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MIM2: Multiple influence maximization across multiple social networks

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Highlights

- A novel MIM2 algorithm is proposed to maximize influence spread across multiple social networks.
- The algorithm is self-decisive in finding the budget of each product.
- The algorithm relies on multiple products and multiple networks simultaneously.
- Classical Linear Threshold and Independent Cascade diffusion models are utilized.
- The proposed algorithm is a trade-off between quality and efficiency.

MIM2: Multiple Influence Maximization across Multiple Social Networks

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Abstract

Influence maximization (IM) is the problem of selecting a small subset of users with the aim of maximizing influence spread to help marketers in promoting their products. None of the existing literature considers the scenario that a marketing company wants to promote multiple products in multiple networks or a network with the different channel of interactions simultaneously. Considering this scenario, we introduce *multiple influence maximization across multiple social networks* (MIM2) problem. This problem considers the assumption that an influential user can accept multiple products for free and non-influential users have enough purchasing power to adopt multiple promotions from their social interactions. It is also important to consider the role of overlapping users to spread the influence across networks. To address these issues, we propose a unified framework to analyze and represent the MIM2 problem. More specifically, first, we perform a mapping to couple a set of networks into a multiplex network via direct linkage strategy. Second, we propose a heuristic method to find the most influential user over multiple product diffusion multiplex networks. Third, we prove that MIM2 problem is NP-hard and expected influence spread function is sub-modular under traditional diffusion models. Finally, the experimental results show that the advantage of proposed IM problem over existing IM problems.

Keywords: Influence Maximization, Multiple Influence Maximization, Information Diffusion, Multiplex Social Networks.

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1. Introduction

In the recent decade, the online social networks like Facebook, Flixster, Twitter, Google+, and Myspace, etc., provide a platform for user's interaction and communication, marketing and promotion for the new product, idea, and innovation. These social networks diffuse and spread information of a product, news, and innovation between users via links using the *word-of-mouth* effect [1]. The word-of-mouth advertising leads to an immense application potential in viral marketing [2], rumor control [3, 4], revenue maximization [5], social recommendation [6], and network monitoring [7], etc. Inspired by the idea of viral marketing, Pedro and Matt [2] were first to introduced *Influence Maximization* (IM) as an optimization problem. IM problem finds a set of k most influential users in a social network so that the aggregate influence is maximized. Kempe et al. [8] proved that IM problem is NP-hard under traditional diffusion models. To solve IM problem, they introduced a greedy algorithm and proved that the greedy solution is approximated to within a factor of $(1 - 1/e - \varepsilon)$.

Nearly most of the previous research works only focused on network topology and ignored some critical factors like multiple product advertisement, users engagement across networks[1], different channels of interaction, and network of networks etc. For example, we consider a scenario that an advertising company wants to promote multiple products across multiple social networks simultaneously. In real-world, distinct users have different influence or interest for distinct product in social networks. Thus, each user have different influence probabilities from their neighbors.

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[1]Networks and social networks are interchangeably used in this article.

Nowadays, many users in networks are actively involved in multiple networks simultaneously. Therefore, *overlapping users*[2] play a pivot role in information spreading across networks. Thus, the study of IM problem in each network separately underestimates the social influence of overlapping users in other networks. Hence, an advertising company needs a promotional strategy that seed users can advocate multiple products together and non-seed users can accept different products across networks simultaneously.

Correspondingly, we propose a framework *multiple influence maximization across multiple social networks* (MIM2) for the above scenario. Compared to traditional IM approaches, multiple influence maximization across multiple social networks introduces several new challenges:

1. How to fix the number of items for each product from budget k and how to find seed users S, $|S| \leq k$.
2. How to identify overlapping users on different social networks and how to coupled these networks into a single one. Also, how to measure the social influence of overlapping and non-overlapping users accurately in a coupled network for each product.
3. How to decide, for which product a user is easily influenced and in which network. Also, in which network information is propagated better.

To tackle these challenges, we present a novel framework to illustrate multiple influence maximization across multiple social networks.

Contribution. To the best of our knowledge, we are the first to introduce MIM2 problem. The major contributions of our work are as follow.

- We propose a novel framework MIM2 for influence maximization across multiple social networks over multiple products simultaneously.
- We prove that MIM2 problem is NP-hard and expected influence spread $\sigma(S)$ is sub-modular under traditional diffusion models.
- We present a model representation for MIM2 problem equivalent to classical IM problem. We also present various coupling strategy to form a multiplex network from multiple single network. The proposed coupling methods can be applied to traditional diffusion models.
- We propose a novel MIM2 algorithm to find seed set across multiple networks with multiple products simultaneously. It also fixes the budget for each product to maximize influence spread.
- We perform experimental analysis to validate the efficiency and influence spread of the proposed algorithm. It also covers the advantage of MIM2 framework over classical IM problem.

Paper organization. The rest of the paper is organized as follows. Section 2 presents a discussion on related work of the IM problem. Section 3 describes primary information regarding problem formulation. Section 4 presents graph notations, influence propagation model, and problem definition. Section 5 explains the proposed MIM2 framework. Section 6 discusses the algorithm with an analysis of their time complexity. Section 7 explains the experimental setup and result analysis of real-world social networks. Section 8 draws the conclusion and future directions.

2. Literature review

Most of the existing research works studied the different variant of IM problem for a single product on separate social networks. Kempe et al. [8] firstly formatted the IM problem and introduced a greedy framework to solve the IM problem. They proposed two widely known influence propagation model, *independent cascade* (IC) model and *linear threshold* (LT) model, where information disseminates in discrete steps, further treated it as a discrete optimization problem and proved it is NP-hard for both two propagation model. Simultaneously, they worked out the greedy algorithm framework with an approximate ratio of $(1-1/e)$. Svirdenko et al. [9] extends IM problem defined by [8] considering non-uniform selection cost. Based on node potential and sub-modular property of the objective

[2]Users who actively engage in multiple networks simultaneously.

function, some more variants of greedy algorithm are introduced [10–12]. The greedy algorithm has low efficiency due to time-consuming Monte-Carlo (MC) simulations and can hardly adapt to large scale online social network, there are so many works to accelerate influence computation and seed selection, such as heuristic-based approaches [10, 13, 14], influence-path based approaches [15–17], community-based approaches [18–20], sampling based [21–23], diffusion probabilities estimation [24, 25], and score estimation based [26–28].

Some of the users in online social networks participate in multiple social networks simultaneously. Therefore, IM approaches of a single network ignore the social influence of users in other participating networks. Hence, these approaches are not able to estimate the social influence of users accurately. Inspired by the above weakness of IM approaches, researchers have started to explore *influence maximization on multiple networks* (IM2). In order to propagate information across social networks, there is a need for a network coupling strategy. Liu et al. [29] presented a coupling strategy to form multiplex. The authors focused on understanding the flow of information and network clustering. Yagan et al. [30] investigated the information diffusion process in over-laying networks. They investigated the outbreak of information using the SIR model on random networks. Considering user involvement and interest, Shen et al. [31] presented information propagation strategy for multiplex networks. The authors used a lossy coupling scheme and ignore individual network properties. Zhang et al. [32] proposed an improved greedy algorithm for influence maximization across multiple social networks. They also proposed new lossless and lossy coupling schemes for linking multiple social networks effectively.

Inspired by the idea that IM for multiple products simultaneously, Sun et al. [33] proposed *multiple influence maximization* (MIM) problem in a single network. They consider a separate product diffusion graph for each product. The authors adopt a greedy framework for seed selection. They assume that the influence of a node is limited to six-hope area. The experimental results show that the advantage of MIM over classical IM problem. Recently, Context-aware influence maximization techniques are introduced to improve the effectiveness of seed. Some research works have been proposed considering contextual features such as location-aware [34, 35], time-aware [36, 37], topic-aware [38–40], and competitive [41, 42]. Inspired by IM2 and MIM, we present a novel framework *multiple influence maximization across multiple social networks* (MIM2). To the best of our knowledge, this is the first work considering MIM2 framework.

3. Preliminaries

3.1. Notations

Notations and Abbreviations that are needed for problem formulations in this paper are given in Table 1 and 2 respectively.

Table 1: Notations

$G(V,E,W)$	$\triangleq$	A social network with vertex set V and edge set E with edge weight set W
$N(u)$	$\triangleq$	The neighbors set of node u
V_a	$\triangleq$	The set of active nodes
S	$\triangleq$	Seed set
k	$\triangleq$	The number of nodes in seed set ($\|S\|$)
p_{xy}	$\triangleq$	The influence probability of x to adjacent node y.
$\sigma(S)$	$\triangleq$	The expected influence of seed set in the network, i.e., $Inf(S)$
m	$\triangleq$	The number of products for advertising
l	$\triangleq$	The number of channel of interaction or relationship
L	$\triangleq$	The set of relationships

3.2. Definitions

Definition 1. ***(Social Network).*** *A social network with N users and M social ties is represented as a weighted-directed graph $G(V,E,W)$. Here, V denotes set of users, $|V| = N$, and E represents a set of relationships, $|E| = M$, and W represents edge-weight. A social network is also known as an influence graph.*

Definition 2. ***(Neighbors).*** *Neighbors $N(u)$ of node u is defined as the set of users v such that $v \in N(u)$ iff $\exists(u,v) \in E$, $v \in V$. $N_{inc}(u)$ and $N_{out}(u)$ denote in and out neighbors of node u respectively.*

Table 2: Abbreviations

IM	$\triangleq$	Influence Maximization
IM2	$\triangleq$	Influence Maximization across Multiple social networks
MIM	$\triangleq$	Multiple Influence Maximization
MIM2	$\triangleq$	Multiple Influence Maximization across Multiple social networks
IC	$\triangleq$	Independent Cascade model
LT	$\triangleq$	Linear Threshold model

Definition 3. ***(Degree centrality).*** *Degree centrality is defined as the number of links incident upon a node i.e. $C_D(u) = |N(u)|$. In directed social network, degree of u is considered as $C_D(u) = |N_{out}(u)|$.*

Definition 4. ***(Seed nodes).*** *Seed nodes (S) are the set of nodes who act as the source of the information propagation process in the social network, $|S| = k$, $S \in V$.*

Definition 5. ***(Active Node).*** *A node $u \in V$ is called active if either $u \in S$ or u adopted the information propagated by previously active nodes $v \in V_A$ under diffusion model. Once u is activated, then $V_A \leftarrow \{V_A \cup u\}$.*

Definition 6. ***(Influence spread).*** *Influence spread $I_S(S)$ of the seed set S is defined as the number of active users after diffusion process under a diffusion model, i.e., $I_S(S) = |V_A(S)|$.*

Definition 7. ***(Information Diffusion Model (IDM/DM)[43].)*** *Given an influence graph $G = (V, E, W)$, a seed set $S \subseteq V$ and an information diffusion model captures the stochastic process for S influence spreading on graph G.*

Definition 8. ***(Influence Maximization (IM) [8]).*** *Given an influence graph $G = (V, E, W)$, an information diffusion model, a positive integer k, then influence maximization process selects a seed set $S \subseteq V$ of k users to maximize the influence spread in G, i.e., $\sigma(S) = argmax_{S^* \subseteq V \wedge |S^*| = k} \sigma(S^*)$.*

3.3. Diffusion model

Kempe et al. [8] incorporated two basic diffusion models, *linear threshold* (LT) and *independent cascade* (IC) for information propagation. In both of these models, each node at any time-stamp t belongs to one of the two state: inactive and active. Nodes that are not influenced by their neighbors or not heard about the product are known as inactive nodes. Initially at time $(t = 0)$, all nodes are inactive. Active nodes are influenced by their neighbors and only such nodes can propagate influence to their neighbors.

