仅供参考!仅供参考!仅供参考!

习题1.1

1. $A \cup B = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$, $AB = \emptyset$,
   $\overline{A} = \{4, 5, 6, 7, 8, 9\}$, $\overline{B} = \{0, 1, 2, 3, 4\}$.

2. $\overline{A} = \{x \in \Omega : x < 1$或$x > 5\}$, $A \cup B = [1, 7)$, $B\overline{C} = B = (3, 7)$,
   $\overline{A} \cap \overline{B} \cap \overline{C} = [0, 1) \cup [7, \infty)$, $(A \cup B)C = \emptyset$.

3. (1) $\overline{A}$=“掷三枚硬币, 至少有一个反面”;
   (2) $\overline{B}$=“抽检一批产品, 至多有两个个次品”;
   (3) $\overline{C}$=“射击三次, 至少命中两次”.

4-7. 略

习题1.2

1. $a + b - c$

2. 0.6

3. 略

4. 当$A \subset B$时, $P(AB)$取到最大值0.4.
   当$A \cup B = \Omega$时, $P(AB)$取到最小值0.1.

5-8. 略

9. 1

习题1.3

1. $\dfrac{C_{15}^3 \cdot C_{30}^7}{C_{45}^{10}} = 0.29$

2. $\dfrac{13^4}{C_{52}^4} = 0.1055$或$\dfrac{13^4 \times 4! \times 48!}{52!} = 0.1055$.

3. $\dfrac{4 \times C_{13}^4}{C_{52}^4} = 0.01056$ 或$\dfrac{4 \times P_{13}^4 \times 48!}{52!} = 0.01056$.

4. 当$k = 0, 1, 2$时, 所求概率依次为0.6935, 0.2972和0.0093.

5. $\dfrac{C_{20}^2 P_{365}^{19}}{365^{20}} = 0.323$.

6. $\dfrac{C_{13}^6 C_{13}^4 C_{13}^2 C_{13}^1}{C_{52}^{13}} = 0.00196$ 或$\dfrac{C_{13}^6 P_{13}^6 C_7^4 P_{13}^4 C_3^2 P_{13}^2 C_1^1 P_{13}^1}{P_{52}^{13}} = 0.00196$.

7. $\dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{6^6} = \dfrac{5}{324} = 0.015$.

8. $\dfrac{C_7^2 6!}{6^7} = \dfrac{35}{648} = 0.054$.

习题1.4

1. $\dfrac{2}{3}$
2. $\dfrac{1}{3}$
3. $\dfrac{7}{12}$
4. $\dfrac{2}{3}$
5. $\dfrac{r}{r+w} \cdot \dfrac{w}{r+w-1}$
6. 0.146
7. $\dfrac{13}{35}$
8. 4
9. $\dfrac{1}{N}$

习题1.5

1-2. 略.

3. (1)独立. (2)不独立.
4. 0.976
5. $1-(1-3/10)^{10}, C_{10}^3(3/10)^3(1-3/10)^7$
6. 略.
7. $1-(1-3\%)(1-5\%)$
8. $(1)\dfrac{47}{250}$, $(2)\dfrac{33}{47}$

习题2.1

1. $P(X < 1) = F(1-0)$, $P(|X-1| \le 2) = F(3) - F(-1-0)$, $P(X^2 > 3) = 1 - F(\sqrt{3}) + F(-\sqrt{3}-0)$, $P(\sqrt{1+X} \ge 2) = P(X \ge 3) = 1 - F(3-0)$.
2. (1) $P(X < 3) = F(3-0) = 11/12$
(2) $P(1 \le X < 3) = F(3-0) - F(1-0) = 11/12 - 1/2 = 5/12$
(3) $P(X > 1/2) = 1 - P(X \le 1/2) = 1 - F(1/2) = 1 - 1/4 = 3/4$
(4) $P(X = 3) = F(3) - F(3-0) = 1 - 11/12 = 1/12$.

