Quiz IV

Wireless Communications, EEEN3008J

Q1. The capacity of a static MIMO channel with only receiver CSI is given by

$$C = B\sum_{i=1}^{r}\log_2\left(1+\lambda_i\frac{P}{M\sigma^2}\right)$$

where M is the number of transmit antennas, P is the total power of the MIMO system, λ_i-s are the singular values or Eigen values of the channel matrix $\mathbf{H}$. Show that if the sum of singular values is bounded as $\sum_{i=1}^{r}\lambda_i \leq \bar{\lambda}$, the capacity expression is maximised when $\lambda_1 = \lambda_2 = \lambda_3 = \cdots = \lambda_r$.

Solution: Given that the sum of singular values is bounded by $\sum_{i=1}^{r}\lambda_i \leq \bar{\lambda}$. Then we want to solve the following Lagrangian equation for all λ_i's such that they maximise C.

$$L = B\sum_{i=1}^{r}\log_2\left(1+\lambda_i\frac{P}{M\sigma^2}\right) - \mu\left(\sum_{i=1}^{r}\lambda_i - \bar{\lambda}\right)$$

which gives

$$\frac{\partial L}{\partial \lambda_i} = \frac{P/(M\sigma^2)}{\log_e 2\left(1+\lambda_i\frac{P}{M\sigma^2}\right)} - \mu \Rightarrow \lambda_i = \frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}$$

Hence all singular values have the same value $\lambda_i = \frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}$. Now $\sum_{i=1}^{r}\lambda_i \leq \bar{\lambda} \Rightarrow r\left(\frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}\right) = \bar{\lambda}$. Therefore, $\mu = \frac{\log_e 2}{\frac{\bar{\lambda}}{r}+\frac{M\sigma^2}{P}}$.

Q2. An LTE eNodeB operates at the 5 MHz spectrum using 64-QAM with no coding for each of the subcarriers. Using normal cyclic prefix length, calculate:

(a) The total number of resource elements (REs) in one subframe.

(b) The peak downlink data rate if a SISO scheme is used.

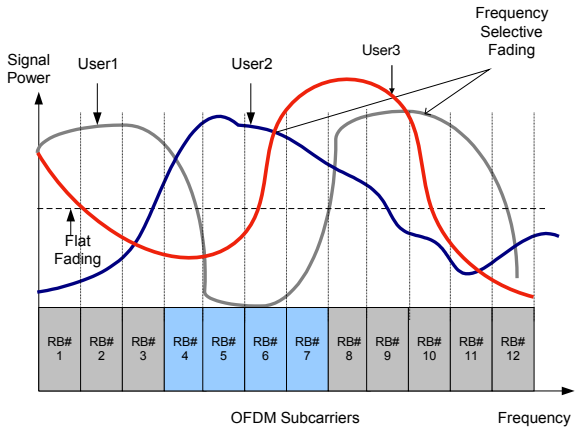
(c) The peak downlink data rate if a 2×2 MIMO scheme is used.

(d) Repeat your calculations in parts (a) - (c) for 1.4, 3, 10, 15, and 20 MHz spectrums.

Solution:

Look here: https://frankrayal.com/2011/06/27/lte-peak-capacity/

Q3. Given below is a figure showing the frequency selectivity of three users.



Comment with appropriate arguments about which resource blocks should be allocated to which user. Hence find the maximum usable data rate for each of the three users using your calculations in question 2(d) if the operating spectrum is 1.4 MHz.

Solution:

Look class notes, this question is intuitive and many solutions can be acceptable as long as they are justified.