3.3.1. Linear threshold model (LT)

In this model, every node x has an activation threshold θ_u and a node y becomes active only if $\Sigma_{x \in N^A_{inc}(y)} w(x,y) \geq \theta_x$ where $N^A_{inc}(y)$ and $w(x,y)$ are set of active incoming neighbors of y and edge weight of (x,y) respectively. For each edge $(x,y) \in E$, we assign edge weight $w(x,y)$ in LT model as follows.

$$w(x,y) = \begin{cases} \frac{1}{indegree(y)} & \text{Uniform} \\ (0,1) & \text{Random} \\ \frac{c(x,y)}{\Sigma c(x,y)} & \text{Parallel} \end{cases} \quad (1)$$

where $c(x,y)$ is the number of parallel edges from x to y in multi-graph.

3.3.2. Independent cascade model (IC)

In this model, when a node x becomes active at time t, it has a only chance to activate its inactive neighbors y with activation probability p_{xy} at the time $(t+1)$. If node y becomes active at the time $(t+1)$ then it will never be inactive in future. The diffusion process terminates if no node is activated at time $(t+1)$. For each edge $(x,y) \in E$, we assign edge weight $w(x,y)$ in IC model as follows.

$$w(x,y) = \begin{cases} [0.01, 0.1] & \text{Constant} \\ \frac{1}{indegree(y)} & \text{Weighted cascade} \\ \{0.001, 0.01, 0.1\} & \text{Tri-valency model} \end{cases} \quad (2)$$

4. Model and problem definition

4.1. Graph notations

Suppose, there are l graphs $G_1, G_2, \ldots, G_l$, each of the graph is represented as $G_i(V_i, E_i, W_i)$, $1 \leq i \leq l$. The set V_i, E_i, and W_i denote the vertex set, edge set, and strength of their relationship respectively in graph G_i. The $N^i_{in}(x)$ and $N^i_{out}(x)$ represent the set of incoming and outgoing neighbors of a user x in network G_i respectively. We modeled these l networks into a single multiplex network $G(V, E, W)$ by a coupling strategy through *overlapping users* to formulate MIM2 problem. The *overlapping users* can be identified using methods in [44–46]. Then, we construct a graph $G^i(V, E, W^i)$, $1 \leq i \leq m$ from G for each product $i \in m$ to propagate each product influence independently.

4.2. Influence propagation model

To compute the influence spread of seed set S in multiplex network G, we incorporated classical diffusion models present in Section 3.3. First, we start the computation of influence spread for IC model. Similar to [8], there are several paths $path_{x,y}$ exist between a pair of nodes x and y. Let $P(path^j_{x,y})$ represents the influence of an individual x to y along a path $path^j_{x,y} = (x = y_1, y_2, \ldots, y_t = y)$, $path^j_{x,y} \in path_{x,y}$, given as follows.

$$P(path^j_{x,y}) = \prod_{i=1}^{i=t-1} p(y_i, y_{i+1}) \tag{3}$$

where $p_{x,y}$ is the influence probability of x to adjacent node y. The influence probability $P(x,y)$ of non-adjacent nodes x to y based on assumption that influence spreads independently along different paths, is given as follows.

$$P(x,y) = 1 - \prod_{path^j_{x,y} \in path_{x,y}} (1 - P(path^j_{x,y})) \tag{4}$$

Now, we can estimate the influence spread of seed set S in the network under IC. The influence spread of node x can be computed by sum up the influence of each reachable path from node x, given as follows.

$$\sigma(x) = 1 + \sum_{y \in V \setminus x} P(x,y) \tag{5}$$

Similarly, we can compute $\sigma(S)$, as follows.

$$\sigma(S) = |S| + \sum_{y \in V \setminus S} P(S,y) \tag{6}$$

4.3. Problem definition

Given l influence graphs $G_i = (V_i, E_i, W_i)$, an information diffusion model, number of products m, and budget k. Influence maximization process selects a seed set $S \in V$ to maximize the influence spread in G, i.e.,

$$\sigma(S) = argmax_{S^* \subseteq V \wedge |S^*| = k} \sigma(S^*) \tag{7}$$

where, $V = \bigcup_{i=1}^{i=l} V_i$, $S^* = \bigcup_{i=1}^{i=m} S^*_i$, $\sum_{i=1}^{i=m} |S^*_i| = k$ and $|S| \leq k$. The objective function $\sigma(S)$ is sub-modular. An arbitrary function is called sub-modular if it satisfies following two properties.

Property 1. ***(Monotone increasing)*** *An objective function $\sigma(S)$ is monotone increasing iff $\sigma(S) \leq \sigma(T)$, $S \subset T$.*

Property 2. ***(Diminishing returns)*** *An objective function $\sigma(S)$ is diminishing return iff $\sigma(S \cup u) - \sigma(S) \geq \sigma(T \cup u) - \sigma(T)$, $\forall u \in T$ and $S \subset T$.*

Theorem 1. *The expected influence spread function $\sigma(S)$ of MIM2 is sub-modular under traditional diffusion models.*

Proof. In MIM2 problem, we incorporated traditional diffusion models LT and IC from [8] for influence diffusion. We use the same propagation strategy as in LT and IC. MIM2 considers multiple products and multiple networks simultaneously to improve the effectiveness of seed nodes. From problem definition, it is clear that if MIM2 consider $l = 1$ and $m = 1$, then MIM2 problem reduces to original IM problem formulated by [8]. Therefore, diminishing return and monotone increasing properties directly inherited from LT and IC diffusion models. Hence, we can conclude that the expected influence spread $\sigma(S)$ of MIM2 remains sub-modular under traditional diffusion models. □

Theorem 2. *The MIM2 problem under traditional influence diffusion models is NP-hard.*

Proof. As we discussed earlier, the MIM2 problem can be reduced in conventional IM problem presented in [8] by considering the number of networks $l = 1$ and the number of products $m = 1$. As we know IM problem is NP-hard under traditional diffusion models proved by [8]. Hence, we can say that MIM2 is NP-hard under diffusion models. □

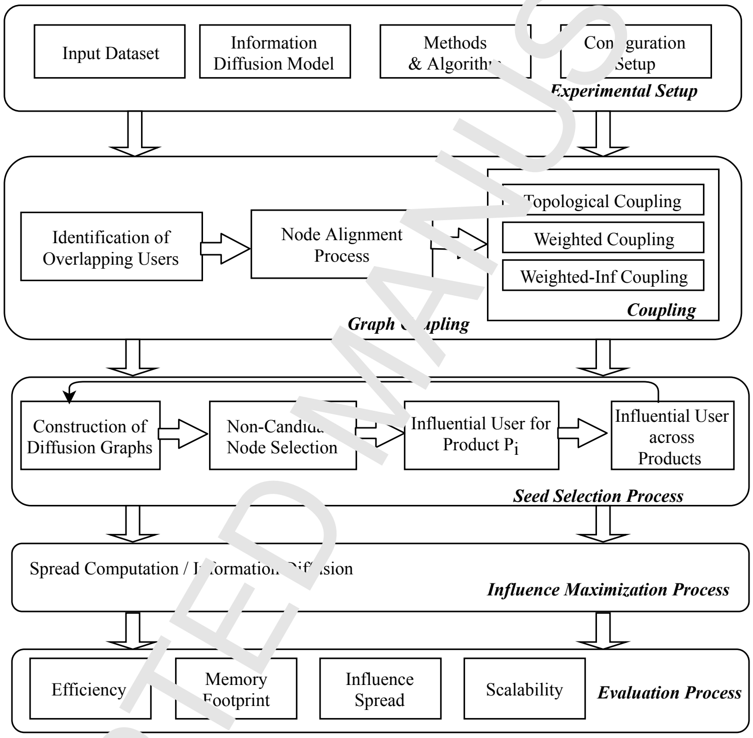


Figure 1: The MIM2 framework

5. MIM2 framework

Suppose, an advertising company wants to select a small set of influential users who interact among a finite set of users through different channels of interaction. For example, the Twitter network consists of three types of interaction networks, such as reply, re-tweet, and mention networks. Each interaction network has the same set of users with the different channel of interaction as shown in Figure 2. Figure 1 presents the proposed MIM2 framework. Let graph $G = (V, E, L)$ represents a complex social network with the different type of relationships, where V is a finite set of users, E is a finite set of relationship between a pair of users with L channels of interaction. The main **Algorithm 1** explains the propose MIM2 framework.

