosstabulation

			location of reader		Total	
			urban	rural		
preference for magazine	Financial Review	Count	32	24	56	
		Expected Count	38.2	17.8	56.0	
		% within preference for magazine	57.1%	42.9%	100.0%	
		% within location of reader	53.3%	85.7%	63.6%	
		% of Total	36.4%	27.3%	63.6%	
	Newsweek	Count	28	4	32	
		Expected Count	21.8	10.2	32.0	
		% within preference for magazine	87.5%	12.5%	100.0%	
		% within location of reader	46.7%	14.3%	36.4%	
		% of Total	31.8%	4.5%	36.4%	
Total		Count	60	28	88	
		Expected Count	60.0	28.0	88.0	
		% within preference for magazine	68.2%	31.8%	100.0%	
		% within location of reader	100.0%	100.0%	100.0%	
		% of Total	68.2%	31.8%	100.0%	

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	8.650 <sup>a</sup>	1	.003	.004	.003
Continuity Correction <sup>b</sup>	7.308	1	.007		
Likelihood Ratio	9.487	1	.002		
Fisher's Exact Test					
Linear-by-Linear Association	8.552	1	.003		
N of Valid Cases	88				

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 10.18.

b. Computed only for a 2x2 table

$H_0$  : Preference for certain publications and the geographic location of the reader are independent.

$H_1$  : Preference for certain publications and the geographic location of the reader are not independent.

$P\text{-value} = 0.003 < \alpha = 0.05$

Reject  $H_0$

Preference for certain publications and the geographic location of the reader are not independent.

## 10.1.2 Independence Test

### ◆ Exercise

A researcher is interested in determining whether drinking preference of coffee and tea is gender related. The data given as follows:

	Preferred tea	Preferred coffee
Male	16	31
Female	21	19

Determine whether the preference is gender related.

### Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Gender * Preference	87	100.0%	0	0.0%	87	100.0%

### Gender \* Preference Crosstabulation

			Preference		Total
			Tea	Coffee	
Gender	Male	Count	16	31	47
		Expected Count	20.0	27.0	47.0
		% within Gender	34.0%	66.0%	100.0%
		% within Preference	43.2%	62.0%	54.0%
		% of Total	18.4%	35.6%	54.0%
	Female	Count	21	19	40
		Expected Count	17.0	23.0	40.0
		% within Gender	52.5%	47.5%	100.0%
		% within Preference	56.8%	38.0%	46.0%
		% of Total	24.1%	21.8%	46.0%
Total	Count		37	50	87
	Expected Count		37.0	50.0	87.0
	% within Gender		42.5%	57.5%	100.0%
	% within Preference		100.0%	100.0%	100.0%
	% of Total		42.5%	57.5%	100.0%

### Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.012 <sup>a</sup>	1	.083	.127	.064
Continuity Correction <sup>b</sup>	2.304	1	.129		
Likelihood Ratio	3.022	1	.082		
Fisher's Exact Test					
Linear-by-Linear Association	2.977	1	.084		
N of Valid Cases	87				

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 17.01.

b. Computed only for a 2x2 table

## 10.1.3 Homogeneity Test

- ◆ Just as Independence test,  $H_0$  and  $H_1$  for this test should also be in a correct position before conducting hypothesis testing.
- ◆ Hypothesis statement for Homogeneity test:  
 $H_0$  : The proportion of **row variables** are same with **column variable**  
 $H_1$  : The proportion of **row variables** are not same with **column variable**



*Need to be changed according to question*

## 10.1.3 Homogeneity Test

200 female owners and 200 male owners of Proton cars selected at random and the colour of their cars are noted. The following data shows the results:

	Car Colour		
	Black	Dull	Bright
Male	40	110	50
Female	20	80	100

Test whether the proportions of colour preference are the same for female and male.

# 10.1.3 Homogeneity Test

- ◆ As the SPSS analysis for this test is the same as Independence test, we will skip the process and focus on the output.

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Gender * Colour	400	100.0%	0	0.0%	400	100.0%

Gender \* Colour Crosstabulation

			Colour			Total
			Black	Dull	Bright	
Gender	Male	Count	40	110	50	200
		Expected Count	30.0	95.0	75.0	200.0
		% within Gender	20.0%	55.0%	25.0%	100.0%
		% within Colour	66.7%	57.9%	33.3%	50.0%
		% of Total	10.0%	27.5%	12.5%	50.0%
	Female	Count	20	80	100	200
		Expected Count	30.0	95.0	75.0	200.0
		% within Gender	10.0%	40.0%	50.0%	100.0%
		% within Colour	33.3%	42.1%	66.7%	50.0%
		% of Total	5.0%	20.0%	25.0%	50.0%
	Total	Count	60	190	150	400
		Expected Count	60.0	190.0	150.0	400.0
		% within Gender	15.0%	47.5%	37.5%	100.0%
		% within Colour	100.0%	100.0%	100.0%	100.0%
		% of Total	15.0%	47.5%	37.5%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	28.070 <sup>a</sup>	2	.000
Likelihood Ratio	28.543	2	.000
Linear-by-Linear Association	25.759	1	.000
N of Valid Cases	400		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 30.00.

$H_0$  : The proportions of colour preference are the same for female and male.

$H_1$  : The proportions of colour preference are not the same for female and male.

$P\text{-value} \approx 0.000 < \alpha = 0.05$

Reject  $H_0$

The proportions of colour preference are not the same for female and male.



## 10.1.3 Homogeneity Test

### ◆ Exercise

Four machines manufacture cylindrical steel pins. The pins are subjected to a diameter specification. A pin may meet the specification or it may be too thin or too thick. Pins are sampled from each machine and the number of pins in each category is counted.

	Too thin	OK	Too Thick
Machine 1	10	102	8
Machine 2	34	161	5
Machine 3	12	79	9
Machine 4	10	60	10

Determine whether the proportion of pins that are too thin, OK, or too thick is the same for all machines.

# Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Machine * Specification	500	100.0%	0	0.0%	500	100.0%

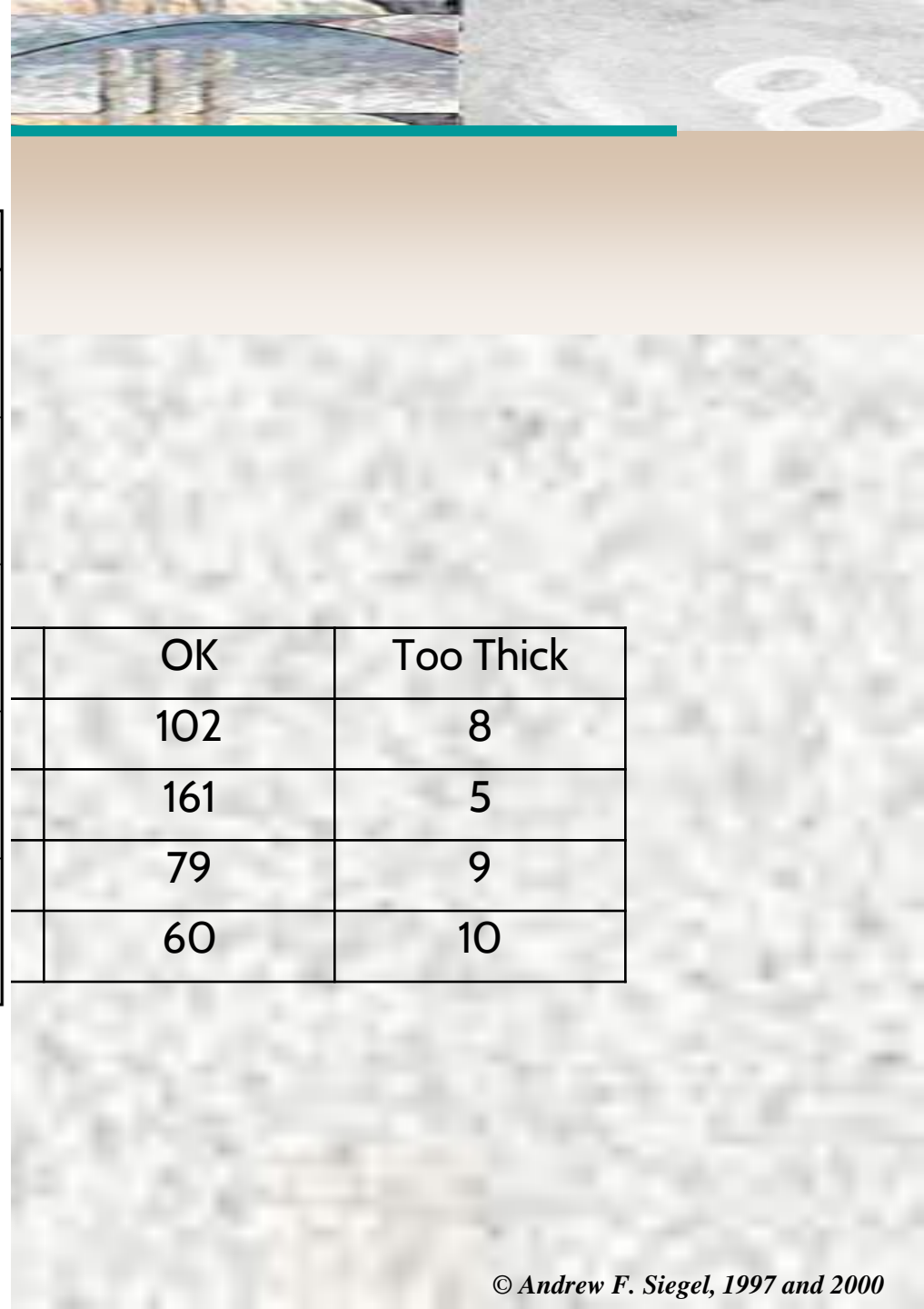
## Machine \* Specification Crosstabulation

			Specification			Total
			Too Thin	OK	Too Thick	
Machine	Machine 1	Count	10	102	8	120
		Expected Count	15.8	96.5	7.7	120.0
		% within Machine	8.3%	85.0%	6.7%	100.0%
		% within Specification	15.2%	25.4%	25.0%	24.0%
		% of Total	2.0%	20.4%	1.6%	24.0%
	Machine 2	Count	34	161	5	200
		Expected Count	26.4	160.8	12.8	200.0
		% within Machine	17.0%	80.5%	2.5%	100.0%
		% within Specification	51.5%	40.0%	15.6%	40.0%
		% of Total	6.8%	32.2%	1.0%	40.0%
	Machine 3	Count	12	79	9	100
		Expected Count	13.2	80.4	6.4	100.0
		% within Machine	12.0%	79.0%	9.0%	100.0%
		% within Specification	18.2%	19.7%	28.1%	20.0%
		% of Total	2.4%	15.8%	1.8%	20.0%
	Machine 4	Count	10	60	10	80
		Expected Count	10.6	64.3	5.1	80.0
		% within Machine	12.5%	75.0%	12.5%	100.0%
		% within Specification	15.2%	14.9%	31.2%	16.0%
		% of Total	2.0%	12.0%	2.0%	16.0%
	Total	Count	66	402	32	500
		Expected Count	66.0	402.0	32.0	500.0
		% within Machine	13.2%	80.4%	6.4%	100.0%
		% within Specification	100.0%	100.0%	100.0%	100.0%
		% of Total	13.2%	80.4%	6.4%	100.0%

## Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	15.584 <sup>a</sup>	6	.016
Likelihood Ratio	16.037	6	.014
Linear-by-Linear Association	.756	1	.385
N of Valid Cases	500		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 5.12.



