Meeting 2

Monday, October 16, 2023 12:02 PM

$$Y_i \sim \text{Normal}(\beta_0 + \beta_1 x_i, \sigma^2)$$

$$\begin{cases} Y_i = \beta_0 + \beta_1 x_i + \varepsilon \\ \varepsilon \sim \text{Normal}(0, \sigma^2) \end{cases}$$

Same thing.

- We learned how to estimate β_0 & β_1 (LS or MLE), call the estimates $\hat{\beta}_0$ & $\hat{\beta}_1$ (and $\hat{\sigma}^2$)

- We know that

$$t = \frac{\hat{\beta}_1 - \beta^*}{SE(\hat{\beta}_1)}$$ has

a t-distribution, with $df = n-2$ (same # of d.f. as for the residual SE).

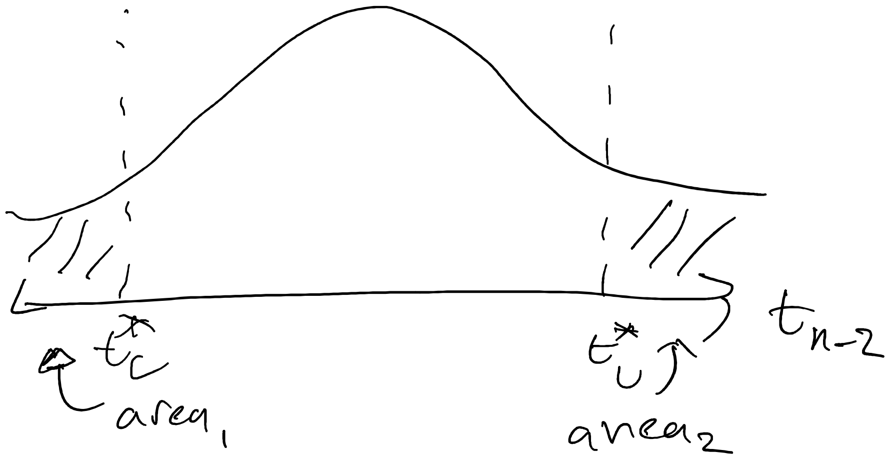
- Assume/let $\beta^* = 0$ (hypothesis test for no slope)

$$t = \hat{\beta}_1 / SE(\hat{\beta}_1)$$

- that mens we can get

$Pr(t = \text{whatever value})$

— we know this b/c we know the dist.



We want area$_1$ + area$_2$ = 0.05

($\alpha = 0.05$).

To find the values of $\hat{\beta}_1$ where we have the right α.

$$t_U^* = \hat{\beta}_1 / SE(\hat{\beta}_1)$$

$$t_L^* = \hat{\beta}_1 / SE(\hat{\beta}_1)$$

Because we know the t-dist., we know t_U^* & t_L^*. we also assume $SE(\hat{\beta}_1)$ is known. The unknown here is $\hat{\beta}_1$.

- (skipping steps) we get to the formula

$$\hat{\beta}_1 \pm t^* SE$$ gives us

our $(1-\alpha)\%$ CI.

So if $\alpha = 0.05$, we find the correct t^*, and this is a 95% confidence interval.

- We get something like

  2.56 (95% CI: 1.32, 3.80)

What does that mean?

- We are 95% confident that β_1 (no hat) is in the interval (1.32, 3.80).

- **It is not true that there is a 95% chance that β_1 is in the interval. We don't know!!!**

- β_1 is in that interval or it isn't. Doesn't make sense to talk about that probability isn't. Doesn't make sense to talk about that probability.

- Law of Large Numbers: if you do something infinity times, you can get the probability that it happen accurately.

- Remember that our sample $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ is only one out of many samples. (there are $\binom{\text{Population size}}{\text{sample size}}$ # of samples we could've gotten).

- The CI tells us what we might expect if we repeated our experiment many, many times (long-run behavior). term

- If we repeat our experiment (or sampling procedure) many, many times, we would expect 95% of those experiments to give us a CI containing the true population parm.

- We can never know (IRL) if our CI for our experiment is in the 95% of (theoretical) CIs in the 95% of (theoretical) CIs that are "good" or in the 5% that are "bad".

<u>What is in today's code example</u>

1. CIs for regression parameters (β_j)
2. CIs for $E(y|x_i)$ (CI for prediction at specific x-value)
3. PIs for $y|x_i$ — CI for a <u>new</u> y value that we haven't seen yet, but know the x-value.