Quiz II

Wireless Communications, EEEN3008J

Q1. An M-sequence is one form of pseudo-noise (PN) sequences. Figure 1 shows an implementation of an M-sequence (binary maximal length shift-register sequence) generated using linear feedback shift-registers and exclusive OR-gate circuits.

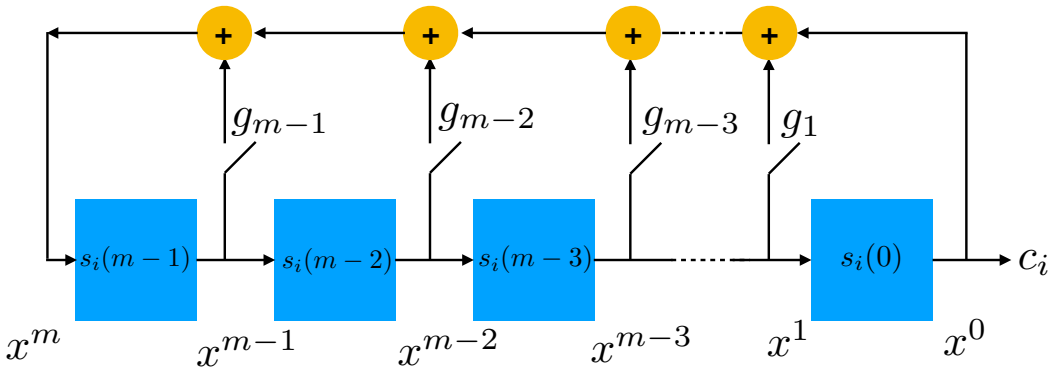

Figure 1: (Problem 1).

The linear generator polynomial $g(x)$ of degree $m > 0$ is given by: $g(x) = g_m x^m + g_{m-1}x^{m-1} + g_{m-1}x^{m-1} + \cdots + g_1 x + g_0$. If $g_j = 1$, then the switch corresponding to the g_j in Fig. 1 is closed else if $g_j = 0$, the corresponding switch is open. Usually, also $g_m = g_0 = 1$. With these information,

(a) Redraw the shift-register implementation of Figure 1, for a generator polynomial of $g(x) = x^4 + x^2 + 1$.

(b) Find the output sequence $\underline{c} = \{c_0, c_1, c_2, \dots\}$ of the shift-register implementation of part (a) if the initial contents of the shift registers are $s_i(3) = 1$, $s_i(2) = 0$, $s_i(1) = 0$, and $s_i(0) = 1$. What is the period of this sequence?

(c) The auto-correlation function of the PN sequence is given by $AC(k) = \frac{1}{N}\sum_{i=1}^{N} c_i c_{i+k}$, where c_{i+k} is the k-th shift of the i-th value of the sequence ($0 \le k \le N-1$) and N is the period of the PN sequence. For the PN sequence generated in part (b) assuming that 0's are encoded as -1 and 1's are encoded as $+1$, find the values of $AC(0)$, $AC(1)$, and $AC(N)$.

Solution: (a)

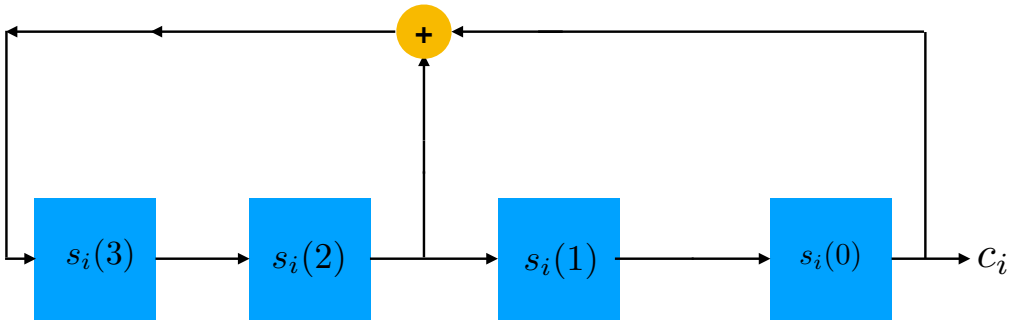

Figure 2: (Solution 1a).

