2.3　　2.4

2.3

2. 由随机变量X满足二项分布. 则 $P(X=0)=C_2^0(1-p)^2=\frac{1}{16}=1-P(X>0)$

解得 $1-p=\frac{1}{4}$

则 $P(Y>0)=1-P(Y=0)=1-C_3^0(1-p)^3=\frac{63}{64}$

3. 设Y为抛掷一枚均匀的硬币5次出现正面的次数 则 $Y\sim b(5,\frac{1}{2})$

设事件A为正面出现偶数次(包括0次)

则 $P(A)=P(Y=0)+P(Y=2)+P(Y=4)$

$=C_5^0(\frac{1}{2})^5+C_5^2(\frac{1}{2})^5+C_5^4(\frac{1}{2})^5$

$=\frac{1}{2}$

5. 由于 $X\sim P(\lambda)$. 则X为离散型随机变量

则 $P(X<\frac{2020}{2021})=P(X=0)=\frac{\lambda^0}{0!}e^{-\lambda}=e^{-\lambda}$

2.4.

2. 由概率密度函数的正则性 $\int_{-\infty}^{+\infty}p(x)dx=\int_0^{+\infty}ce^{-\sqrt{x}}dx$

设 $y=\sqrt{x}$ 则 $\int_0^{+\infty}ce^{-\sqrt{x}}dx=c\int_0^{+\infty}e^{-y}dy^2=2c\int_0^{+\infty}ye^{-y}dy$

$=-2c(y+1)e^{-y}\Big|_0^{+\infty}=2c=1$ 解得 $c=\frac{1}{2}$

$P(X>2)=\int_2^{+\infty}\frac{1}{2}e^{-\sqrt{x}}=-(\sqrt{x}+1)e^{-\sqrt{x}}\Big|_2^{+\infty}=(\sqrt{2}+1)e^{-\sqrt{2}}$

3. 当$x\le 0$时 $F(x)=P(X\le x)=\int_{-\infty}^{x}p(t)dt=\int_{-\infty}^{x}\frac{1}{2}e^{t}dt=\frac{1}{2}e^{x}$

当$x>0$时 $F(x)=\int_{-\infty}^{0}p(t)dt+\int_0^{x}p(t)dt=\frac{1}{2}+\int_0^{x}\frac{1}{2}e^{-t}dt=1-\frac{1}{2}e^{-x}$

$$\therefore F(x)=\begin{cases}\frac{1}{2}e^{x} & x\le 0\\ 1-\frac{1}{2}e^{-x} & x>0\end{cases}$$

4. $P(\frac{1}{2}<X<\frac{3}{2})=\int_{\frac{1}{2}}^{1}xdx+\int_1^{\frac{3}{2}}(2-x)dx=\frac{1}{2}x^2\Big|_{\frac{1}{2}}^{1}+(2x-\frac{1}{2}x^2)\Big|_1^{\frac{3}{2}}$

$=\frac{3}{8}+\frac{3}{8}=\frac{3}{4}$

2.5

1. $p(x)=\frac{1}{\sqrt{\pi}}e^{-(x-1)^2}=\frac{1}{\sqrt{2\pi}\cdot\frac{1}{\sqrt{2}}}e^{-\frac{(x-1)^2}{2\cdot(\frac{1}{\sqrt{2}})^2}}$ 则 $X\sim N(1,\frac{1}{2})$

$\therefore P(0\le X\le 2)=\Phi(\frac{2-1}{\frac{1}{\sqrt{2}}})-\Phi(\frac{0-1}{\frac{1}{\sqrt{2}}})=\Phi(\sqrt{2})-\Phi(-\sqrt{2})=2\Phi(\sqrt{2})-1$

由 $\Phi(1.41)\approx 0.9207$

$P(0\le X\le 2)\approx 0.9207\times 2-1=0.8414$

2. $\because X\sim N(10,4)$ $\therefore \mu=10.\ \sigma=2$

$\therefore P(6<X\le 9)=\Phi(\frac{9-10}{2})-\Phi(\frac{6-10}{2})=\Phi(0.5)-\Phi(-2)$

$=\Phi(2)-\Phi(0.5)=0.9772-0.6915=0.2857$

$P(7\le X<12)=\Phi(\frac{12-10}{2})-\Phi(\frac{7-10}{2})=\Phi(1)-\Phi(-1.5)$

$=\Phi(1)+\Phi(1.5)-1=0.8413+0.9332-1=0.7745$

$P(X>c)=1-P(X\le c)=P(X\le c)$ 则 $P(X\le c)=\frac{1}{2}$

$P(X\le c)=\Phi(\frac{c-10}{2})=\frac{1}{2}$

$\therefore \frac{c-10}{2}=\frac{1}{2}$ $c=10$

4. $P(-1<X<3)=\Phi(\frac{3-1}{\sigma})-\Phi(\frac{-1-1}{\sigma})=\Phi(\frac{2}{\sigma})-\Phi(\frac{-2}{\sigma})=2\Phi(\frac{2}{\sigma})-1=\frac{1}{2}$

解得 $\Phi(\frac{2}{\sigma})=\frac{3}{4}$

查表得 $\Phi(0.67)\approx 0.7486$ $\therefore \frac{2}{\sigma}\approx 0.7486$ $\sigma\approx 2.67$

5. $P(X>1)=\int_1^{+\infty}\lambda e^{-\lambda x}dx=e^{-\lambda}$ $P(X>2)=\int_2^{+\infty}\lambda e^{-\lambda x}=e^{-2\lambda}$

$\therefore e^{-\lambda}=2\cdot e^{-2\lambda}$ 解得 $\lambda=\ln 2$