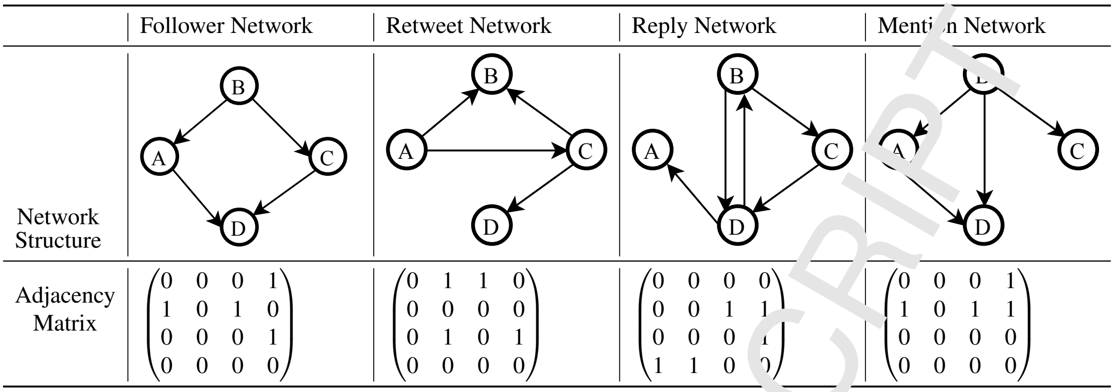


Figure 2: An example graph of Twitter network (Follower (FN) network) with three type of of interaction networks (Retweet (RTN), Reply(REN), and Mention (MEN) networks) with their adjacency matrix.

Algorithm 1: MIM2 (G_i, k, l, m)

Input: $G_i(V_i, E_i, W_i)$, $1 \leq i \leq l$: Social graphs, k: Size of the seed set, l: Number of social graphs, m: Number of products

Output: S: Seed set

```
S ← φ
Remaining ← k
G ← Apply one of the coupling strategy based on Algorithm 2, 3, or 4
Find product diffusion graph G^i of multiplex graph G for each product i, 1 ≤ i ≤ m
while Remaining > 0 do
    seed_node ← NextSeedB(G^i, update_i, Acc_i, seed_i)        ▷ See Algorithm 7
    seed_node.Graph.update ← 0
    seed_node.deleted = True
    S[seed_node.Graph].insert(seed_node)
    Remaining ← Remaining − 1
Return S
```

5.1. Identification of overlapping users

MIM2 problem considers users with accounts on multiple social networking sites like Facebook, Twitter, etc., and it brings the opportunity to account the user's influence across multiple networks. This leads to the generation of more effective seed nodes. To perform information diffusion across multiple networks simultaneously, we need to find users who actively involved in multiple networks. These users are useful in graph coupling and known as overlapping or identical users. The identification of overlapping users is a challenging task because of the diversification of users information across networks and also network data is noisy, big, unstructured and incomplete. The overlapping users can be identified using methods present in [44–46]. The identification of overlapping users is not in the focus of this paper. We consider an assumption that a user has a single account in a network.

5.2. Node alignment process

Node alignment is the process of reassigning a *universal identification number* (Uid) to every node u in each network so that every overlapping user has the same Uid across networks. In general, every network has its own node naming system. Therefore, it might be possible that a user has different identification in the different networks. As a consequence, we need to take care of user states across networks repeatedly and it results in extra effort and a complicated mechanism. Therefore, We ease this problem by reassigning a universal Uid to each individual in each network.

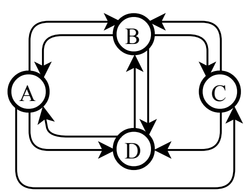
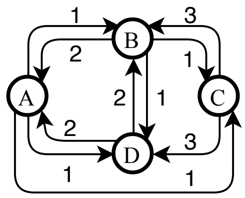
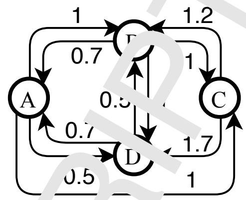
	Topological Coupling	Weighted Coupling	Weighted-influence Coupling
Coupled Multiplex Network			
Multiplex Adjacency Matricx	$\begin{pmatrix}0&1&1&1\\1&0&1&1\\0&1&0&1\\1&1&0&0\end{pmatrix}$	$\begin{pmatrix}0&1&1&1\\2&0&1&1\\0&3&0&3\\2&2&0&0\end{pmatrix}$	$\begin{pmatrix}0&1&1&0.5\\0.7&0&1&0.5\\0&1.2&0&1.7\\0.7&1&0&0\end{pmatrix}$
Normalized Adjacency Matricx	$\begin{pmatrix}0&1&1&1\\1&0&1&1\\0&1&0&1\\1&1&0&0\end{pmatrix}$	$\begin{pmatrix}0&0.3&0.3&0.3\\2&0&0.3&0.3\\0&1&0&1\\0.6&0.6&0&0\end{pmatrix}$	$\begin{pmatrix}0&0.5&0.5&0.2\\0.4&0&0.5&0.2\\0&0.7&0&1\\0.4&0.5&0&0\end{pmatrix}$

Figure 3: The coupling of networks.

5.3. Graph coupling scheme: Direct linkage

In the linkage strategy, each relationship graph $L_i \in L$ is considered as an individual network. In order to transform these relationship networks into a single multiplex network $G = (V, E, W)$, we need to aggregate all relationships between each pair u and v to form a single relationship. Let us consider a complex system $G = (V, E, L)$, $L = \{L_1, L_2, \ldots, L_l\}$ with l types of relationship. Each interaction network $L_i \in L$ is integrated with an adjacency matrix $A^{L_i} = \{a_{uv}^{L_i}\}$. Therefore, such a complex system can be expressed as $A = [A^{L_1}, A^{L_2}, \ldots, A^{L_l}]$. Similarly, the degree of an individual u can be represented as a vector $D(u) = [D^{L_1}(u), D^{L_2}(u), \ldots, D^{L_l}(u)]$, where $D^{L_i}(u) = \sum_{v \in V} a_{uv}^{L_i}$. Both vector A and $D(u)$ are requisite to represent the complex system correctly. Figure 2 shows an example Twitter graph with four types of relationship and also presents their corresponding adjacency matrices. From Figure 2, we can see that $l = 4$ and $D(A) = [1, 2, 0, 1]$.

Algorithm 2: Topological_Coupling($G_i(V_i, E_i, W_i), l$)

Input: $G_i(V_i, E_i, W_i)$, $1 \leq i \leq l$: Social graphs
Output: $G(V, E, W)$: Topological multiplex social graph

$V\_set \leftarrow \phi$
$Edge\_set \leftarrow \phi$
$Weight \leftarrow \phi$
for *each graph* G_i, $1 \leq i \leq l$ **do**
  for *each edge* $(u, v) \in E_i$ **do**
    $Edge\_set \leftarrow Edge\_set \cup (u, v)$
    $V\_set \leftarrow V\_set \cup \{u, v\}$
for *each edge* $(u, v) \in Edge\_set$ **do**
  $Weight[u, v] \leftarrow 1$
Return $G(V\_set, Edge\_set, Weight)$

In order to aggregate these relationships to a single relationship, the direct linkage strategy presents two types of adjacency matrix: topological and weighted adjacency matrices. The topological adjacency matrix ignores the fact that different type of relationships may present in such a network between a pair of individuals. It ignores the possibility of multi-link and it shows a connection between u and v if a link present in any one of the interaction network. The topological adjacency matrix $A_{TMN} = \{a_{uv}\}$ of a coupled multiplex network is defined as similar to [47], where

$$a_{uv} = \begin{cases} 1 & if \quad \exists a_{uv}^{L_i} = 1, L_i \in L \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

For example, Figure 3 shows the topological multiplex network of example graph as shown in Figure 2 with their corresponding adjacency matrix.

Algorithm 3: Weighted_Coupling$(G_i(V_i, E_i, W_i), l)$

Input: $G_i(V_i, E_i, W_i)$, $1 \le i \le l$: Social graphs
Output: $G(V, E, W)$: Weighted multiplex social graph

```
V_set ← φ
Edge_set ← φ
Weight ← φ
for each graph G_i, 1 ≤ i ≤ l do
    for each edge (u,v) ∈ E_i do
        if (u,v) ∈ Edge_set then
            Weight[u,v] ← Weight[u,v] + 1
        else
            Edge_set ← Edge_set ∪ (u,v)
            V_set ← V_set ∪ {u,v}
            Weight[u,v] ← 1
Max_weight ← max(Weight)                    ▷ Normalize edge weight
for each edge (u,v) ∈ Edge_set do
    Weight[u,v] ← Weight[u,v] / Max_weight
Return G(V_set, Edge_set, Weight)
```

The weighted adjacency matrix of coupled multiplex network considers the possibility of multi-link between a pair of individual. The weighted adjacency matrix $A_{WMN} = \{w(u,v)\}$ is defined as similar to [47], where

$$w(u,v) = \sum_{L_i \in L} a_{uv}^{L_i}; 0 \le w(u,v) \le l, \forall u, v \in V \tag{9}$$

For example, Figure 3 shows the weighted multiplex network of example graph as shown in Figure 2 with their corresponding weighted adjacency matrix.

To construct real dynamics of information propagation in a social network, we need to identify which type of relationship is best suited for information diffusion and which is worst. For this purpose, we uses a probability vector $\alpha = [\alpha^{L_1}, \alpha^{L_2}, \ldots, \alpha^{L_l}]$ to represent importance of each type of relationship. Therefore, weight index $w(u,v)$ of each link (u,v) is calculated as follows.

$$w(u,v) = \sum_{L_i \in L} a_{uv}^{L_i}.\alpha^{L_i}; 0 \le \alpha^{L_i} \le 1, 0 \le \sum_{L_i \in L} \alpha^{L_i} \le l \tag{10}$$

For example, let us consider that probability vector $\alpha = [\alpha^{SN}, \alpha^{RTN}, \alpha^{REN}, \alpha^{MTN}] = [0.2, 1, 0.5, 0.5]$. Figure 3 shows the weighted-influence multiplex network with probability vector and corresponding weighted matrix. We need to perform normalization of edge weight $w(u,v) \in [0,1]$ of every individual u and v in weighted and weighted-influence coupling. The overall time complexity of the coupling scheme is $O(\sum_{i=1}^{i=l}(|V_i| + |E_i|))$.