Solution: (b)

$$g(x) = g_m x^m + g_{m-1} x^{m-1} + \cdots + g_1 x + g_0$$

Output sequence recurrence condition according to g(x)

$$c_{i+m} = g_{m-1} c_{i+m-1} + g_{m-2} c_{i+m-2} + \cdots + g_1 c_{i+1} + c_i \pmod 2$$

$$g(x) = x^4 + x^2 + 1$$

$$\text{Therefore, } g_4 = 1, g_3 = 0, g_2 = 1, g_1 = 0, g_0 = 1$$

Using the recurrence relation above and setting $c_0 = 1, c_1 = 0, c_2 = 0, c_3 = 1$ we get $c_4 = g_3 c_3 + g_2 c_2 + g_1 c_1 + c_0 = 1$
continuing in this manner we get: $c_5 = g_3 c_4 + g_2 c_3 + g_1 c_2 + c_1 = 1$
$c_6 = 0$, $c_7 = 0$, $c_8 = 1$, $c_9 = 1$,$c_{10} = 1$,$c_{11} = 1$,$c_{12} = 0$,$c_{13} = 0$
$c_{14} = 1$, and $c_{15} = 1$. Therefore the sequence is $1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1$ and the period is 5.

Solution: (c) Applying the formula it is straightforward. Check if $AC(0) = AC(N)$.

Q2. A certain area is covered by a cellular radio system with 84 cells and a cluster size N. 300 voice channels are available for the system. Users are uniformly distributed over the area covered by the cellular system, and the offered traffic per user is 0.04 Erlang. Assume that blocked calls are cleared and the designated blocking probability is $P_b = 1\%$.

(a) Determine the maximum carried traffic per cell if cluster size $N = 4$ is used. Repeat for cluster sizes $N = 7$ and $N = 12$.

(b) Determine the maximum number of users that can be served by the system for a blocking probability of 1% and cluster size $N = 4$. Repeat for cluster sizes $N = 7$ and $N = 12$.

Solution:

No of cells=84
Total voice channels=300
Offered traffic per user (A_u) =0.04 Erl.
Blocking Probability (P_B)=0.01

N=4
Total voice channels per cell =(300/4)=75
Traffic supported by 1 cell (offered)=A=60.73 Erl.
Carried traffic per cell=A(1-P_B)=60.73(1-0.01)= **60.123 Erl.**

N=7
Total voice channels per cell =(300/7)= 43
Traffic supported by 1 cell (offered)=A=31.66 Erl.
Carried traffic per cell=A(1-P_B)=31.66(1-0.01)= **31.343 Erl.**

N=12
Total voice channels per cell =(300/12)= 25
Traffic supported by 1 cell (offered)=A=16.13 Erl.
Carried traffic per cell=A(1-P_B)=16.13(1-0.01)= **15.969 Erl.**

(b)
for a blocking probability of 1% and cluster size $N = 4$
Maximum number of users served per cell =A/A_u

$$= \frac{60.73}{0.04} = 1518.25 \approx 1518$$

Maximum number of users served per cell that can be served by the system
$=1518 \times 84$
=127512

for a blocking probability of 1% and cluster size $N = 7$
Maximum number of users served per cell =A/A_u

$$= \frac{31.66}{0.04} = 791.5 \approx 791$$

Maximum number of users served per cell that can be served by the system
$=791 \times 84$
=66444

for a blocking probability of 1% and cluster size $N = 12$
Maximum number of users served per cell =A/A_u

$$= \frac{16.13}{0.04} = 403.25 \approx 403$$

Maximum number of users served per cell that can be served by the system
$=403 \times 84$
=33852

