2. $E(X+Y)^2=\int_0^1\int_0^1(x+y)^2dxdy=\int_0^1(\frac{1}{3}x^3+x^2+x)dx=\frac{7}{6}$

$E|X-Y|^2=\int_0^1\int_0^1(x-y)^2dxdy=\int_0^1(\frac{1}{3}x^3-x^2+x)dx=\frac{1}{6}$

$E|X-Y|=\int_0^1\int_0^1|x-Y|dxdy=2\int_0^1dx\int_0^x(x-y)dy=2\int_0^1x^2-\frac{1}{2}x^2dx=\frac{1}{3}$

则 $Cov(max(X,Y),min(X,Y))=Cov(\frac{X+Y+|X-Y|}{2},\frac{X+Y-|X-Y|}{2})$

$=\frac{1}{4}Cov(X+Y+|X-Y|,X+Y-|X-Y|)=\frac{1}{4}(Cov((X+Y)^2-(X-Y)^2)-E((X+Y)+|X-Y|)E((X+Y)-|X-Y|)$

$=\frac{1}{4}(\frac{7}{6}-\frac{1}{6}-(\frac{1}{2}+\frac{1}{2}+\frac{1}{3})(\frac{1}{2}+\frac{1}{2}-\frac{1}{3}))=\frac{1}{4}\times(1-1+\frac{1}{9})=\frac{1}{36}$

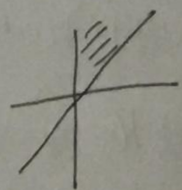
4. 由 $Var(X+Y)=VarX+VarY+2Cov(X,Y)$

$Cov(X,Y)=\frac{1}{2}(VarX+VarY-Var(X+Y))$

由于 $\sum_{i=1}^{100}X_i$ 与 $\sum_{i=101}^{150}X_i$ 互相独立（由 $X_1,X_2\cdots$ 独立同分布可知）

则 $Var\sum_{i=1}^{100}X_i+Var\sum_{i=101}^{150}VarX_i=Var\sum_{i=1}^{150}X_i$

$\therefore Cov(\sum_{i=1}^{100}X_i,\sum_{i=101}^{50}X_i)=\frac{1}{2}\times0=0$



8. $EX=\int_0^{+\infty}xdx\int_x^{+\infty}e^{-y}dy=\int_0^{+\infty}xe^{-x}dx=-(x+1)e^{-x}|_0^{+\infty}=1$

$EY=\int_0^{+\infty}dx\int_x^{+\infty}ye^{-y}dy=\int_0^{+\infty}(x+1)e^{-x}dx=-(x+2)e^{-x}|_0^{+\infty}=2$

$EXY=\int_0^{+\infty}dx\int_y^{+\infty}xye^{-y}dy=\int_0^{+\infty}(x^2+x)e^{-x}dx=-(x^2+3x+3)e^{-x}|_0^{+\infty}=3$

$EX^2=\int_0^{+\infty}x^2dx\int_x^{+\infty}e^{-y}dy=\int_0^{+\infty}x^2e^{-x}dx=-(x^2+2x+2)e^{-x}|_0^{+\infty}=2$

$EY^2=\int_0^{+\infty}dx\int_x^{+\infty}y^2e^{-y}dy=\int_0^{+\infty}(x^2+2x+2)e^{-x}=-(x^2+4x+6)e^{-x}|_0^{+\infty}=6$

$\therefore Cov(X,Y)=EXY-EXEY=3-1\times2=1$

$VarX=EX^2-(EX)^2=2-1=1\quad VarY=EY^2-(EY)^2=6-2^2=2$

$Corr(X,Y)=\frac{Cov(X,Y)}{\sqrt{VarX}\sqrt{VarY}}=\frac{1}{\sqrt{1}\sqrt{2}}=\frac{\sqrt{2}}{2}$

1. 由原点矩 $\mu_k = EX^k = \int_0^1 x^k p(x)dx = \int_0^1 x^k = \frac{1}{k+1}x^{k+1}\Big|_0^1 = \frac{1}{k+1}$