Algorithm 4: Weighted_Influence_Coupling(G_i, l, α)

Input: $G_i(V_i, E_i, W_i)$, $1 \le i \le l$: Social graphs, α: Probability vector
Output: $G(V, E, W)$: Weighted influence multiplex graph

```
V_set ← φ
Edge_set ← φ
Weight ← φ
for each graph G_i, 1 ≤ i ≤ l do
    for each edge (u,v) ∈ E_i do
        if (u,v) ∈ Edge_set then
            Weight[u,v] ← Weight[u,v] + α^{L_i}
        else
            Edge_set ← Edge_set ∪ (u,v)
            V_set ← V_set ∪ {u,v}
            Weight[u,v] ← α^{L_i}
Max_weight ← max(Weight)                        ▷ Normalize edge weight
for each edge (u,v) ∈ Edge_set do
    Weight[u,v] ← Weight[u,v] / Max_weight
Return G(V_set, Edge_set, Weight)
```

5.4. Multiple influence maximization

In this section, we discuss the IM process for multiple products in a multiplex network achieved from the above discussed coupling strategy.

5.4.1. Generation of product diffusion graph

To maximize the influence spread in the multiplex network for m products simultaneously, MIM2 constructs a social graph corresponding to each product. The edge weight in the social graph of each product follow a probability distribution for information diffusion and all graphs share the same set of users. The influence propagation for each product is independent in the network. The objective of MIM2 is to select k seed nodes from these weighted product diffusion graphs to maximize the overall influence spread of products. The overall complexity of generating m diffusion graph is $O(m(|V|+|E|))$.

5.4.2. Finding non-candidate nodes

The non-candidate nodes are those who less likely to be influential in the network. We find the fixed percentage of non-candidate nodes from the network to trace back to maximum influencing seeds using the reverse graph. To identify non-candidate nodes, first, we remove all the edges with weight less than $w\%$ of the average weight of the network. Second, assign the estimated influence of nodes already selected as seed equal to infinity. Next, assign the estimated influence of remaining nodes equal to the sum of weights of outgoing edges. Finally, we select t least weighted nodes as non-candidate. The overall time complexity of finding non-candidate set in a graph is $O(|E|+|V|\log|V|)$.

5.4.3. Identifying most influential user for product P_i

To identify the most influential user for a product P_i, MIM2 selects the corresponding product diffusion graph and performs backward propagation to find the most influential user. First, the algorithm selects non-candidate nodes and then it traverses the graph in reverse order from these nodes using Breadth First Search (BFS) to accumulate the expected influence spread of each node. Finally, the algorithm selects the node with the maximum expected influence as the most influential user for the the corresponding product. The time complexity of finding the most influential user over a graph is $O(|E|+|V|\log|V|)$.

Algorithm 5: Finding_Non_Candidates(G_i, t, w)

Input: $G_i(V_i, E_i, W_i)$: Social graph, w: Threshold weight percentage, t: Number of non-candidate nodes
Output: Non_Cand: Non-candidate nodes

$Sum \leftarrow 0$
for *each edge* $(u, v) \in E_i$ **do**
  $Sum \leftarrow Sum + W_i[u, v]$
$Average \leftarrow \frac{Sum}{|E|}$
$Threshold \leftarrow Average * w$
$Est\_inf \leftarrow$ Initialize estimated influence of each node to 0
for *each edge* $(u, v) \in E_i$ **do**
  if $W_i[u, v] > Threshold$ **then**
    $Est\_inf[u] \leftarrow Est\_inf[u] + W_i[u, v]$
$V\_Sort \leftarrow G_i.V_i$ in ascending order with respect to estimated influence Est_inf
$Non\_Cand \leftarrow$ Select first t nodes of V_Sort as non-candidate nodes
Return Non_Cand

Algorithm 6: NextSeedA($G^i, update, Acc, seed$)

Input: $G_i(V, E, W^i)$: Social graph, $update$: Vector indicated whether influence value is updated, Acc: Vector store accumulated influence so far, $seed$: Current seed set so far
Output: x: Most influential seed

if $G^i.update = 1$ **then**
  Return max(Acc)
$Nds \leftarrow$ Finding_Non_Candidates(G^i, t, w)
Queue Q
for *each node* $u \in Nds$ **do**
  $Q.push(u)$
  $u.visited \leftarrow 1$
Initialize Acc value of each node to 1
while *(!Q.empty())* **do**
  $r \leftarrow Q.front()$
  $Q.pop()$
  for *each edge* $(v, r) \in E$ **do**
    $x \leftarrow v$
    if $x \notin seed$ **then**
      if $!(x.visited == 1)$ **then**
        $x.visited = 1$
        $Q.push(x)$
      $Acc[x] \leftarrow Acc[x] + Acc[r] * W^i[v, r]$
$G^i.update \leftarrow 1$
Return max(Acc)

5.4.4. Identifying most influential user among all product diffusion graph

The algorithm finds seed nodes by selecting most influential node among all product diffusion graphs iteratively. To find most influential user among all product, it selects most influential user for each product P_i, $1 \leq i \leq m$ by reverse tracing using BFS and compare with each other to find maximal spread node. After finding most influential node, add it

---

Algorithm 7: NextSeedB$(G^i, update_i, Acc_i, seed_i)$

Input: $G^i(V,E,W^i)$, $1 \leq i \leq m$: Social graphs, $update_i$: Vector indicated whether influence value is updated, Acc_i: Accumulated influence so far, $seed_i$: Current seed set so far

Output: x: Most influential seed among all m-graphs

$x \leftarrow Null$

for *each graph* G^i, $1 \leq i \leq m$ **do**

  $x \leftarrow \max\big(x, \text{NextSeedA}(G^i, update_i, Acc_i, seed_i)\big)$ ▷ See **Algorithm 6**

Return x

---

to seed set and delete this node from the corresponding product diffusion graph. This process is continued until k nodes are selected.

5.4.5. *Influence estimation*

To evaluate the performance of the proposed algorithm, we need to compute the influence spread of selected seed nodes in the coupled multiplex network. We use the same propagation strategy as discussed in Section 4.2 under traditional diffusion models.

6. Algorithm

In this section, we provide a detailed description of each algorithm. The main **Algorithm 1** MIM2 takes four inputs, l social graphs, budget k, number of graphs l, and number of products m. Line 1 initializes seed set S with an empty set. Line 2 assigns *Remaining* to budget k. Line 3 performs coupling on l social graphs and gets a multiplex network G. Line 4 constructs product diffusion graph using normal distribution. Lines 5-10 iteratively finds the most influential node across diffusion graph and adds it to seed set S. Line 11 returns the seed set S as the final output.

The **Algorithm 2** Topological_Coupling takes l influence graphs as input and it returns a coupled multiplex network based on topological structure as output. First of all, lines 1-3 assign an empty set to V_set, $Edge\_set$, and $Weight$. The **for** loop in lines 4-7 add edge and vertex of individual graphs in $Edge\_set$ and V_set. The **for** loop in lines 8-9 perform coupling based on their topological structure. Line 10 returns a coupled multiplex network.

The **Algorithm 3** Weighted_Coupling takes l influence graphs as input and it returns a coupled multiplex network based on their weighted topological structure as output. Lines 1-3 assign an empty set to V_set, $Edge\_set$, and $Weight$. Lines 4-11 perform coupling based on their weighted topological structure by considering the different type of relationships. Line 12 finds the maximum possible weight of an edge. Line 13-14 performs normalization of edge weight. Line 15 returns a coupled multiplex network.

The **Algorithm 4** Weighted_Influence_Coupling takes l influence graphs as input and it returns a coupled multiplex network based on their weighted topological structure as output. Lines 1-3 assign an empty set to V_set, $Edge\_set$, and $Weight$. Lines 4-11 perform coupling based on their weighted influence by considering their different diffusion behavior. Line 12 finds the maximum possible weight of an edge. Line 13-14 performs normalization of edge weight. Line 15 returns a coupled multiplex network.

The **Algorithm 5** Finding_Non_Candidates takes three inputs, a social graph G_i, threshold weight percentage w, and the number of non-candidates t. Line 1 initializes the *Sum* with 0. Lines 2-3 calculate the sum of influence weight of each edge in the graph G_i. Line 4 estimates the average influence of an edge in the graph. Line 5 sets a threshold value for the selection of non-candidate nodes. Line 6 initializes the estimated influence Est_inf of each node u to 0. Lines 7-9 compute Est_inf for each node u iteratively. Line 10 performs sorting in ascending order based on Est_inf. Line 11 selects t non-candidate seed nodes and store in a vector Non_Cand. Line 12 returns the Non_Cand as an output.

The **Algorithm 6** NextSeedA takes four inputs, a social graph G_i, update status vector $update$, accumulated influence vector Acc, and current seed set $seed$. Line 1-2 checks that the influence values of nodes are already updated or not. Line 3 calls Finding_Non_Candidates algorithm and finds non-candidate nodes Nds. Line 4 maintains a queue Q. Lines 5-7 marks non-candidate nodes as visited and push Nds nodes into Q. Line 8 initializes accumulated influence Acc of each node to 1. Line 9-19 updates Acc iteratively for each node. Line 10-11 pop the node from the front and store it to r, if queue Q is not empty. Lines 12-18 updates Acc iteratively based on each edge. Line 19 marks the graph as updated. Line 20 returns maximum accumulated influence node as most influential node in the graph.

The **Algorithm 7** NextSeedB takes four inputs, m social graphs, update status vector $update_i$, accumulated influence vector Acc_i, and current seed set $seed_i$ for each graph G_i. Line 1 initializes most influential node x to null. Lines 2-3 find the most influential user across m social graphs and store it to x. Line 4 returns x as most influential user.

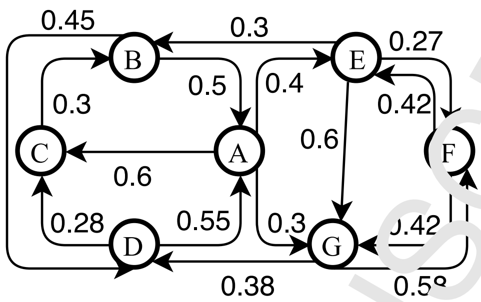


Figure 4: A running example.