	OK	Too Thick
Machine 1	102	8
Machine 2	161	5
Machine 3	79	9
Machine 4	60	10

## 10.2 Nonparametric Statistics

- ◆ This subchapter introduce statistical techniques that deal with ordinal data.
- ◆ Shown below are the list of nonparametric techniques that can be used as an alternative to their respective parametric counterpart.

Nonparametric test	Parametric counterpart
10.2.1 Sign test	One sample t-test
	Paired sample t-test
10.2.2 Mann-Whitney Test	Two sample independent t-test
10.2.3 Kruskal Wallis Test	One way ANOVA

## 10.2.1 Sign Test

- ◆ Sign test is a nonparametric alternative to the hypotheses testing for one and paired population mean.
- ◆ It based on the direction of the + and – sign of the observation and not their numerical magnitude.
- ◆ It also called the binomial sign test with the null proportion is 0.5 (Uses the binomial distribution as the decision rule).
- ◆ There are two types of sign test: **One sample sign test** and Paired sample sign test.



Our focus will be on this test only

## 10.2.1 Sign Test

- ◆ One sample sign test example:

The following data constitute a random sample of 15 measurement of the octane rating of a certain kind gasoline:

99.0	102.3	99.8	100.5	99.7	96.2	99.1	102.5
103.3	97.4	100.4	98.9	98.3	98.0	101.6	

Test the null hypothesis  $m = 98.0$  against the alternative hypothesis  $m > 98.0$  at the 0.01 level of significance.

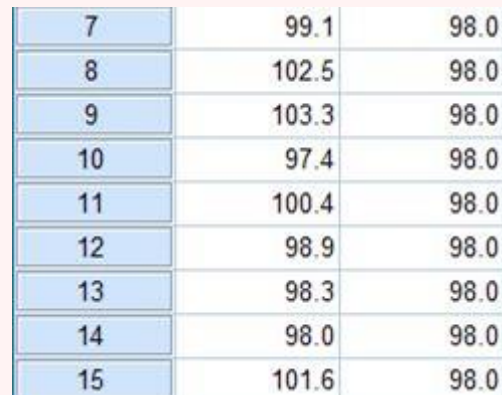


## 10.1.1 Sign Test

### ◆ In SPSS (Data view)

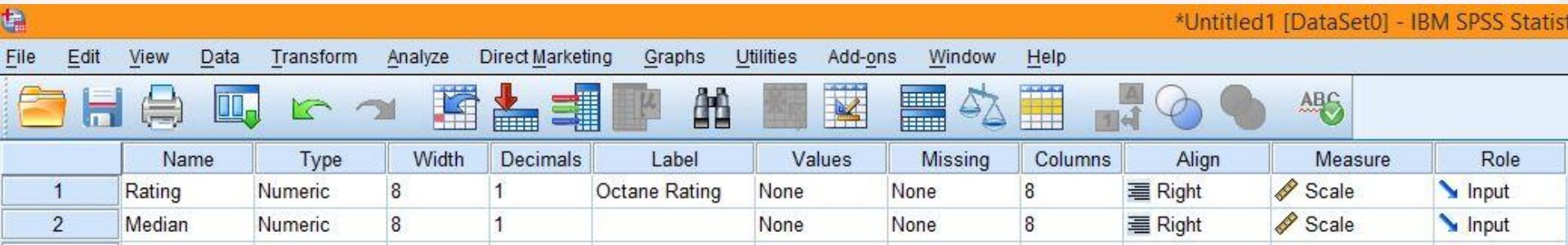


	Rating	Median
1	99.0	98.0
2	102.3	98.0
3	99.8	98.0
4	100.5	98.0
5	99.7	98.0
6	96.2	98.0



7	99.1	98.0
8	102.5	98.0
9	103.3	98.0
10	97.4	98.0
11	100.4	98.0
12	98.9	98.0
13	98.3	98.0
14	98.0	98.0
15	101.6	98.0