3.

$$F_{X^+}(x) = \begin{cases} F_X(x), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{X^-}(x) = \begin{cases} 1 - F_X(-x-0), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{|X|}(x) = \begin{cases} F_X(x) - F_X(-x-0), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{aX+b}(x) = \begin{cases} F_X((x-b)/a), & a > 0; \\ 1 - F_X((x-b)/a - 0), & a < 0; \\ 1, & a = 0, x \geq b \\ 0, & a = 0, x < b. \end{cases}$$

4.

$$F(x) = \begin{cases} 1, & x \geq 1; \\ 1/2, & 0 \leq x < 1; \\ 0, & x < 0. \end{cases}$$

5. 是.

6-8. 略.

习题2.2

1.

X	3	4	5
P	1/10	3/10	3/5

2. 1/2

3.

$$F(x) = \begin{cases} 0, & x < a, \\ \dfrac{k-a+1}{b-a+1}, & k \leq x < k+1, \quad k = a, \cdots, b-1. \\ 1, & x \geq b. \end{cases}$$

4. 1

5. 1/3

6.

$$F(x)=\begin{cases}0, & x<1,\\ 2/5, & 1\le x<2,\\ 7/10, & 2\le x<3,\\ 9/10, & 3\le x<4,\\ 1, & x\ge 4.\end{cases}$$

7.

X	-1	0	1	3
P	0.2	0.4	0.3	0.1

8. 对.

习题2.3

1. $P(X<6)=\sum_{k=0}^{5}p_k=\sum_{k=0}^{5}C_{15}^{k}0.5^k(1-0.5)^{15-k}=$
2. $63/64$.
3. $1/2$.
4. 略.
5. $e^{-\lambda}$.

习题2.4

1. (1)$1/2$, (2)1, (3)$1/\pi$.
2. $1/2, (\sqrt{2}+1)e^{-\sqrt{2}}$.
3.

$$F(x)=\begin{cases}1-1/2e^{-x}, & x\ge 0,\\ 1/2e^{x}, & x<0.\end{cases}$$

4.

$$F(x)=\begin{cases}0, & x\le 0,\\ x^2/2, & 0<x\le 1,\\ -x^2/2+2x-1, & 1<x\le 2,\\ 1, & x>2.\end{cases}$$

$P(1/2<X<3/2)=3/4$.

5. $P(X\le 3)=1-e^{-9}$, $P(X>1)=e^{-3}$.

习题2.5

1. 0.8414
2. 0.2857, 0.7745, 10.
3. -0.817.
4. 2.96
5. $\ln 2$
6. 略.

习题3.1

1. 略.
2. (1)$1-F(a,\infty)-F(\infty,b)+F(a,b)$
   (2) $F(c,\infty)-F(a-0,\infty)-F(c,d-0)+F(a-0,d-0)$
   (3) $F(a,\infty)-F(a,b-0)$
   (4) $F(a,\infty)-F(a-0,\infty)-F(a,b)+F(a-0,b)$
3. (1)1/16, (2)9/16, (3) 15/16.
4.

$X \backslash Y$	1	2	3
1	4/9	2/9	0
2	1/9	1/9	1/9

5. (1)1/2, (2)0, (3)$\frac{3}{4}$.
6. $\frac{255}{256}$.
7. (1)5/4, (2)$\frac{13}{16}$, (3)$1-\frac{\sqrt{2}}{2}$.

习题3.2

1. $c=1/40$.
   X的可能取值有$-2,-1,0,1,2$, 且
   $$P(X=i)=\sum_{j=-2}^{2}p_{ij}=\sum_{j=-2}^{2}c|i+j|=(6+i^2)/40,\qquad i=-2,-1,0,1,2.$$
2. X与Y的边际分布列分别为

X	0	1
P	7/8	1/8

Y	0	1	2
P	9/16	3/16	1/4

3. X与Y的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 1, & 0 \le x \le 1, \\ 0, & x < 0\text{或}x > 1. \end{cases} \qquad p_Y(y) = \begin{cases} 1/2, & 0 \le y \le 2, \\ 0, & y < 0\text{或}y > 2. \end{cases}$$

4. X与Y的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 3x^2, & 0 < x < 1, \\ 0, & x \le 0\text{或}x \ge 1. \end{cases} \qquad p_Y(y) = \begin{cases} 3/2(1-y^2), & 0 < y < 1, \\ 0, & y \le 0\text{或}y \ge 1. \end{cases}$$

5. X的边际概率密度函数为$p_X(x) = \dfrac{1}{2}e^{-|x|}$.

6. Y的边际概率密度函数为$p_Y(y) = \dfrac{1}{\sqrt{\pi}}e^{-y^2}$. 所求概率$P(Y > \sqrt{2}) = 1 - \Phi(2)$

7-8. 略

习题3.3

1. $a = 1/18, b = 1/6, c = 2/9$.