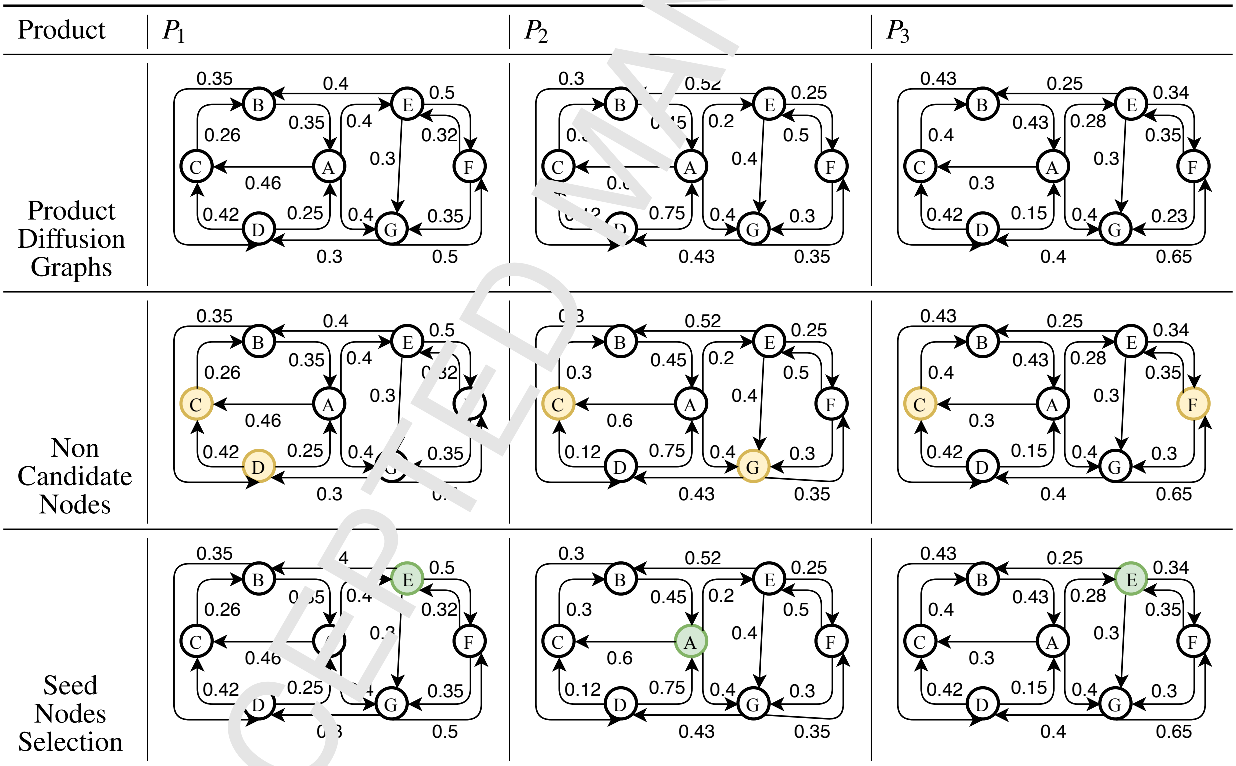


Figure 5: The working of seed selection process in MIM2 framework.

6.1. Applying the Algorithm

In this section, we present a running example to explain the working of the proposed algorithm for MIM2 problem. Let consider a coupled multiplex network and $m = 3$, which is constructed using the weighted-influence coupling scheme and shown in Figure 4. In the seed selection process of MIM2, first, we construct the product diffusion graphs for each product P_i. Let assume that the product diffusion graphs are constructed using normal distribution and given in Figure 5. Next, MIM2 selects non-candidate nodes in each product diffusion graph G^i. Each edge (u,v) in G^i has edge weight $W^i[u,v] \leftarrow W_i[u,v] * W_D^{P_i}[u,v]$, where $W_i[u,v]$ and $W_D^{P_i}[u,v]$ denote edge weight of edge (u,v) in the coupled multiplex graph and normal distribution edge in diffusion graph of product P_i. To find non-candidate nodes, we consider $w = 80\%$, $t = 2$ and calculates expected influence $Average$ for P_1 as $Average \leftarrow Sum/|E|$, i.e. $Average \leftarrow ((0.4+0.4+0.46)+(0.35+0.35)+(0.26)+(0.25+0.42)+(0.5+0.4+0.3)+(0.32+0.35)+(0.5+0.3))/15 = 0.37$. Before calculating estimated influence $Est\_Inf[u]$ of a node u, we compute threshold value $Threshold \leftarrow 0.37 * 0.8 = 0.29$. Now, we compute $Est\_Inf[u]$ of each node u. The Est_Inf is calculated as $Est\_Inf \leftarrow [0.4+0.4+0.46, 0.35+0.35, 0, 0.42, 0.5+0.4+0.3, 0.32+0.35, 0.5+0.3] = [1.26, 0.7, 0, 0.42, 1.2, 0.67, 0.8]$. Next, we select least-$t$ nodes based on Est_Inf value as non-candidate nodes N_C, i.e. $N_C \leftarrow \{C,D\}$ for product diffusion graph G^1. Let non-candidate nodes for graph G^2 and G^3 are $\{C,G\}$ and $\{C,F\}$ respectively. The yellow colored nodes represent non-candidate nodes as in Figure 5.

Next, MIM2 selects seed by reverse tracing using BFS and assume budget $k = 3$. To select the most influential node in graph G^1, initially, queue Q has N_C nodes, i.e. $Q \leftarrow \{C,D\}$ and marks these nodes as visited. The accumulated influence Acc for each node is 1 at start of reverse tracing. The accumulated influence $Acc \leftarrow [2.9, 2, 1.52, 1.78, 3.32, 2.2, 2.52]$ of G^1 is computed based on algorithm NextSeedA as shown in Table 3. Therefore, most influential user in G^1 is E. We assume that most influential user for graph G^2 and G^3 are A and F respectively. Using algorithm NextSeedB, MIM2 selects most influential user among all products is E for P_1. Therefore, E is added to seed set S and removes from G^1. Similarly, remaining seed nodes are selected. Node E and A are selected from graph G^3 and G^2 respectively as seed. The green colored nodes represent seed nodes in the respected diffusion graph, as shown in Figure 5. Finally, MIM2 procedure combines the all seed set S_i for G_i. The overall seed set $S \leftarrow \bigcup_{i=1}^{i=m} = \{E\} \cup \{A\} \cup \{E\} = \{A,E\}$, $\sum_{i=1}^{i=m} |S_i^*| = k = 3$ and $|S| = 2 \leq k$. Figure 6 shows the final seed nodes in green color under the MIM2 framework.

Table 3: Computation of accumulated influence Acc based on algorithm NextSeedA for product diffusion graph G^1

Queue Q	$< Acc, visited >$ value						
	A	B	C	D	E	F	G
ϕ	1,0	1,0	1,0	1,0	1,0	1,0	1,0
$\{C,D\}$	1,0	1,0	1,1	1,1	1,0	1,0	1,0
$\{D,A\}$	1.46,1	1,0	1,1	1.42,1	1,0	1,0	1,0
$\{A,B,G\}$	1.46,1	1.49,1	1,1	1.42,1	1,0	1,0	1.42,1
$\{B,G\}$	1.46,1	2,1	1,1	1.78,1	1,0	1,0	1.42,1
$\{G,E\}$	1.46,1	2,1	1.52,1	1.78,1	1.8,1	1,0	1.42,1
$\{E,F\}$	2.02,1	2,1	1.52,1	1.78,1	2.22,1	1.49,1	1.42,1
$\{F\}$	2.9,1	2,1	1.52,1	1.78,1	2.22,1	2.2,1	1.42,1
ϕ	2.9,1	2,1	1.52,1	1.78,1	3.32,1	2.2,1	2.52,1

6.2. Complexity analysis

In this section, we analyze the worst-case time complexity of proposed algorithm MIM2. Lines 1-2 take $O(1)$ basic operations to initialize S and *Remaining*. Line 3 performs coupling procedure in $O(\sum_{i=1}^{i=l}(|V_i|+|E_i|))$ basic operations, i.e. $O(|V|+|E|)$, where V and E denotes the node and edge set of a multiplex network (also known as coupled network). Line 4 constructs m product diffusion graph in $O(m(|V|+|E|))$ basic operations. Lines 5-10 performs seed selection procedure iteratively. First, it selects non-candidate nodes using $O(\sum_{i=1}^{i=m}(|E|+|V|\log|V|))$ basic operations. Then, it finds overall seed set from MIM perspective and take $O(k(|E|+|V|\log|V|))$ basic operations. So overall complexity of lines 5-10 is $O((m+k)(|E|+|V|\log|V|))$. Therefore, the worst-case time complexity of MIM2 algorithm is $O((l+m)(M+N)+(k+m)(M+N\log N))$, where $|V| = N$ and $|E| = M$.

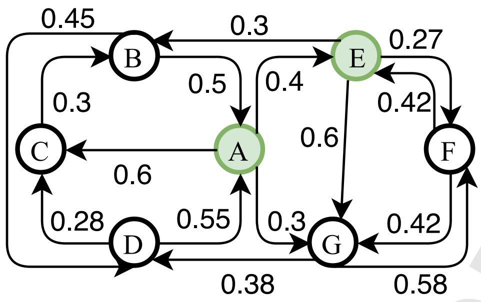


Figure 6: The final seed nodes.

6.3. Compared with the state-of-the-art algorithms

We categorize the classical IM problem presented in [8] into four classes: IM, IM2, MIM, and MIM2 problem. We categorize existing state-of-the-art algorithms into these four problems and compare the characteristics of these algorithms with the proposed algorithm. Table 4 provides a theoretical analysis of above-discussed IM approaches. Column 1 categorizes the existing approaches in four classes: IM, IM2, MIM, and MIM2. The column *Algorithm* gives the name of the algorithm with reference. The column *Time Complexity* gives the worst-case time complexity of the algorithm. The column *Approximation* states the approximation ratios of the algorithm (N.A. indicates no approximation guaranteed). The column *Problem Solving Perspective* gives the framework used in the algorithm. The column *State-of-the-art Algorithm* gives the name of the state-of-the-art methods. The column *Base Algorithm* gives the name of the base algorithm for the corresponding algorithm. Table 5 compare the characteristic such as diffusion models applicable, category, and network, etc., of MIM2 algorithm with the existing algorithms.