则 $EX = \mu_1 = \frac{1}{2}$

中心矩 $V_k = E(X-\mu_1)^k = E(X-\frac{1}{2})^k = \int_0^1 (x-\frac{1}{2})^k dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} x^k dx$

当 k 为奇数 $V_k = 0$ k 为偶数 $V_k = 2\int_0^{\frac{1}{2}} x^k dx = 2\cdot\frac{1}{k+1}(\frac{1}{2})^{k+1} = \frac{1}{(k+1)2^k}$

$\therefore V_k = \begin{cases} 0 & k\text{为奇数} \\ \frac{1}{(k+1)2^k} & k\text{为偶数} \end{cases}$

3. $F(x) = \begin{cases} 0 & x<0 \\ x & 0\le x<1 \\ 1 & x\ge 1 \end{cases}$

由 $0<\alpha<1$ 可知 $F(\alpha)=\alpha$ 则 $x_\alpha = \alpha = \inf\{x, F(x)\ge a\}$

$\therefore$ 均匀分布 $(0,1)$ 的 α 分位数为 α

4. 由题意 $F(x_{0.975}) = 0.975$

又 $X \sim N(10,4)$ 则 $\frac{X-10}{2} \sim N(0,1)$ 记 $Y\sim N(0,1)$ 反查表得 $Y_{0.975}=1.96$

则 $\frac{X_{0.975}-10}{2} = Y_{0.975}$ $X_{0.975} = 2Y_{0.975}+10 = 13.92$

2. 设第i次掷得的点数为X_i，则X_i ($i=1,2\cdots100$) 独立同分布

$EX_i=\frac{6+5+4+3+2+1}{6}=\frac{7}{2}$

$VarX_i=E(X_i-\frac{7}{2})^2=\frac{1}{6}((1-\frac{7}{2})^2+(2-\frac{7}{2})^2+(3-\frac{7}{2})^2+(4-\frac{7}{2})^2+(5-\frac{7}{2})^2+(6-\frac{7}{2})^2)=\frac{35}{12}$

由中心极限定理 $P(3\le\bar{X}\le4)=P(300\le\sum_{i=1}^{100}X_i\le400)=\Phi(\frac{400-100\times\frac{7}{2}}{\sqrt{100\cdot35/12}})-\Phi(\frac{300-100\cdot\frac{7}{2}}{\sqrt{100\cdot35/12}})$

$=0.9966$

4. 设学生答对的题数为X，则由题意 $X\sim(100,\frac{1}{2})$

当学生通过考试时 $X\ge60$，~~由于每题~~由中心极限定理

则 $P(X\ge60)=1-\Phi(\frac{60-0.5-100\times\frac{1}{2}}{\sqrt{100\cdot\frac{1}{2}\cdot\frac{1}{2}}})=1-\Phi(1.9)=1-0.9713=0.0287$

5. 设X表示n个螺丝钉中合格的螺丝钉个数 则 $X\sim b(n,0.99)$

$EX=0.99n$ $VarX=0.99n\cdot0.01$

则 $P(X\ge100)\ge95\%$

即 $1-\Phi(\frac{100-0.5-0.99n}{\sqrt{n\cdot0.99\cdot0.01}})\ge95\%$

$\Phi(\frac{99.5-0.99n}{\sqrt{n\cdot0.99\cdot0.01}})\le5\%$

$\frac{99.5-0.99n}{\sqrt{n\cdot0.99\cdot0.01}}\le-1.65$

化简 $0.9801n^2-197.0337n+9900.25\ge0$

解得 $n\ge102.18$

则一盒中至少装103个螺丝钉

7. (用马尔科夫) 令 $S_n=\sum_{i=1}^{n}X_i$ 记 $VarX=C$

$VarS_n=\sum_{i=1}^{n}VarX_i+2\sum_{i<j}Cov(X_i,X_j)\le\sum_{k=1}^{n}VarX_k=nVarX_1=nC$

则 $\frac{VarS_n}{n^2}=\frac{nC}{n^2}=\frac{C}{n}\to0$

由马尔科夫大数定律可知 $\{X_n,n\ge1\}$服从大数定律