2. 不独立.

3. X与Y的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 4x^3, & 0 < x < 1, \\ 0, & x \le 0\text{或}x \ge 1. \end{cases} \qquad p_Y(y) = \begin{cases} 12y^2(1-y), & 0 < y < 1, \\ 0, & y \le 0\text{或}y \ge 1. \end{cases}$$

   X与Y不独立.

4. 独立.

5. 不独立.

6. X与Y的边际概率密度函数分别为

$$p_X(x) = \begin{cases} \dfrac{1}{2}, & |x| < 1, \\ 0, & |x| \ge 1. \end{cases} \qquad p_Y(y) = \begin{cases} \dfrac{1}{2}, & |y| < 1, \\ 0, & |y| \ge 1. \end{cases}$$

   X与Y不独立.

7. (1)(X, Y)的联合概率密度函数为

$$p(x, y) = \begin{cases} e^{-y}, & 0 < x < 1, y > 0, \\ 0, & \text{其他}. \end{cases}$$

   (2)e^{-1}, (3)$1 - e^{-1}$.

习题3.4

1. Y的概率密度函数为$p_Y(y)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\sigma}\exp\left(-\dfrac{(\ln y-\mu)^2}{2\sigma^2}\right)y^{-1}, & y>0,\\ 0, & y\le 0.\end{cases}$

2. 略.

3. $(U,V)\sim N\left(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2;\mu_1-\mu_2,\sigma_1^2+\sigma_2^2;\dfrac{\sigma_1^2-\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)$

4. 当$\lambda_1=\lambda_2=\lambda$时, $Z=X+Y$的概率密度函数为$p_Z(z)=\begin{cases}\lambda^2 ze^{-\lambda z}, & z>0,\\ 0, & z\le 0.\end{cases}$

   当$\lambda_1\ne\lambda_2$时, $Z=X+Y$的概率密度函数为$p_Z(z)=\begin{cases}\dfrac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\left(e^{-\lambda_2 z}-e^{-\lambda_1 z}\right), & z>0,\\ 0, & z\le 0.\end{cases}$

   $T=\max(X,Y)$的分布函数为$F_T(t)=\begin{cases}(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t}), & t>0,\\ 0, & t\le 0.\end{cases}$

   $S=\min(X,Y)$的分布函数为$F_S(s)=\begin{cases}(1-e^{-(\lambda_1+\lambda_2)s}), & s>0,\\ 0, & s\le 0.\end{cases}$ 即$S=\min(X,Y)$服从指数分布$Exp(\lambda_1+\lambda_2)$.

5. (1)$T=X-Y$的概率密度函数$p_T(t)=\begin{cases}2(t+1), & -1<t<0,\\ 0, & \text{其他}.\end{cases}$

   (2)所求概率$P\left(Y-X\le\dfrac{1}{2}\right)=3/4$.

6. $Y\sim N(0,1)$.

7. 负二项分布$Nb(r,p)$.

8. 略.

9. 1/2.

10. $Y+1$服从几何分布$Ge(1-e^{-\lambda})$.

习题4.1

1. 略

2. $\lambda^3+3\lambda^2+\lambda$.

3. $\dfrac{1}{p}$.

4. $\dfrac{1}{4}, e-1, \dfrac{1}{12}$.

5. $\dfrac{\Gamma(n+\alpha)}{\lambda^n\Gamma(\alpha)}$.

6. 4.
7. $\dfrac{2}{3}, \dfrac{1}{3}$.
8. $\dfrac{e^{-\lambda}}{1-e^{-\lambda}}$.

习题4.2

1. $\dfrac{2-p}{p^2} - \left(\dfrac{1}{p}\right)^2$.
2. $\dfrac{\alpha}{\lambda^2}$.
3. $nm, 2mn(m+n-4)$.
4. $9/113$.
5. $e^{1/2}, e^2 - e$.
6. $8/9$.
7. $\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2,\ \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$.
8. $1, 2, 3, 1, 2, 5$
9. 略.
10. $0, 2$.