Table 4: Comparison of the characteristics of the existing IM algorithms with the proposed algorithm − I

	Algorithm	Time Complexity	Approximation	Problem Solving Perspective	State-of-the-art Algorithms	Base Algorithm
IM	Greedy [8]	$O(kNMI)$	$1-1/e-\varepsilon$	Spread Simulation	MaxDegree, Central & Random	–
	Knapsack Greedy [9]	$O(N^5)$	$1-1/e-\varepsilon$	Spread Simulation	–	Greedy
	SP1M [15]	$O(kNM)$	$1-1/e$	Influence Path	Degree, PageRank & Closeness	–
	CELF [11]	$O(kNMI)$	$1-1/e-\varepsilon$	Sub-modularity	Greedy	Greedy
	Degree Discount [13]	$O(k\log N+M)$	N.A.	Heuristic based	CELF, Greedy & Random	High Degree
	NewGreedy [13]	$O(kIM)$	$1-1/e-\varepsilon(r)$	Snapshots	CELF, Greedy & Random	High Degree
	TW Greedy [10]	$O(kNMI)$	$1-1/e-\varepsilon$	Spread Simulation	SCG, KKG & High Degree	Greedy
	MIA / PMIA [16]	$O(Nt_{i\theta}+kn_{o\theta}n_{i\theta}(n_{i\theta}+\log N))$	$1-1/e$	Influence Path	Greedy, Random, DD & PageRank	SP1M
	LDAG [26]	$O(Nt_\theta+kn_\theta m_\theta(m_\theta+\log N))$	N.A.	Score Estimation	Greedy,SPIN, DD & PageRank	–
	CGA [18]	$O(M+IM_C(N_C-C)+k(C+N_C)))$ [illegible]	$1-e^{-1/(1+\delta_C)}$	Community Based	DD, MG & Random	–
	CELF++ [12]	$O(kNMI)$	$1-1/e-\varepsilon$	Sub-modularity	CELF	CELF
	Diffusion Degree [14]	$O(N+M)$	N.A.	Centrality Based	DD & High Degree	High Degree
	SIMPATH [27]	$O(k^lNP_\theta)$	N.A.	Score Estimation	High Degree, CELF & PageRank	LDAG
	IRIE [28]	$O(k(n_{o\theta}k+M))$	N.A.	Score Estimation	Greedy & PMIA	–
	IPA [17]	$O(\frac{NO_vn_v}{c}+k^2(\frac{O_vn_{vu}}{c}+(c-1)))$	N.A.	Influence Path	Greedy, DD & Random	PMIA
	StaticGreedy [21]	$O(\frac{kMN\log\binom{N}{k}}{\varepsilon^2})$	$1-1/e-\varepsilon$	Snapshots	CELF, SP1M, DD & High Degree	PMIA
	PRUNEDMC [22]	$O(\frac{kMN\log\binom{N}{k}}{\varepsilon^2})$	$1-1/e-\varepsilon$	Snapshots	IRIE, Random, PMIA & Degree	Greedy
	TIM [23]	$O(\frac{k(M+N)\log N}{\varepsilon^2})$	$1-1/e-\varepsilon$	Reverse Reachability	CELF++, IRIE & SIMPATH	–
	DPSO [48]	$O(k^2\log k n\bar{D}^2)$	N.A.	Swam Optimization	Degree, CELF++ & SAEDV	–
IM2	BP-Greedy [49]	–	$1-1/e$	Spread Estimation	Betweenness, Degree, & Closeness	Greedy
	MPMN-CELF++ [50]	$O(kNMI)$	N.A.	Spread Simulation	SIMPATH & CELF++	CELF++
	MPMN-SIMPATH [50]	$O(klNP_\theta)$	N.A.	Influence Ranking	SIMPATH & CELF++	SIMPATH
	ASMTC [[illegible]]	$O(\lvert V^s\rvert^2+\lvert V^s\rvert)$	N.A.	Reverse Reachability	–	–
	SeedSelectionIM [52]	–	N.A.	Rank Refinement	Degree, K-Shell & VoteRank	–
	LCI [32]	$O((N+M)N.d)$	N.A.	Sub-modularity	Greedy	Greedy
MIM	MIM-Greedy [[illegible]]	$O(kmNMI)$	$1-1/e$	Spread Simulation	MaxDegree, Init-First & Random	Greedy
MIM2	**MIM2**	$O((l+m)(M+N)+(k+m)(M+N\log N))$	N.A.	Heuristic based	MaxDegree, DD, & MIM-Greedy	C2IM

Table 5: Comparison of the characteristics of the existing IM algorithms with the proposed algorithm − II

	Algorithm	Diffusion Model				Simulation	Heuristic	Meta-heuristic	Sketch	Network	
		Linear Threshold	Independent Cascade	Triggering	Continuous Time-aware					Single	Multiple
IM	Greedy [8]	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗
	Knapsack Greedy [9]	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗
	SP1M [15]	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗
	CELF [11]	✓	✓	✓	✓	✓	✗	[illegible]	✗	✓	✗
	Degree Discount [13]	✓	✓	✓	✓	✗	✓	✗	✗	✓	✗
	NewGreedy [13]	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗
	TW Greedy [10]	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗
	MIA / PMIA [16]	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗
	LDAG [26]	✓	✗	✗	✗	✗	✓	✗	✗	✓	✗
	CGA [18]	✗	✓	✗	✗	✓	✗	✗	✗	✓	✗
	CELF++ [12]	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗
	Diffusion Degree [14]	✓	✓	✓	✓	✗	✓	✗	✗	✓	✗
	SIMPATH [27]	✓	✗	✗	✗	✗	✓	✗	✗	✓	✗
	IRIE [28]	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗
	IPA [17]	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗
	StaticGreedy [21]	✗	✓	✗	✗	✗	✓	✗	✓	✓	✗
	PRUNEDMC [22]	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗
	TIM [23]	✓	✓	✓	✗	✗	✗	✗	✓	✓	✗
	ACO [53]	✓	✓	✓	✓	✗	✗	✓	✗	✓	✗
	DPSO [48]	✓	✓	✓	✓	✗	✗	✓	✗	✓	✗
IM2	BP-Greedy [49]	✓	✓	✓	✗	✓	✗	✗	✗	✗	✓
	MPMN-CELF++ [50]	✓	✓	✓	✓	✓	✗	✗	✗	✗	✓
	MPMN-SIMPATH [50]	✓	✗	✗	✗	✗	✓	✗	✗	✗	✓
	ASMTC [51]	✗	✓	✗	✗	✗	✗	✗	✓	✗	✓
	SeedSelection-M [52]	✗	✓	✗	✗	[illegible]	✓	✗	✗	✗	✓
	LCI [32]	✓	✓	✗	✗	✓	✗	✗	✗	✗	✓
MIM	MIM-Greedy [33]	✓	✓	✓	[illegible]	✓	✗	✗	✗	✓	✗
MIM2	**MIM2**	✓	✓	✓	✓	✗	✓	✗	✗	✗	✓

7. Experimental evaluation

In this section, we explain the experimental setup and the results of the conducted experiments. We evaluate the proposed algorithm MIM2 with the state-of-the-art seeding strategies from the following aspects: (a) comparison of the coupling schemes; (b) advantage of coupled networks; (c) advantage of MIM2 framework; (d) the effectiveness in terms of influence spread; (b) the efficiency in terms of running time.

7.1. Experimental setup

To evaluate the proposed seeding strategy, we compare MIM2 with best suited existing algorithm under different coupling scheme. To perform experiments, we employ real-world networks. The propagation probabilities and activation probabilities are produced from well-known distributions, as described later in the section.

7.1.1. Network structure

In order to perform experiment, we use two datasets: Higgs Twitter networks[3] [54] and co-author networks[4] [32, 55]. Higgs dataset extracted from Twitter network based on user activities on elusive Higgs boson discovery between 1st and 7th July 2012. The Twitter network is follower-follow relationship network (FN) and it consists three types of interaction networks: Retweet network (RTN), Reply network (REN) and Mention network (MEN). The authors of [54] perform experiments on different periods and divide network based on these experiment time period. The experiment periods are: before 1 PM GMT on 2nd July (Period I), after 2nd July and before the announcement on 4th July (Period II), and after announcement to 7th July (Period III). The statistical information of Higgs dataset is given in Tables 6 and 7.

[3] http://snap.stanford.edu/data/higgs-twitter.html

[4] https://www.cise.ufl.edu/research/OptimaNetSci/tools/id_inter.html

Table 6: Statistical information of Higgs dataset

	FN	RTN	RE	MEN
Nodes $\|V_i\|$	456626	256491	38918	116408
Edges $\|E_i\|$	14855842	328132	32523	150818

Table 7: Statistical information of follower network on Higgs dataset over different period

	Period I	Period II	Period III
Nodes $\|V_i\|$	6061	87457	247440
Edges $\|E_i\|$	7712	169526	[illegible]78243

The co-author dataset consists of three citation network in the field of Condensed Matter(CM), Network Science(NS), and High-Energy Theory(HT). The statistical information of co-author dataset is given in Tables 8. To identify overlapping users in co-author dataset, we use authors name for matching. The number of overlapping users are 2860, 90, and 517 for network pairs CM-HT, HT-NS, and CM-NS respectively.