No. of Trunks (N)	Traffic (A) in erlangs for P = 0.1%	0.2%	0.5%	1%	1.2%	1.3%	1.5%	2%	3%	5%
1	0.001	0.002	0.005	0.010	0.012	0.013	0.02	0.020	0.031	0.053
2	0.046	0.065	0.105	0.153	0.168	0.176	0.19	0.223	0.282	0.381
3	0.194	0.249	0.349	0.455	0.489	0.505	0.53	0.602	0.715	0.899
4	0.439	0.535	0.701	0.869	0.922	0.946	0.99	1.09	1.26	1.52
5	0.762	0.900	1.13	1.36	1.43	1.46	1.52	1.66	1.88	2.22
6	1.15	1.33	1.62	1.91	2.00	2.04	2.11	2.28	2.54	2.96
7	1.58	1.80	2.16	2.50	2.60	2.65	2.73	2.94	3.25	3.74
8	2.05	2.31	2.73	3.13	3.25	3.30	3.40	3.63	3.99	4.54
9	2.56	2.85	3.33	3.78	3.92	3.98	4.08	4.34	4.75	5.37
10	3.09	3.43	3.96	4.46	4.61	4.68	4.80	5.08	5.53	6.22
11	3.65	4.02	4.61	5.16	5.32	5.40	5.53	5.84	6.33	7.08
12	4.23	4.64	5.28	5.88	6.05	6.14	6.27	6.61	7.14	7.95
13	4.83	5.27	5.96	6.61	6.80	6.89	7.03	7.40	7.97	8.83
14	5.45	5.92	6.66	7.35	7.56	7.65	7.81	8.20	8.80	9.73
15	6.08	6.58	7.38	8.11	8.33	8.43	8.59	9.01	9.65	10.6
16	6.72	7.26	8.10	8.88	9.11	9.21	9.39	9.83	10.5	11.5
17	7.38	7.95	8.83	9.65	9.89	10.0	10.19	10.7	11.4	12.5
18	8.05	8.64	9.58	10.4	10.7	10.8	11.00	11.5	12.2	13.4
19	8.72	9.35	10.3	11.2	11.5	11.6	11.82	12.3	13.1	14.3
20	9.41	10.1	11.1	12.0	12.3	12.4	12.65	13.2	14.0	15.2
21	10.1	10.8	11.9	12.8	13.1	13.3	13.48	14.0	14.9	16.2
22	10.8	11.5	12.6	13.7	14.0	14.1	14.32	14.9	15.8	17.1
23	11.5	12.3	13.4	14.5	14.8	14.9	15.16	15.8	16.7	18.1
24	12.2	13.0	14.2	15.3	15.6	15.8	16.01	16.6	17.6	19.0
25	13.0	13.8	15.0	16.1	16.5	16.6	16.87	17.5	18.5	20.0
26	13.7	14.5	15.8	17.0	17.3	17.5	17.72	18.4	19.4	20.9
27	14.4	15.3	16.6	17.8	18.2	18.3	18.59	19.3	20.3	21.9
28	15.2	16.1	17.4	18.6	19.0	19.2	19.45	20.2	21.2	22.9
29	15.9	16.8	18.2	19.5	19.9	20.0	20.32	21.0	22.1	23.8
30	16.7	17.6	19.0	20.3	20.7	20.9	21.19	21.9	23.1	24.8
31	17.4	18.4	19.9	21.2	21.6	21.8	22.07	22.8	24.0	25.8
32	18.2	19.2	20.7	22.0	22.5	22.6	22.95	23.7	24.9	26.7
33	19.0	20.0	21.5	22.9	23.3	23.5	23.83	24.6	25.8	27.7
34	19.7	20.8	22.3	23.8	24.2	24.4	24.72	25.5	26.8	28.7
35	20.5	21.6	23.2	24.6	25.1	25.3	25.60	26.4	27.7	29.7
36	21.3	22.4	24.0	25.5	26.0	26.2	26.49	27.3	28.6	30.7
37	22.1	23.2	24.8	26.4	26.8	27.0	27.39	28.3	29.6	31.6
38	22.9	24.0	25.7	27.3	27.7	27.9	28.28	29.2	30.5	32.6
39	23.7	24.8	26.5	28.1	28.6	28.8	29.18	30.1	31.5	33.6
40	24.4	25.6	27.4	29.0	29.5	29.7	30.08	31.0	32.4	34.6
41	25.2	26.4	28.2	29.9	30.4	30.6	30.98	31.9	33.4	35.6
42	26.0	27.2	29.1	30.8	31.3	31.5	31.88	32.8	34.3	36.6
43	26.8	28.1	29.9	31.7	32.2	32.4	32.79	33.8	35.3	37.6
44	27.6	28.9	30.8	32.5	33.1	33.3	33.69	34.7	36.2	38.6
45	28.4	29.7	31.7	33.4	34.0	34.2	34.60	35.6	37.2	39.6
46	29.3	30.5	32.5	34.3	34.9	35.1	35.51	36.5	38.1	40.5
47	30.1	31.4	33.4	35.2	35.8	36.0	36.42	37.5	39.1	41.5
48	30.9	32.2	34.2	36.1	36.7	36.9	37.34	38.4	40.0	42.5
49	31.7	33.0	35.1	37.0	37.6	37.8	38.25	39.3	41.0	43.5
50	32.5	33.9	36.0	37.9	38.5	38.7	39.17	40.3	41.9	44.5
51	33.3	34.7	36.9	38.8	39.4	39.6	40.08	41.2	42.9	45.5
52	34.2	35.6	37.7	39.7	40.3	40.6	41.00	42.1	43.9	46.5
53	35.0	36.4	38.6	40.6	41.2	41.5	41.92	43.1	44.8	47.5
54	35.8	37.2	39.5	41.5	42.1	42.4	42.84	44.0	45.8	48.5
55	36.6	38.1	40.4	42.4	43.0	43.3	43.77	44.9	46.7	49.5

