1. Compute the order of β =(13)(247) and $\gamma = \alpha^{-1}\beta\alpha$, where α =(36)(254).

2. Special orthogonal matrices SO(2,$\mathbb{R}$) is a set of elements A_θ in the form below, equipped with matrix production.

$$A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \theta \in \mathcal{R}$$

1) Show SO(2,$\mathbb{R}$) is an Abelian group;

2) Give an appropriate definition of $A_\theta{}^k$ by matrix production, and prove $A_\theta{}^k = A_{k\theta}$, for $k \in \mathbb{Z}$;

3) Find all the elements of finite order in SO(2,$\mathbb{R}$).

3. GL(2,$\mathbb{Q}$) denotes the set of the *nonsingular* (having nonzero determinant, 行列式非 0) rational square matrix of length 2 under matrix product.

1) Show that it's a group

2) Compute the order of A=[0, -1; 1, 0] and B=[0, 1; -1, 1] .

3) What's the order of AB ?

4) GL(k,$\mathbb{R}$) denotes the set of the nonsingular k$\times$k real matrix under matrix product. Show that it is a group.

4. 1) The affine set Aff(1,$\mathbb{R}$) consists of all functions $\mathbb{R}\rightarrow\mathbb{R}$ of the form $f_{a,b}(x)=ax+b$, where the coefficients a and b are real numbers

with $a \neq 0$. Show that Aff(1,$\mathbb{R}$) is a group under the function composition;

2) The affine set Aff(k,$\mathbb{R}$) consists of all functions $\mathbb{R}^k \rightarrow \mathbb{R}^k$ of the form $f_{A,b}(X)=AX+b$, where A is a $k \times k$ matrix with $|A| \neq 0$, b is a $k \times 1$ vector. Show that Aff(k,$\mathbb{R}$) is a group under the function composition.