Table 8: Statistical information of co-author dataset

	CM	NS	HT
Nodes $\|V_i\|$	40420	1588	6360
Edges $\|E_i\|$	175692	2742	15751

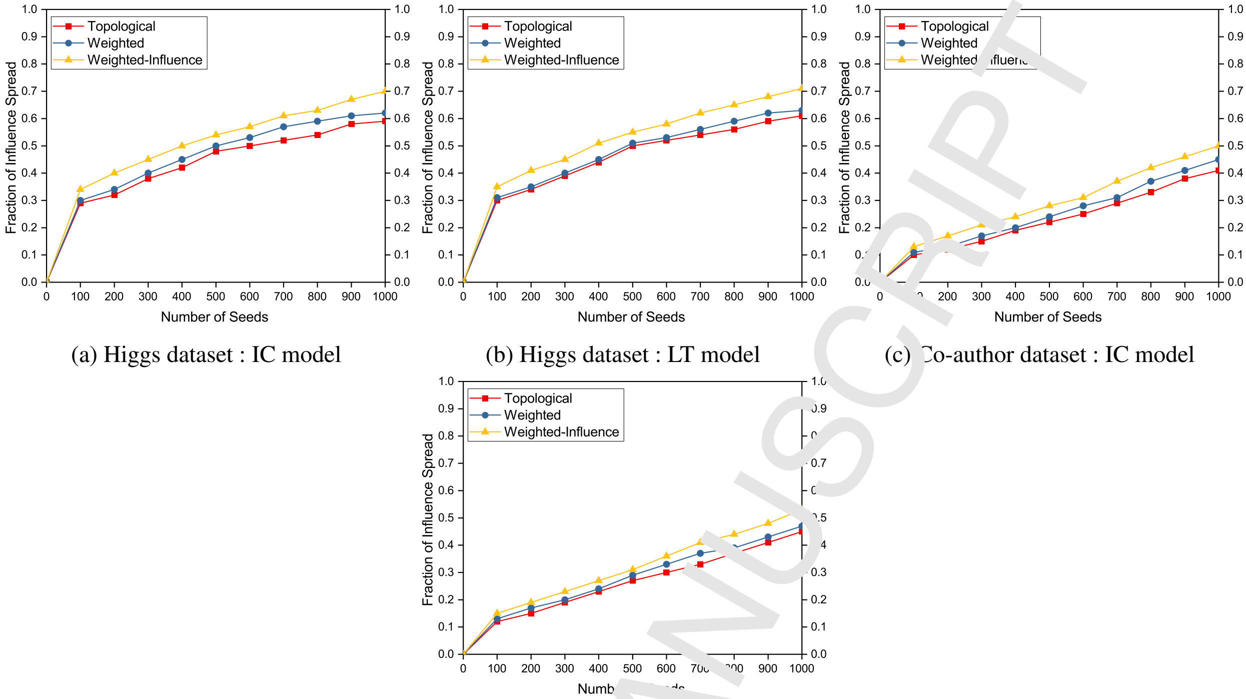
7.1.2. Probability distribution

The Higgs dataset only provides topology information so we need to assign edge weight or propagation probability of each edge (u,v). In IC model, the propagation probability follows uniform discrete distribution over $\{0.1, 0.01, 0.001\}$. In LT, the propagation probability of (u,v) is assigned to $\frac{1}{indegree(v)}$. In co-author dataset edge weights are given. The propagation probability of (u,v) of product diffusion graph for both datasets follows a normal distribution. The assignment of activation probability of each node u follows a uniform distribution.

7.1.3. Seeding strategies to compare

We adopt following state-of-the-art algorithms to evaluate the performance of the proposed algorithm under traditional diffusion models.

- **Random.** This method selects k seed nodes randomly [8].
- **MaxDegree.** This method selects k highest out-degree nodes as seed nodes [8].
- **Degree Discount.** This approach is based on the concept that after a node is selected as a seed node then it will no longer be influenced by its neighbors [13].
- **MIM-Greedy.** This approach performs IM based on a greedy approach for multiple products simultaneously [33]. It takes the assumption that influence of a node is spread up to 6 hope.
- **MIM2.** This is our proposed heuristic algorithm. It selects seed by reverse tracing from non-candidate nodes across multiple networks for multiple products simultaneously.



(a) Higgs dataset : IC model
(b) Higgs dataset : LT model
(c) Co-author dataset : IC model
(d) Co-author dataset : LT model

Figure 7: Comparison of the coupling schemes over MIM2 influence spread for $m = 1$.

7.2. *Comparison of coupling schemes*

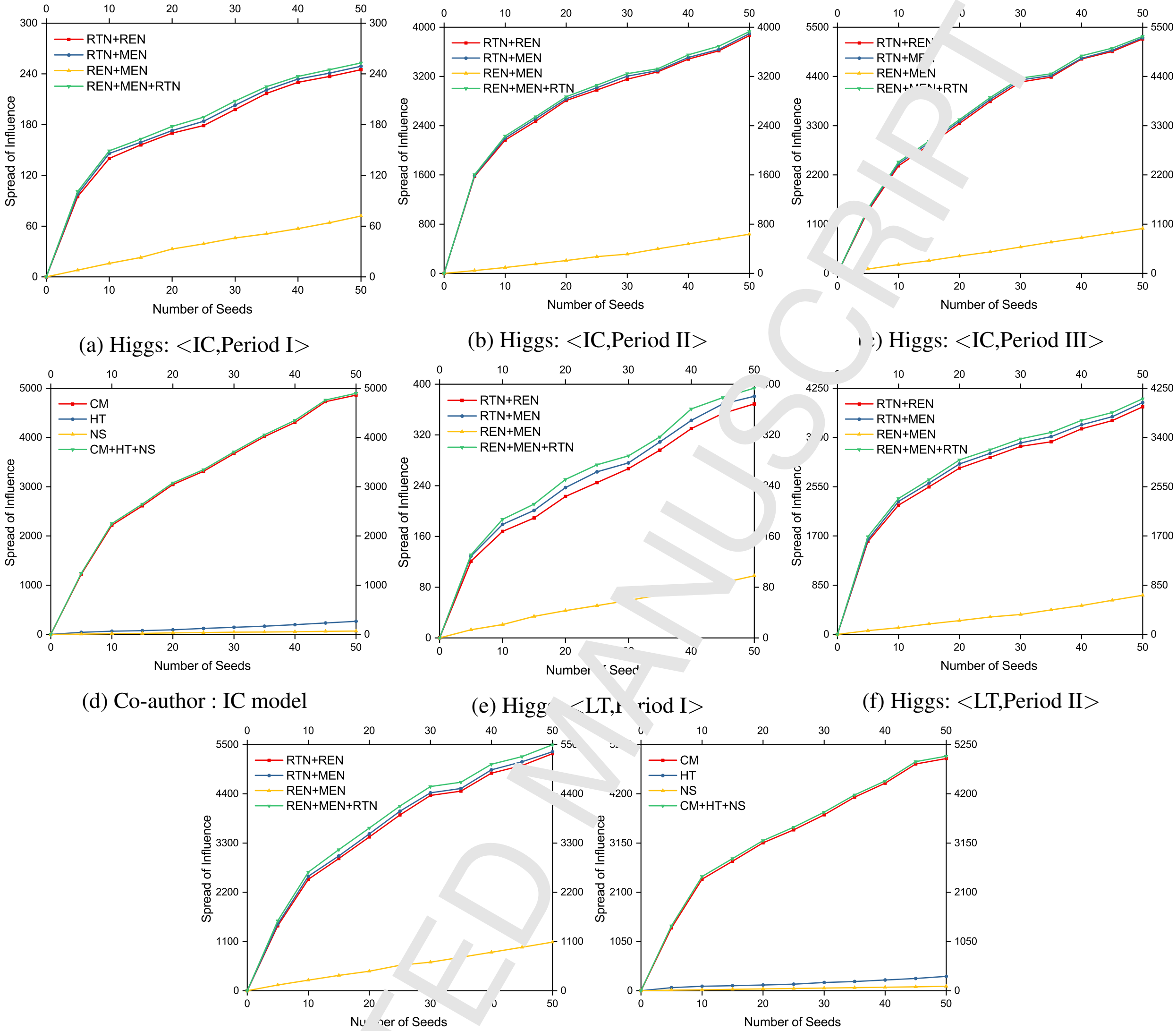
We evaluate the performance of coupling schemes on the influence spread of MIM2 algorithm. Figure 7 shows that the weighted-influence coupling scheme activates more users than other two coupling schemes under traditional diffusion models for both Twitter and co-author networks. This is because of weighted-influence coupling considers the importance of different types of relationships for information diffusion along with the topology structure of the network.

Table 9: The influence spread comparison of algorithm under IM2 framework for Higgs Twitter dataset at $k = 50$

		Influence Spread			
		RTN+REN	RTN+MEN	REN+MEN	RTN+REN+MEN
IC	Period I	245	249	72	253
	Period II	3864	3896	637	3929
	Period III	5238	5262	1003	5298
LT	Period I	369	381	98	394
	Period II	3929	4004	677	4069
	Period III	5298	5342	1085	5498

7.3. *Advantages of using IM2*

In order to understand the benefit of coupled multiplex networks under IM2 framework, we compare the influence spread of proposed algorithm for different seed size over coupled networks. Figure 8 shows the effect of avoiding one type of relationship in Higgs Twitter networks and co-author networks under a traditional diffusion model. In all three



(a) Higgs: <IC,Period I> (b) Higgs: <IC,Period II> (c) Higgs: <IC,Period III>

(d) Co-author : IC model (e) Higgs: <LT,Period I> (f) Higgs: <LT,Period II>

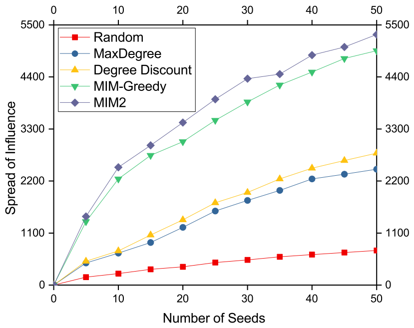
(g) Higgs: <LT,Period III> (h) Co-author : LT model

Figure 8: Comparison of the influence spread over IM2 framework for $m = 1$ under traditional diffusion models.

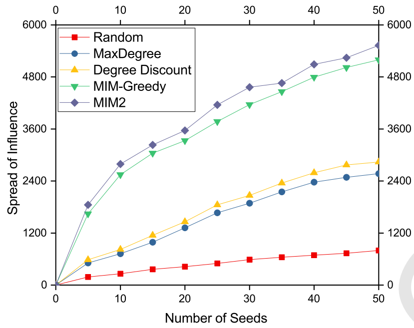
Table 10: The influence spread comparison of algorithm under IM2 framework for co-author dataset at $k = 50$

Diffusion	Influence Spread			
Model	CM	HT	NS	CM+HT+NS
IC	4762	265	68	4857
LT	4952	305	94	5063

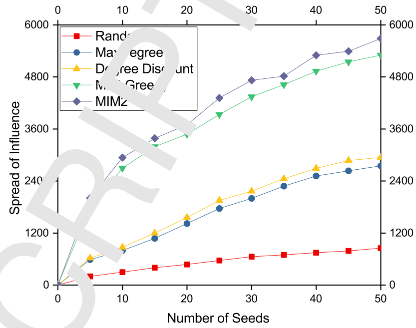
periods of Higgs dataset under both diffusion models, the avoidance of RTN edges causes the most significant difference in influence spread. The influence spread significantly deteriorate when RTN edges not considered in the influence diffusion process. Figure 8 also illustrates the effect of the coupled network in co-author networks. The influence spread is very less when we consider only HT or only NS network. Table 9 and 10 provide the influence spread of MIM2 algorithm in coupled multiplex networks under IM2 framework at $k = 50$ for Higgs Twitter and co-author datasets respectively.