### ◆ In SPSS (Variable view)



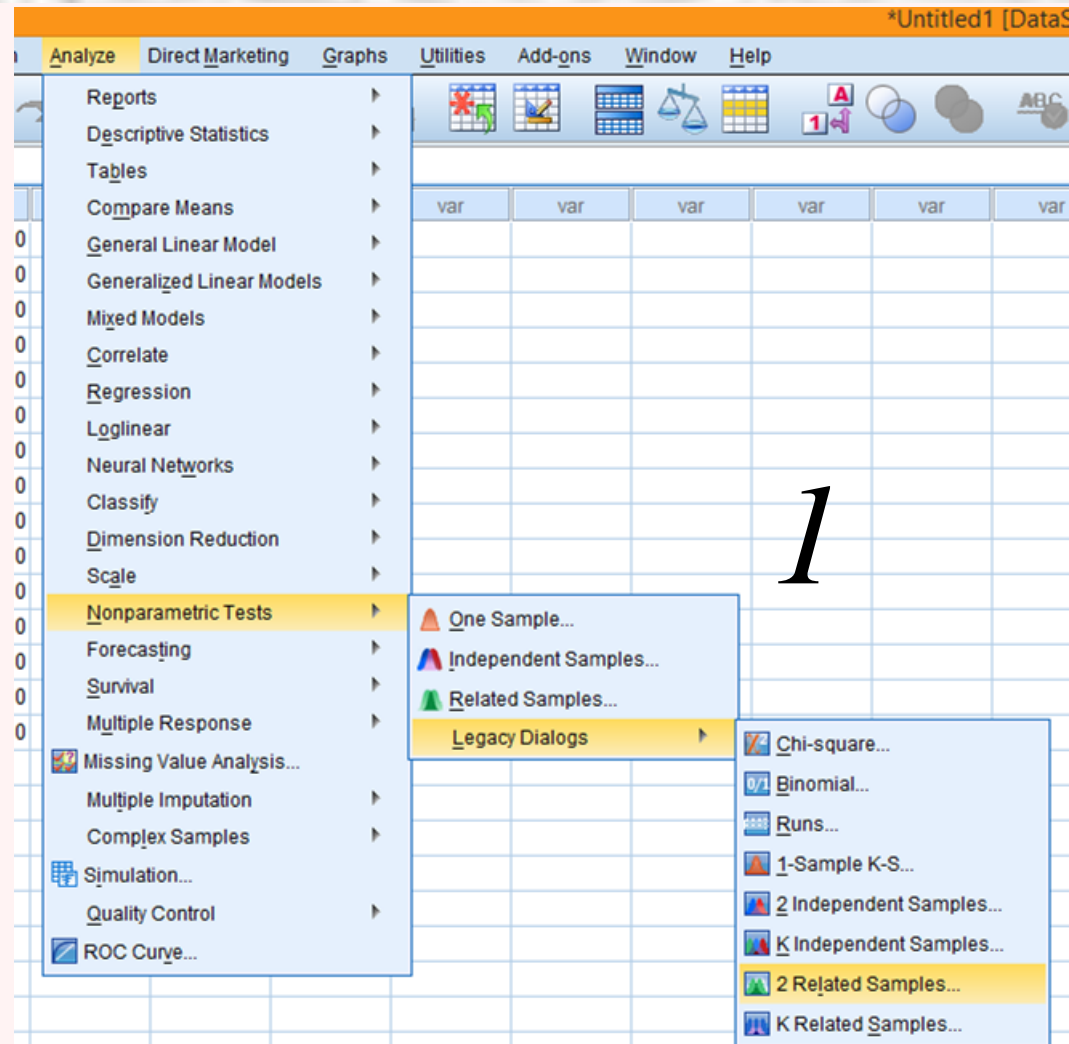
\*Untitled1 [DataSet0] - IBM SPSS Statistics

	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure	Role
1	Rating	Numeric	8	1	Octane Rating	None	None	8	Right	Scale	Input
2	Median	Numeric	8	1		None	None	8	Right	Scale	Input



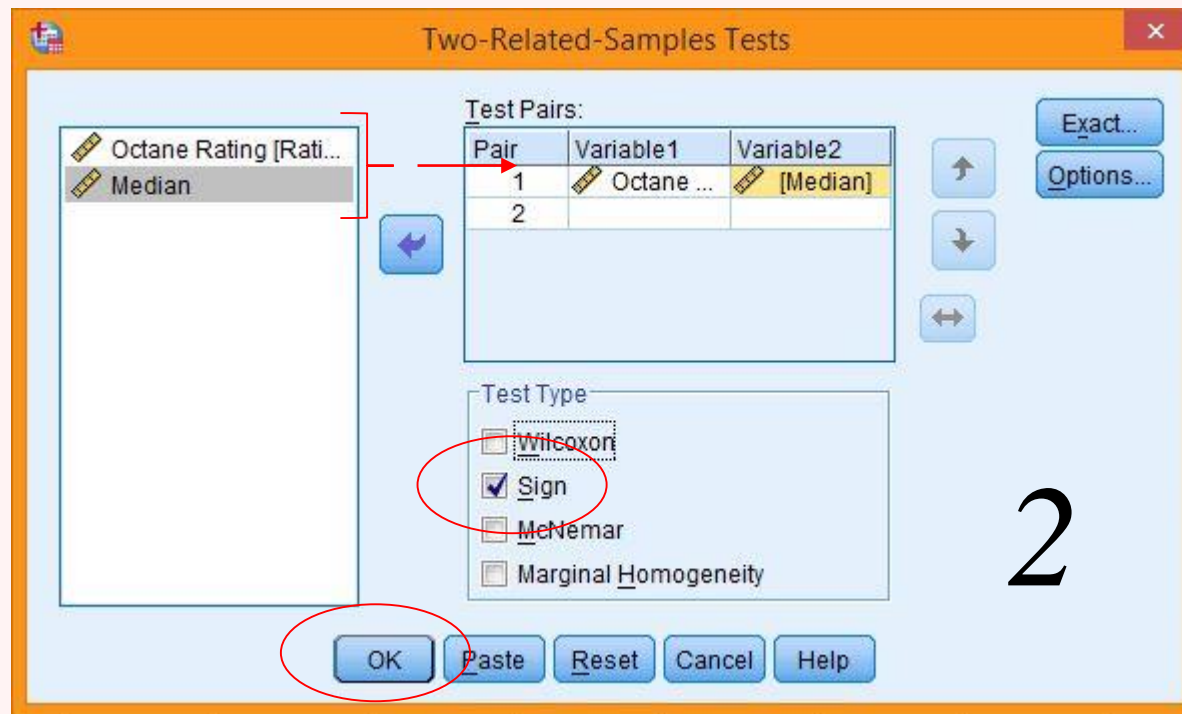
# 10.1.1 Sign Test

## ◆ Analysis



## 10.1.1 Sign Test

### ◆ Analysis



# 10.1.1 Sign Test

## ◆ Output

### → NPar Tests

#### Sign Test

Frequencies		N
Median - Octane Rating	Negative Differences <sup>a</sup>	12
	Positive Differences <sup>b</sup>	2
	Ties <sup>c</sup>	1
	Total	15

a. Median < Octane Rating

b. Median > Octane Rating

c. Median = Octane Rating

#### Test Statistics<sup>a</sup>

	Median - Octane Rating
Exact Sig. (2-tailed)	.013 <sup>b</sup>

a. Sign Test

b. Binomial distribution used.

$$H_0 : m = 98.0$$

$$H_1 : m > 98.0$$

$$P\text{-value} = \frac{0.013}{2} = 0.0065 < \alpha = 0.01$$

Reject  $H_1$

The median octane rating of the given kind of gasoline exceeds 98.0

## 10.2.1 Sign Test

### ◆ Exercise:

A bank manager claims that the median number of customers per day is no more than 750. A teller doubts the accuracy of this claim. The numbers of bank customers per day for 16 randomly selected days are listed below.  $\alpha = 0.05$ , can the teller reject the manager's claim?