习题4.3

1. 略.
2. $\dfrac{1}{36}$
3. 0.
4. 0.
5. 0. X与Y不独立.
6. $\dfrac{1}{9}, \dfrac{1}{3}$.
7. $\dfrac{\sigma_1^2 - \sigma_2^2}{\sqrt{(\sigma_1^2 + \sigma_2^2)^2 - 4\rho^2\sigma_1^2\sigma_2^2}}$.
8. $2, \dfrac{\sqrt{2}}{2}$.

习题4.4

1. $\mu_k = \dfrac{1}{k+1}, \quad \nu_k = \begin{cases} 0, & k\text{为奇数}, \\ \dfrac{1}{(k+1)2^k}, & k\text{为偶数}. \end{cases}$

2. $\beta_s = 0, \quad \beta_k = -\dfrac{6}{5}, \quad C_v = \dfrac{\sqrt{3}}{3}.$

3. α

4. 13.92.

5. $\chi^2_{0.05}(8) = 2.7326, \quad \chi^2_{0.95}(10) = 18.3070, \quad \chi^2_{0.975}(12) = 23.3367.$

习题4.5

1. 略

2. 0.9966

3. 0.6443.

4. 0.0287.

5. 由

$$\frac{100 - 0.5 - n \cdot 0.99}{\sqrt{n \cdot 0.99 \cdot 0.01}} \leq 0.5199,$$

解得$n \geq ?$.

6-7. 略.

习题5.1

1-2. 略

3.

$$P(X_1 = k_1, X_2 = k_2, ..., X_m = k_m) = \prod_{i=1}^{m}\left[\binom{n}{k_i} p^{k_i}(1-p)^{n-k_i}\right] = \prod_{i=1}^{m}\binom{n}{k_i} p^{\sum_{i=1}^{m} k_i}(1-p)^{nm-\sum_{i=1}^{m} k_i}$$
$$k_i = 0, 1, ..., n, \ i = 1, 2, ..., m.$$

4.

$$P(X_1 = k_1, X_2 = k_2, ..., X_n = k_n) = \frac{\lambda^{\sum_{i=1}^{n} k_i}}{\prod_{i=1}^{n} k_i!} e^{-n\lambda}, \quad k_i \in \mathbb{Z}^+, \quad i = 1, 2, ..., n.$$

5.

$$P(X_1 = k_1, X_2 = k_2, ..., X_n = k_n) = \begin{cases} \frac{1}{\theta^n}, & k_i \in (0, \theta), \ i = 1, 2, ..., n \\ 0. & \text{其他} \end{cases}$$

6.

$$P(X_1 = k_1, X_2 = k_2, ..., X_n = k_n) = \begin{cases} n - e^{-\lambda \sum_{i=1}^{n} k_i}, & k_i \geq 0, \ i = 1, 2, ..., n \\ 0. & \text{其他} \end{cases}$$

7.

$$F(x)=\begin{cases}0 & x\in(-\infty,-0.5)\\ 0.1 & x\in[-0.5,-0.2)\\ 0.2 & x\in[-0.2,0.2)\\ 0.4 & x\in[0.2,0.5)\\ 0.7 & x\in[0.5,0.7)\\ 0.9 & x\in[0.7,1.5)\\ 1 & x\in[1.5,\infty)\end{cases}$$

习题5.2

1. T_1, T_2, T_4 是统计量, T_3不是统计量.

2. (1) $\overline{X}$=4.6, S^2=5.3.
   (2) $\overline{X}$=104.6, S^2=5.3.

3. (1)

$$P(X_1=k_1,X_2=k_2,...,X_n=k_n)=\prod_{i=1}^{n}p^{k_i}(1-p)^{1-k_i}=p^{\sum\limits_{i=1}^{n}k_i}(1-p)^{n-\sum\limits_{i=1}^{n}k_i}$$

$$k_i=0,1;\ i=1,2,...,n.$$

   (2)

$$P\left(\sum_{k=1}^{n}X_k=m\right)=\binom{n}{m}p^m(1-p)^{n-m},\quad m=0,1,2,...,n.$$