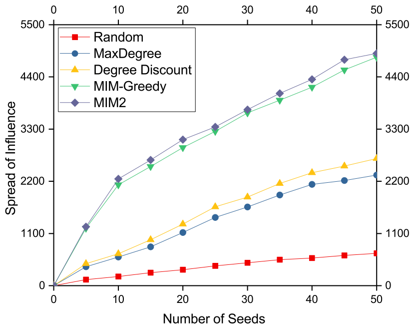
(a) Higgs dataset: $m = 1$



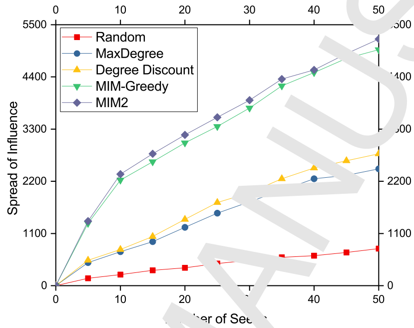
(b) Higgs dataset: $m = 2$



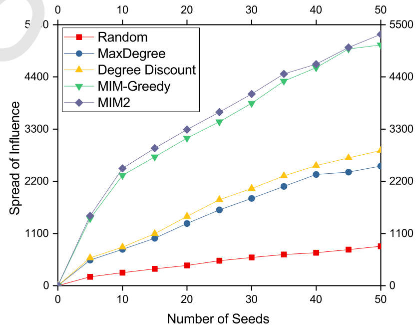
(c) Higgs dataset: $m = 3$



(d) Co-author: $m = 1$



(e) Co-author: $m = 2$



(f) Co-author: $m = 3$

Figure 9: Comparison of the influence spread over MIM framework for different number of product m under IC model.

Table 11: The influence spread comparison of compared algorithms under MIM framework for different m at $k = 50$

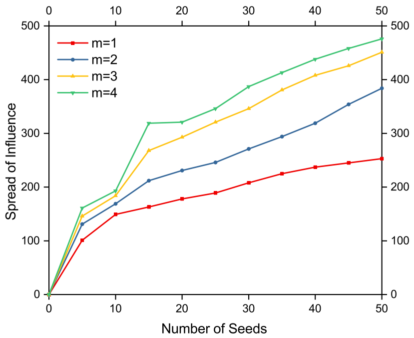
Dataset	m	Influence Spread				
		Random	MaxDegree	Degree Discount	MIM-Greedy	MIM2
Twitter	1	[illegible]	2451	2789	4954	5298
	2	798	2571	2839	5194	5528
	3	[illegible]	2751	2945	5304	5688
Co-author	1	681	2331	2679	4814	4897
	2	[illegible]	2461	2779	4974	5197
	3	831	2521	2849	5074	5297

7.4. Advantages of using MIM

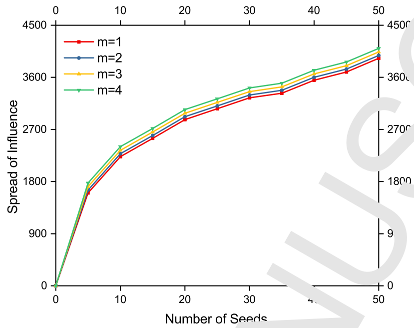
In our experiments, we assume number of products $m = 1, 2, 3$ and seed size $k = 5, 10, \ldots, 50$. Figure 9 shows the influence spread of compared algorithms for different size seed set and for different value of m under IC model. We can see that MIM2 algorithm outperforms other algorithms under MIM framework for the different number of products. This is because of Random, MaxDegree, and Degree Discount heuristic methods are only dependent on network topology and edge weight. MIM-Greedy is based on the influence matrix which considers influence up to six hope. Our MIM2 algorithm estimates the diffusion degree of each node by reverse tracing the whole network. Figures 9a to 9c illustrate that the influence spread of algorithms are increases with the increase of the number of products in Higgs dataset. Similarly, Figures 9d to 9f compares the influence spread of algorithm for different value of m. From these experiments we can observe that the influence spread of MIM2 algorithm is increases with increment in number of products. This influence spread increment is highly evident until k reaches a certain threshold. This is because of the remaining nodes have less social source, in-neighbors and out-neighbors. Table 11 gives the influence spread of compared algorithm under IC diffusion model for MIM framework at $k = 50$.

7.5. *Advantages of using MIM2*

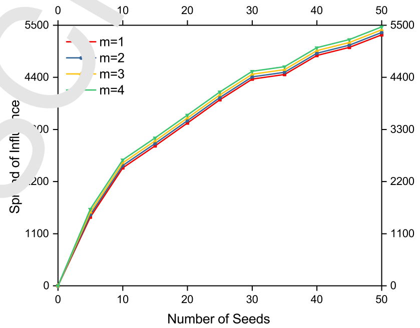
To understand the benefit of proposed MIM2 framework, we perform experiments on influence spread and running time of proposed algorithm for different diffusion graphs under traditional diffusion models.



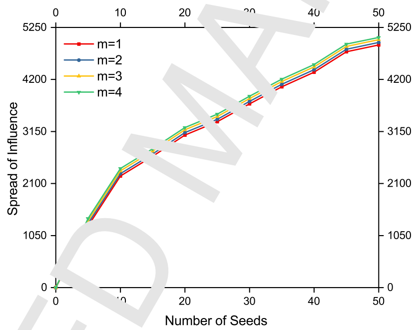
(a) Higgs dataset: Period I



(b) Higgs dataset: Period II



(c) Higgs dataset: Period III



(d) Co-author dataset

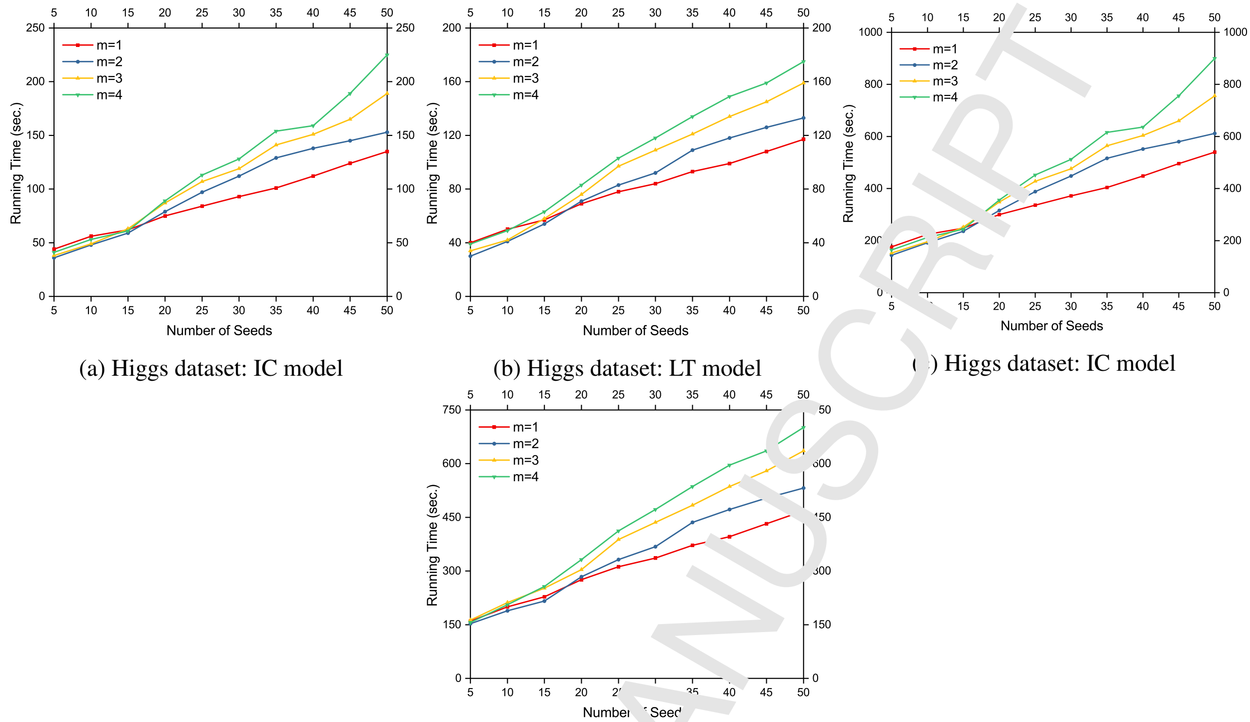
Figure 10: Comparison of the influence spread of MIM2 framework for different number of product m under IC model.

7.5.1. *Influence spread*

Figure 10 shows the influence spread of MIM2 algorithm for different m values under IC model. We can see that the influence spread increases with growth in seed size. It is also evident that the influence spread of MIM2 algorithm increases with increment in the number of diffusion graphs k. These experimental results show that our influence model and MIM2 framework are full of effectiveness.

7.5.2. *Running time*

Apart from the influence spread comparison, we also compare the running time of proposed algorithm under MIM2 framework for different m ranges from 1 to 4. Figure 11 shows the running time of our algorithm for both datasets. We can see that the running time increases with the growth of seed set k as well as the number of product diffusion graphs m. The running time of selecting seed under MIM2 framework takes very few minutes. This shows that MIM2 algorithm has good performance. Therefore, we can say that MIM2 algorithm has good quality solution and efficiency.



(a) Higgs dataset: IC model

(b) Higgs dataset: LT model

(c) Higgs dataset: IC model

(d) Co-author: LT model

Figure 11: Comparison of the running time of MIM2 framework for different number of product *m*.

8. Conclusion and future directions

In this paper, we present a *multiple influence maximization across multiple networks* (MIM2) problem to tackle the scenario that multiple products can be promoted in a network with different channel of interactions. To effectively solve this problem, we presented MIM2 algorithm. First, we introduced a coupling scheme to map multiple interaction networks into a single multiplex network. Then, we presented a procedure to find non-candidate nodes for backward tracing. Next, we constructed product diffusion graphs for each product. Finally, we provided a seed selection process across product diffusion graphs. Exhaustive experiments provided new insights into the MIM2 problem and the benefits of MIM2 framework. This work opens to several future work directions such as the investigation of MIM2 problem in multiplex networks with heterogeneous diffusion models with some constraints like competitiveness, budget, location, time, etc.

9. Compliance with Ethical Standards

The authors declare no conflicts of interest. The article does not contain any studies with human or animal subjects. This article present an algorithm MIM2.

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