775	765	801	742	754	753	739	751
745	750	777	769	756	760	782	789

## 10.2.1 Sign Test: Output

**Frequencies**

	N
Median - Customer    Negative Differences <sup>a</sup>	12
Positive Differences <sup>b</sup>	3
Ties <sup>c</sup>	1
Total	16

a. Median < Customer

b. Median > Customer

c. Median = Customer

**Test Statistics<sup>a</sup>**

	Median - Customer
Exact Sig. (2-tailed)	.035 <sup>b</sup>

a. Sign Test

b. Binomial distribution used.

$$\alpha = 0.05$$

753	739	751
760	782	789

## 10.2.2 Mann-Whitney Test

- ◆ Mann Whitney Test is equivalent to independent group t-test in parametric test.
- ◆ Example: The productivity level of two factories, A and B, are to be compared. The monthly output, in kilogram, is recorded for two consecutive months:

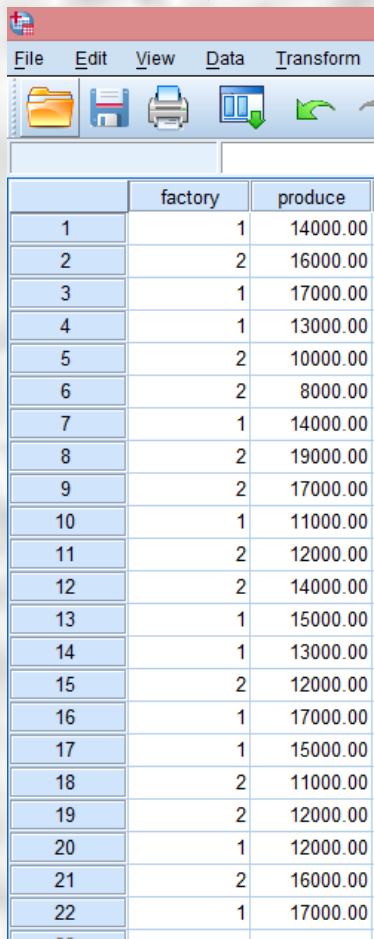
Factory	
A	B
14000	16000
17000	10000
13000	8000
14000	19000
11000	17000
15000	12000
13000	14000
17000	12000
15000	11000
12000	12000
17000	16000

The data has been identified violating the parametric assumptions.

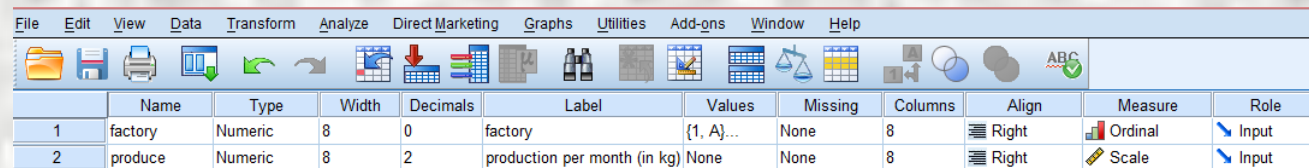


# 10.2.2 Mann-Whitney Test

- ◆ In SPSS (Data view)
- ◆ In SPSS (Variable view)



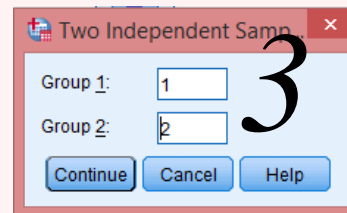
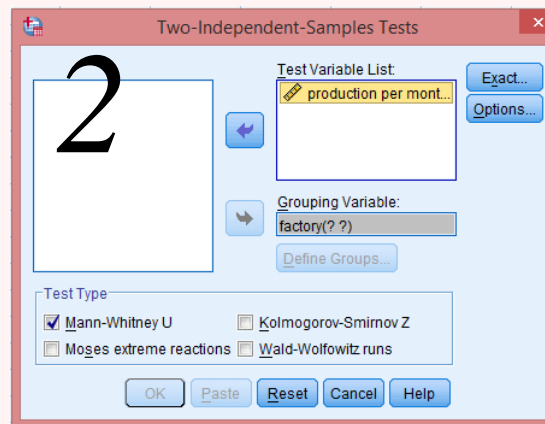
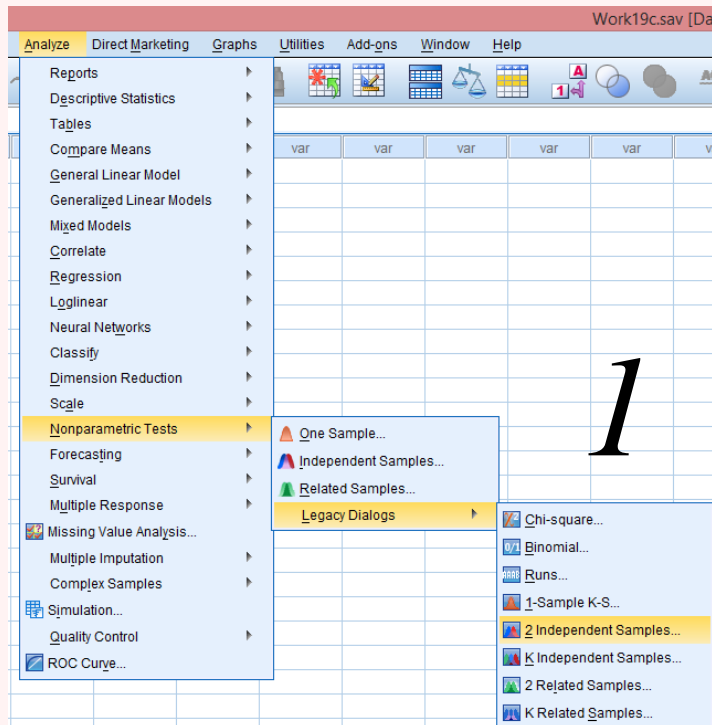
	factory	produce
1	1	14000.00
2	2	16000.00
3	1	17000.00
4	1	13000.00
5	2	10000.00
6	2	8000.00
7	1	14000.00
8	2	19000.00
9	2	17000.00
10	1	11000.00
11	2	12000.00
12	2	14000.00
13	1	15000.00
14	1	13000.00
15	2	12000.00
16	1	17000.00
17	1	15000.00
18	2	11000.00
19	2	12000.00
20	1	12000.00
21	2	16000.00
22	1	17000.00



