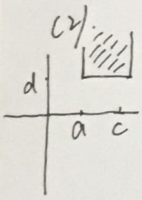
2. (1). $P(X>a, Y>b) = 1 - P(X\le a) - P(Y\le b) + P(X\le a, Y\le b)$

$= 1 - F(a,+\infty) - F(+\infty, b) + F(a,b)$

(2).



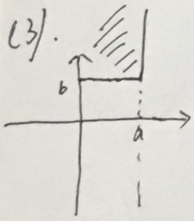
$P(a\le X\le c, Y\ge d)$ 若 $c<a$ $P(a\le X\le c, Y\ge d)=0$

若 $a\le c$ $P(a\le X\le c, Y\ge d) = P(X\le c) - P(X<a) - P(X\le c, Y<d)$

$+ P(X<a, Y<d) = F(c,+\infty) - F(a-0,+\infty) - F(c, d-0)$

$+ F(a-0, d-0)$

(3).



$P(X\le a, Y\ge b) = P(X\le a) - P(X\le a, Y<b)$

$= F(a,+\infty) - F(a, b-0)$

(4). $P(X=a, Y>b) = P(X=a) - P(X=a, Y\le b)$

$= P(X\le a) - P(X<a) - (P(X\le a, Y\le b) - P(X<a, Y\le b))$

$= F(a,+\infty) - F(a-0,+\infty) - F(a,b) + F(a-0, b)$

3. (1). 由联合分布列的正则性 $\frac{1}{2}+\frac{1}{8}+\frac{1}{4}+\frac{1}{16}+c=1$ 解得 $c=\frac{1}{16}$

(2). $P(X=Y) = P(X=0,Y=0) + P(X=1,Y=1) = \frac{1}{2} + \frac{1}{16} = \frac{7}{16}$

(3). $P(X\le Y) = P(X=0,Y=0) + P(X=0,Y=1) + P(X=0,Y=2) + P(X=1,Y=1) + P(X=1,Y=2)$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{4} + \frac{1}{16} = \frac{15}{16}$

4. X可取1.2 Y可取1.2.3

当$X=1$时.两个小球在两个盒子

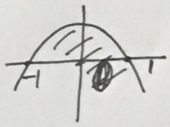
若$Y=1$. 可能在1.2或者1.3盒子 $P(X=1, Y=1)=\frac{A_2^2\times 2}{3^2}=\frac{4}{9}$

若$Y=2$. 在2.3盒子 $P(X=1, Y=2)=\frac{A_2^2}{3^2}=\frac{2}{9}$

Y不可取3

当$X=2$时.两个小球在一个盒子 则 $P(X=1, Y=i)=\frac{1}{3^2}=\frac{1}{9}$ $(i=1,2,3)$

X \ Y	1	2	3
1	$\frac{4}{9}$	$\frac{2}{9}$	0
2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

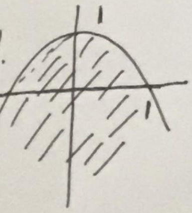


7. (1). 设 $D^*=\{(x,y): 0\le y\le 1-x^2\}$

于是 $1=\iint p(x,y)\,dx\,dy=\iint_{D^*} c(x^2+y)\,dx\,dy=\cancel{\int_0^{1-x^2}dy\int_{-1}^{1}c(x^2+y)dx}$

$=\cancel{\int_0^{1-x^2}2c(\frac{1}{3}+y)dy}=\int_{-1}^{1}dx\int_0^{1-x^2}c(x^2+y)dy=\int_{-1}^{1}\frac{c(1-x^4)}{2}dx=\frac{4}{5}c$

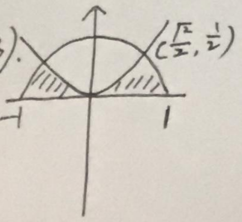
解得 $c=\frac{5}{4}$

(2). 

$P(Y\le X+1)=\int_{-1}^{0}dx\int_0^{x+1}\frac{5}{4}(x^2+y)dy+\int_0^1 dx\int_0^{1-x^2}c(x^2+y)dy$

$=\frac{5}{4}\int_{-1}^{0}(x^3+\frac{3}{2}x^2+x+\frac{1}{2})dx+\frac{1}{2}$

$=\frac{13}{16}$

(3). 

$\frac{1}{2}P(Y\le X^2)=\int_0^{\frac{\sqrt{2}}{2}}dx\int_0^{x^2}\frac{5}{4}(x^2+y)dy+\int_{\frac{\sqrt{2}}{2}}^{1}dx\int_0^{1-x^2}\frac{5}{4}(x^2+y)dy$

$=\frac{5}{4}\left(\int_0^{\frac{\sqrt{2}}{2}}\frac{3}{2}x^4dx+\int_{\frac{\sqrt{2}}{2}}^{1}\frac{1-x^4}{2}dx\right.$

$=\frac{5}{4}\left(\frac{3}{80}\sqrt{2}+\frac{2}{5}-\frac{19}{80}\sqrt{2}\right)$

$\therefore P(Y\le X^2)=1-\frac{\sqrt{2}}{2}$