Quiz III

Wireless Communications, EEEN3008J

Q1. Consider a 2×2 MIMO system with channel gain matrix given by:

$$\mathbf{H} = \begin{bmatrix} 3 & 1+i \\ -1+i & -3 \end{bmatrix}$$

The singular value decomposition of the above channel matrix is given as:

$$\mathbf{H} = \begin{bmatrix} -0.7071 & x \\ 0.5-0.5i & -0.5+0.5i \end{bmatrix} \times \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix} \times \begin{bmatrix} -0.7071 & -0.7071 \\ -0.5+0.5i & y \end{bmatrix}$$

(a) Determine the values of x, y, λ_1, and λ_2. Show all calculations, DO NOT use MATLAB.

(b) If the above MIMO system is used with $\mathbf{H}$ known to both transmitter and receiver, calculate the channel capacity of the equivalent MIMO system. Assume total transmit power $P = 20$ mW across the two antennas, AWGN with $N_0 = 10^{-9}$ W/Hz at each receive antenna and bandwidth $B = 200$ KHz.

(c) Calculate the capacity of an equivalent single-input-single-output (SISO) channel and comment on the capacity enhancing ability of MIMO.

Solution: Start with the U and V matrices, apply their unitary properties i.e., $UU^H = U^H U = I_2$ and $VV^H = V^H V = I_2$ to solve for x and y. Then solve for the λ's either by multiplying all three matrices and writing equations in λ's or solving the eigenvalue equations $|\lambda I - H| = 0$.
For rest of the solution follow lecture notes.
Note: I have wrongly put the V matrix in the above problem and I have been a bit lenient in marking this question because of that.

Q2. Following the same methods as in pages 10-12 of the Lecture Notes 11, derive expressions of the output SNR in terms of the coefficients of the channel matrix $\mathbf{H}$ for a 2×2 MIMO system for (a) MGC and (b) EGC. (**Hint:** Use the received signal covariance matrix $\mathbf{R_{yy}}$ to find the signal and noise powers.)

Solution: (a) $SNR = \frac{E_b(\sum_{i=1}^{M} \alpha_i h_{i1}^2 + \sum_{i=1}^{M} \alpha_i h_{i2}^2)}{\sigma^2 \sum_{i=1}^{M} \alpha_i^2} \leq \frac{E_b(\sum_{i=1}^{M} \alpha_i^2)(\sum_{i=1}^{M} h_{i1}^2) + (\sum_{i=1}^{M} \alpha_i^2)(\sum_{i=1}^{M} h_{i2}^2)}{\sigma^2 \sum_{i=1}^{M} \alpha_i^2}$, using Cauchy-Schwartz inequality.
Therefore, simplifying the above expression we get,

$$SNR \leq \frac{E_b}{\sigma^2}[\sum_{i=1}^{M}(h_{i1}^2 + h_{i2}^2)] = \frac{E_b}{\sigma^2}\,\mathrm{Tr}\,(\mathbf{H}\mathbf{H}^{\mathbf{H}})$$

Thus, for MGC: $SNR = \frac{E_b}{\sigma^2}\,\mathrm{Tr}\,(\mathbf{H}\mathbf{H}^{\mathbf{H}})$.

(b) For EGC: $SNR = \frac{E_b}{2\sigma^2}\,\mathrm{Tr}\,(\mathbf{H}\mathbf{H}^{\mathbf{H}})$

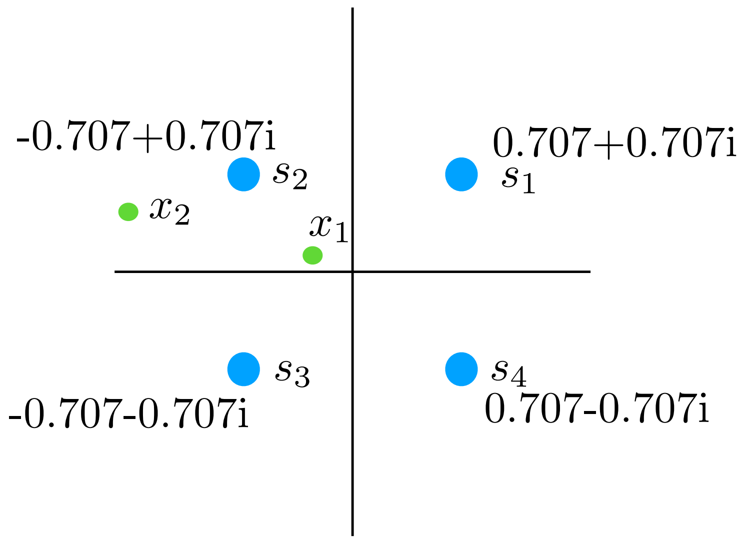
Q3. Suppose we have a CDMA system and that a 0 is transmitted as a positive pulse +L and 1 is transmitted as a negative pulse - L. A certain station is assigned the code 101111 and another station can choose among the codes 001110, 000101, and 100111 respectively. Which code should the second station use and why?

Solution: The code 000101 will be used as it is orthogonal to the code 101111 and hence the interference between the two stations will be cancelled out.

Q4. Consider a 2×1 MIMO system with channel coefficients are $h_1 = 0.5129 + 0.5054i$ and $h_2 = -0.0446 - 0.1449i$; If the received symbols are $y_1 = -0.0222 + 0.1423i$, and $y_2 = 0.4673 + 1.0784i$, what are the transmitted symbols. Consider 4-QAM modulation is used and the modulated symbols are multiplied by a factor of $1/\sqrt{2}$ to ensure average transmit power of 1 per symbol.

Solution: $x_1 = \frac{h_1^* y_1 + h_2 y_2^*}{|h_1|^2 + |h_2|^2} = -0.2665 + 0.0480i$ and $x_2 = \frac{h_2^* y_1 - h_1 y_2^*}{|h_1|^2 + |h_2|^2} = -1.4688 + 0.5758i$

From the figure below, both transmitted symbols fall within the decision region of symbol s_2. So s_2 was transmitted at each time interval as per space-time coding scheme.



Note: This problem won't be there in your exam as I did not teach Space time coding in detail. I had only two slides on this concept. I graded this question with utmost leniency. Whoever attempted this I gave almost 50% marks on this problem.