

Shun-Qing Shen

Topological Insulators

Dirac Equation in Condensed Matters

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Dirac Equation in Condensed Matters

With 54 Figures



Springer

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Preface

In recent years, we have seen rapid emergence of topological insulators and superconductors. The field is an important advance of the well-developed band theory in solids since its birth in the 1920s. The band theory or Fermi liquid theory and Landau's theory of spontaneously broken symmetry are two themes for most collective phenomena in many-body systems, such as semiconductors and superconductors. Discovery of the integer and fractional quantum Hall effects in the 1980s opens a new window to explore the mystery of condensed matters: topological order has to be introduced to characterize a large class of quantum phenomena. Topological insulator is a triumph of topological order in condensed matter physics.

The book grew out of a series of lectures I delivered in an international school on “Topology in Quantum Matter” at Bangalore, India, in July 2011. The aim of this book is to provide an introduction of a large family of topological insulators and superconductors based on the solutions of the Dirac equation. I believe that the Dirac equation is a key to the door of topological insulators. It is a line that could thread all relevant topological phases from one to three dimensions and from insulators to superconductors or superfluids. This idea actually defines the scope of this book on topological insulators. For this reason, a lot of topics in topological insulators are actually not covered in this book, for example, the interacting systems and topological field theory. Also I have no ambition to review rapid developments of the whole field and consequently no intention to introduce all topics in this introductory book.

I would like to express my gratitude to my current and former group members, and various parts of the manuscript benefited from the contributions of Rui-Lin Chu, Huai-Ming Guo, Jian Li, Hai-Zhou Lu, Jie Lu, Hai-Feng Lv, Wen-Yu Shan, Rui Yu, Yan-Yang Zhang, An Zhao, Yuan-Yuan Zhao, and Bin Zhou. Especially I would like to thank Hai-Zhou Lu for critically reading the manuscript and replotting all figures. I benefited from numerous discussions and collaborations with Qian Niu, Jainendra K. Jain, Jun-Ren Shi, Zhong Fang, and Xin Wan on the relevant topics. I am grateful

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Shun-Qing Shen

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List of Abbreviations

ARPES:	angle-resolved photoemission spectroscopy
BCS:	Bardeen-Cooper-Schrieffer
BdG:	Bogoliubov-de Gennes
DOS:	density of states
ESP:	equal spin pairing
FQHE:	fractional quantum Hall effect
HH:	heavy hole
IQHE:	integer quantum Hall effect
LH:	light hole
NMR:	nuclear magnetic resonance
PHS:	particle-hole symmetry
QAHE:	quantum anomalous Hall effect
QSHE:	quantum spin Hall effect
STM:	scanning tunneling microscopy
TAI:	topological Anderson insulator
TKNN:	Thouless-Kohmoto-Nightingale-Nijs
TRS:	time reversal symmetry

Note: elementary charge: $-e$ (e is positive)