	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure	Role
1	factory	Numeric	8	0	factory	{1, A}...	None	8	Right	Ordinal	Input
2	produce	Numeric	8	2	production per month (in kg)	None	None	8	Right	Scale	Input

# 10.2.2 Mann-Whitney Test

## ◆ Analysis



## 10.2.2 Mann-Whitney Test

### ◆ Output

#### Mann-Whitney Test

Ranks				
	factory	N	Mean Rank	Sum of Ranks
production per month (in kg)	A	11	12.77	140.50
	B	11	10.23	112.50
	Total	22		

#### Test Statistics<sup>a</sup>

	production per month (in kg)
Mann-Whitney U	46.500
Wilcoxon W	112.500
Z	-.927
Asymp. Sig. (2-tailed)	.354
Exact Sig. [2*(1-tailed Sig.)]	.365 <sup>b</sup>

a. Grouping Variable: factory

b. Not corrected for ties.

$H_0$  : The productivity level of two factories, A and B, are the same.

$H_1$  : The productivity level of two factories, A and B, are different.

$$P\text{-value} = 0.354 > \alpha = 0.05$$

Fail to reject  $H_0$

The productivity level of two factories, A and B, are the same.

## 10.2.2 Mann-Whitney Test

### ◆ Exercise

Data below show the marks obtained by electrical engineering students in an examination:

Gender	Marks
Male	60
Male	62
Male	78
Male	83
Female	40
Female	65
Female	70
Female	88
Female	92

Can we conclude the achievements of male and female students identical?

# 10.2.2 Mann-Whitney Test: Output

**Ranks**

	Gender	N	Mean Rank	Sum of Ranks
Marks	1	4	4.50	18.00
	2	5	5.40	27.00
	Total	9		

**Test Statistics<sup>a</sup>**

	Marks
Mann-Whitney U	8.000
Wilcoxon W	18.000
Z	-.490
Asymp. Sig. (2-tailed)	.624
Exact Sig. [2*(1-tailed Sig.)]	.730 <sup>b</sup>

a. Grouping Variable: Gender

b. Not corrected for ties.

**Marks**

60  
62  
78  
83  
40  
65  
70  
88  
92



## 10.2.3 Kruskal Wallis Test

- ◆ The Kruskal Wallis Test is equivalent to One-Way ANOVA, and therefore allow possible differences between two or more groups to be examined.



## 10.2.3 Kruskal Wallis Test

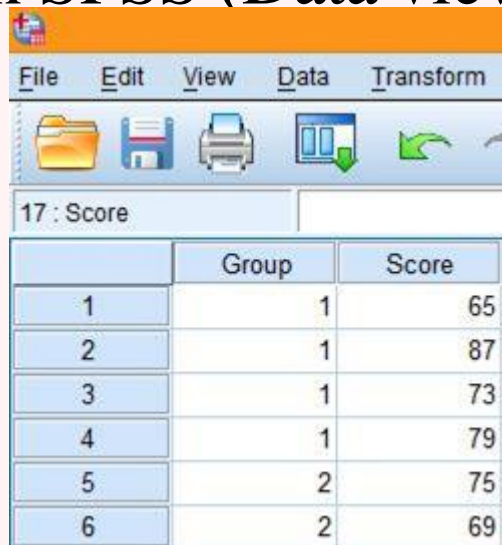
### ◆ Example:

Four groups of students were randomly assigned to be taught with four different techniques, and their achievement test scores were recorded. At the 0.05 level, are the distributions of test scores the same?

1	2	3	4
65	75	59	94
87	69	78	89
73	83	67	80
79	81	62	88

## 10.2.3 Kruskal Wallis Test

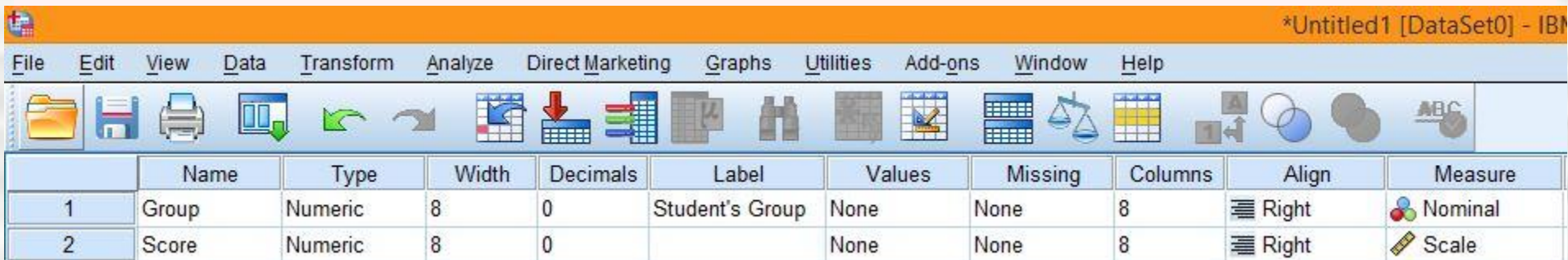
### ◆ In SPSS (Data view)