   (3) $p,\ \dfrac{p(1-p)}{n},\ p(1-p)$

4. $\lambda,\ \dfrac{\lambda}{n},\ \lambda$

5. 略

6. $\dfrac{n\overline{X}+X_{n+1}}{n+1},\quad \dfrac{n-1}{n}S^2+\dfrac{1}{n+1}\left(X_{n+1}-\overline{X_n}\right)^2$

习题5.3

1. $\dfrac{1}{20},\quad \dfrac{1}{100}$

2. 0.1

3. 0.705, 注意$\dfrac{(X+Y)^2}{(X-Y)^2}\sim F(1,1)$

4. $\sqrt{\dfrac{3}{2}}$

5. 1.3304, -1.3304

6. 0.5

7-8. 略

9. $t(n-1)$

10. 0, $\quad \dfrac{2}{n(n+1)}$

习题6.1

1. $\dfrac{1}{\overline{x}}$

2. $\left(\dfrac{\overline{X}}{1-\overline{X}}\right)^2$

3. $\dfrac{\overline{X}}{m}$

4. $1-\frac{S_n^2}{\overline{X}}$, $\quad \left[\frac{\overline{X}^2}{\overline{X}-S_n^2}\right]$, 其中[ ]表示取整.

5. $3\overline{X}$

6. $2\overline{X}$

7. 74.002, $\quad 6*10^{-6}$

习题6.2

1. $\frac{1}{\overline{X}}$
2. $\left(\frac{1}{n}\sum_{k=1}^{n}\ln X_k\right)^{-2}$
3. $\frac{\overline{X}}{m}$
4. $\frac{\sum_{i=1}^{2} X_i^2}{2n}$
5. $\frac{-1+\sqrt{1+\frac{4\sum_{i=1}^{n} X_i^2}{n}}}{2}$
6. $X_{(n)}$
7. $74.002, \quad 6 * 10^{-6}$
8. $\overline{X}, \frac{1}{\overline{X}}$

习题6.3

1. 略
2. 不是
3. $\overline{X}^2 - \frac{\overline{X}}{n}$
4. 当$n = 1, 2, ..., 8$时, $\hat{\theta}_1$)较有效.
   当$n \geq 9$时, $\hat{\theta}_3$与$\hat{\theta}_2$较有效, 且有效性相同, $\hat{\theta}_1$有效性较差.
5. $k_1 = \frac{1}{3}, k_2 = \frac{2}{3}$.
6. $\hat{\theta}_2$

习题6.5

1. $[2.08, 9.92]$
2. $[2.6895, 2.7205]$
3. $n \geq \left(\frac{2}{L}\right)^2 \sigma^2 u_{1-\alpha/2}^2$
4. $[6562.618, 6877.382]$
5. $[0.0566, 2.4523]$
6. $[7.4300, 21.0736]$

7. $[2.6762, 2.9238]$, $[0.0268, 0.1244]$

8. (1) $[-0.98, 0.98]$, (2)$[e^{-0.48}, e^{1.48}]$

习题7.1

1. 存伪(第二类错误), 拒真(第一类错误)

2. 控制犯第一类错误的概率

3. (1)0.0037, 0.0367.
   (2)略
   (3)34

4. 0.0016

习题7.2

1. $H_0: \mu = \mu_0 = 2.64$, $\quad H_1: \mu \neq \mu_0$
   拒绝H_0, 即有显著影响.

2. $H_0: \mu = \mu_0 = 350$, $\quad H_1: \mu \neq \mu_0$
   不能拒绝H_0, 即同意该厂的说法.

3. $H_0: \mu \leq \mu_0 = 32$, $\quad H_1: \mu > \mu_0$
   拒绝H_0, 即今年家庭平均每月耗电量有所提高.

4. $H_0: \mu = \mu_0 = 800$, $\quad H_1: \mu \neq \mu_0$
   不能拒绝H_0, 即可以认为这批产品的平均重量为800克.

5. $H_0: \sigma = \sigma_0 = 0.04$, $\quad H_1: \sigma \neq 0.04\%$
   不能拒绝H_0, 即总体方差为$\sigma = 0.04\%$.

6. $H_0: \sigma = \sigma_0 = 1.2$, $\quad H_1: \sigma \neq 1.2$
   拒绝原假设H_0, 即纱的均匀度显著变劣.