	Group	Score
1	1	65
2	1	87
3	1	73
4	1	79
5	2	75
6	2	69

7	2	83
8	2	81
9	3	59
10	3	78
11	3	67
12	3	62
13	4	94
14	4	89
15	4	80
16	4	88

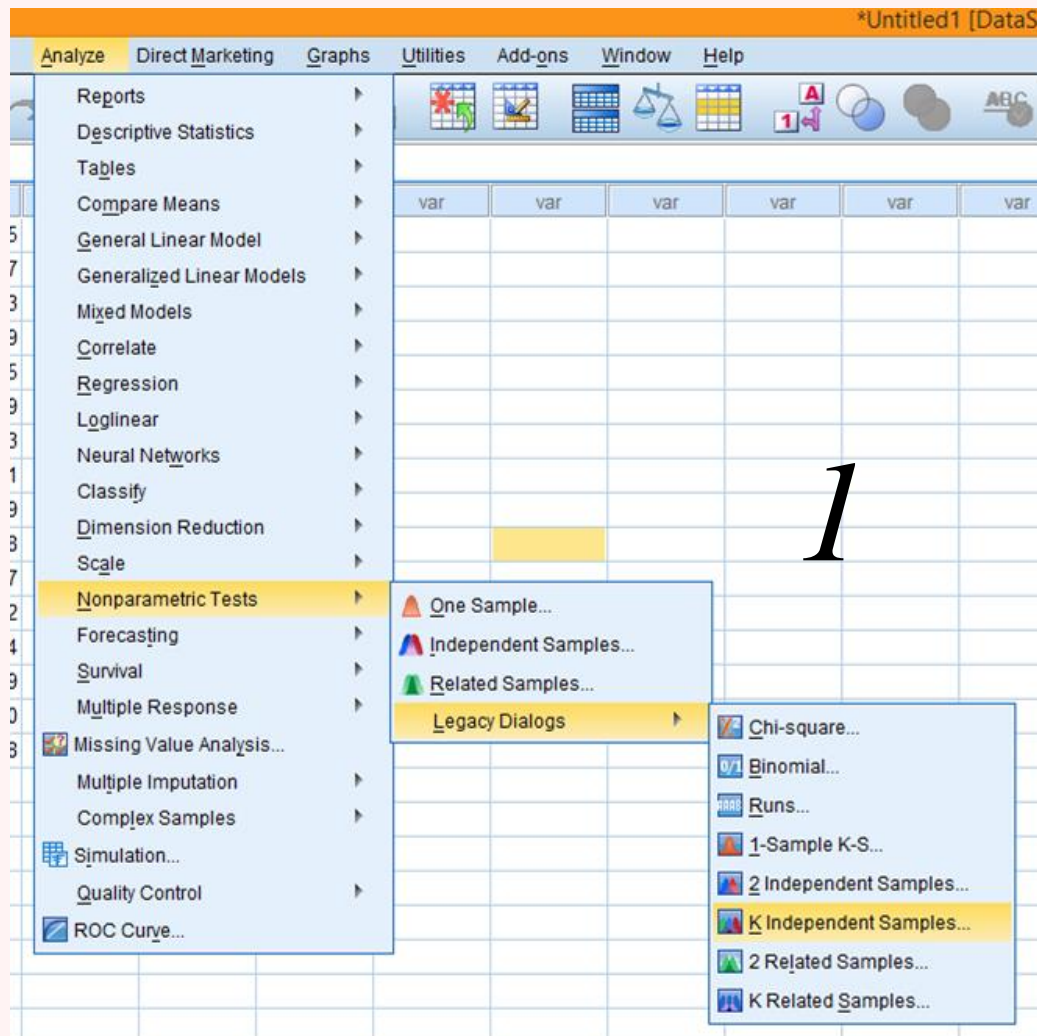
### ◆ In SPSS (Variable view)



	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
1	Group	Numeric	8	0	Student's Group	None	None	8	Right	Nominal
2	Score	Numeric	8	0		None	None	8	Right	Scale

## 10.2.3 Kruskal Wallis Test

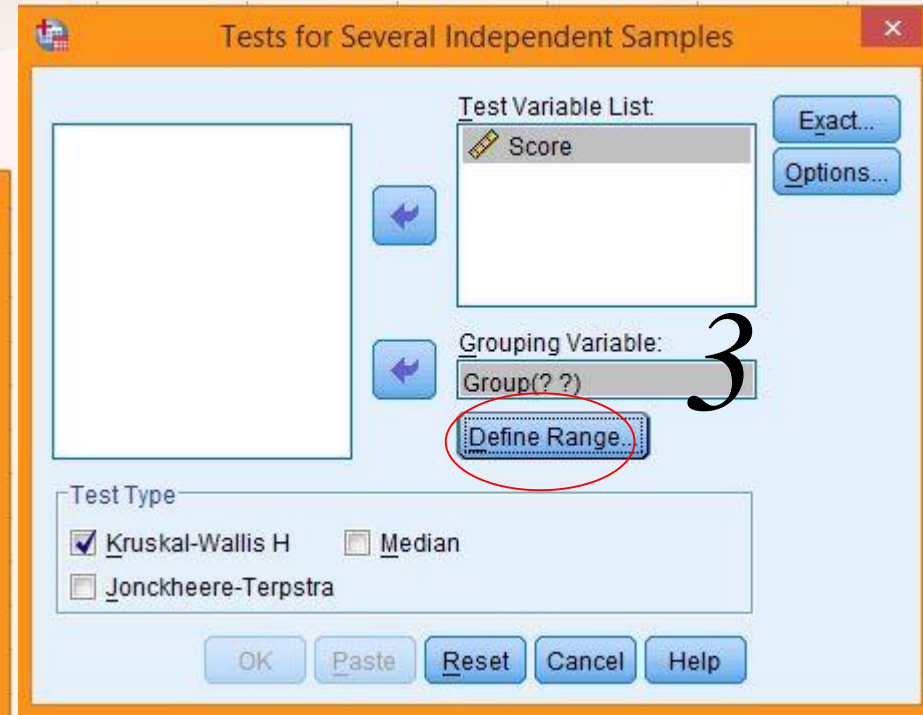
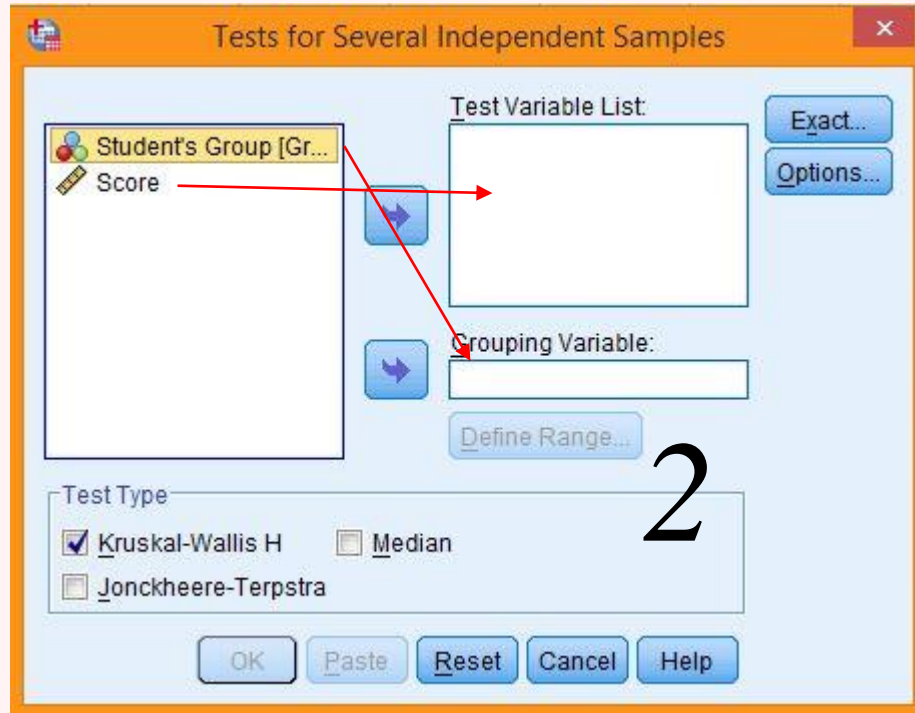
### ◆ Analysis



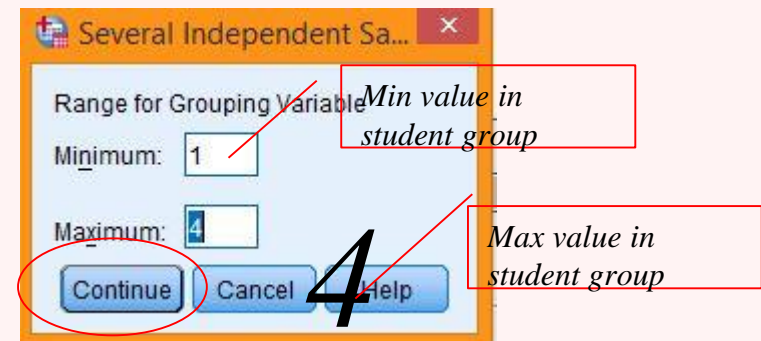


## 10.2.3 Kruskal Wallis Test

### ◆ Analysis



### ◆ Continue and OK



## 10.2.3 Kruskal Wallis Test

### ◆ Output

#### → NPar Tests

##### Kruskal-Wallis Test

Ranks		
	Student's Group	N
Score	1	4
	2	4
	3	4
	4	4
Total		16
		Mean Rank
		7.75
		8.75
		3.75
		13.75

##### Test Statistics<sup>a,b</sup>

	Score
Chi-Square	8.956
df	3
Asymp. Sig.	.030

a. Kruskal Wallis Test

b. Grouping Variable:  
Student's Group

$$H_0 : m_1 = m_2 = m_3 = m_4$$

$H_1$  : at least one  $m_i$  differs from others

$$P\text{-value} = 0.03 < \alpha = 0.05$$

Reject  $H_0$

The distributions of test scores differ from at least one of the student's groups.

## 10.2.3 Kruskal Wallis Test

### ◆ Exercise:

Each of three aerospace companies has randomly selected a group of technical staff workers to participate in a training conference sponsored by a supplier firm. The three companies have sent 6, 5 and 7 employees respectively. At the beginning of the session. A preliminary test is given, and the scores are shown in the table below. At the 0.05 level, can we conclude that the median scores for the three population of technical staff workers could be the same?

Test score		
Firm 1	Firm 2	Firm 3
67	64	75
57	73	61
62	72	76
59	68	71
70	65	78
67		74
		79



# 10.2.3 Kruskal Wallis Test: Output

**Ranks**

	Firms	N	Mean Rank
Score	1	6	5.33
	2	5	9.00
	3	7	13.43
	Total	18	

**Test Statistics<sup>a,b</sup>**

	Score
Chi-Square	7.497
df	2
Asymp. Sig.	.024

a. Kruskal Wallis Test

b. Grouping Variable: Firms

**Test score**

	Firm 2	Firm 3
	64	75
	73	61
	72	76
	68	71
	65	78
		74
		79