No. of Trunks (N)	Traffic (A) in erlangs for P = 0.1%	0.2%	0.5%	1%	1.2%	1.3%	1.5%	2%	3%	5%
56	37.5	38.9	41.2	43.3	43.9	44.2	44.69	45.9	47.7	50.5
57	38.3	39.8	42.1	44.2	44.8	45.1	45.62	46.8	48.7	51.5
58	39.1	40.6	43.0	45.1	45.8	46.1	46.54	47.8	49.6	52.6
59	40.0	41.5	43.9	46.0	46.7	47.0	47.47	48.7	50.6	53.6
60	40.8	42.4	44.8	46.9	47.6	47.9	48.40	49.6	51.6	54.6
61	41.6	43.2	45.6	47.9	48.5	48.8	49.33	50.6	52.5	55.6
62	42.5	44.1	46.5	48.8	49.4	49.7	50.26	51.5	53.5	56.6
63	43.3	44.9	47.4	49.7	50.4	50.7	51.19	52.5	54.5	\\
64	44.2	45.8	48.3	50.6	51.3	51.6	52.12	53.4	55.4	58.6
65	45.0	46.6	49.2	51.5	52.2	52.5	53.05	54.4	56.4	59.6
66	45.8	47.5	50.1	52.4	53.1	53.5	53.99	55.3	57.4	60.6
67	46.7	48.4	51.0	53.4	54.1	54.4	54.92	56.3	58.4	61.6
68	47.5	49.2	51.9	54.3	55.0	55.3	55.86	57.2	59.3	62.6
69	48.4	50.1	52.8	55.2	55.9	56.2	56.79	58.2	60.3	63.7
70	49.2	51.0	53.7	56.1	56.8	57.2	57.73	59.1	61.3	64.7
71	50.1	51.8	54.6	57.0	57.8	58.1	58.67	60.1	62.3	65.7
72	50.9	52.7	55.5	58.0	58.7	59.0	59.61	61.0	63.2	66.7
73	51.8	53.6	56.4	58.9	59.6	60.0	60.55	62.0	64.2	67.7
74	52.7	54.5	57.3	59.8	60.6	60.9	61.49	62.9	65.2	68.7
75	53.5	55.3	58.2	60.7	61.5	61.8	62.43	63.9	66.2	69.7
76	54.4	56.2	59.1	61.7	62.4	62.8	63.37	64.9	67.2	70.8
77	55.2	57.1	60.0	62.6	63.4	63.7	64.32	65.8	68.1	71.8
78	56.1	58.0	60.9	63.5	64.3	64.7	65.26	66.8	69.1	72.8
79	56.9	58.8	61.8	64.4	65.2	65.6	66.20	67.7	70.1	73.8
80	57.8	59.7	62.7	65.4	66.2	66.5	67.15	68.7	71.1	74.8
81	58.7	60.6	63.6	66.3	67.1	67.5	68.09	69.6	72.1	75.8
82	59.5	61.5	64.5	67.2	68.0	68.4	69.04	70.6	73.0	76.9
83	60.4	62.4	65.4	68.2	69.0	69.4	69.99	71.6	74.0	77.9
84	61.3	63.2	66.3	69.1	69.9	70.3	70.93	72.5	75.0	78.9
85	62.1	64.1	67.2	70.0	70.9	71.2	71.88	73.5	76.0	79.9
86	63.0	65.0	68.1	70.9	71.8	72.2	72.83	74.5	77.0	80.9
87	63.9	65.9	69.0	71.9	72.7	73.1	73.78	75.4	78.0	82.0
88	64.7	66.8	69.9	72.8	73.7	74.1	74.73	76.4	78.9	83.0
89	65.6	67.7	70.8	73.7	74.6	75.0	75.68	77.3	79.9	84.0
90	66.5	68.6	71.8	74.7	75.6	76.0	76.63	78.3	80.9	85.0
91	67.4	69.4	72.7	75.6	76.5	76.9	77.58	79.3	81.9	86.0
92	68.2	70.3	73.6	76.6	77.4	77.8	78.53	80.2	82.9	87.1
93	69.1	71.2	74.5	77.5	78.4	78.8	79.48	81.2	83.9	88.1
94	70.0	72.1	75.4	78.4	79.3	79.7	80.43	82.2	84.9	89.1
95	70.9	73.0	76.3	79.4	80.3	80.7	81.39	83.1	85.8	90.1
96	71.7	73.9	77.2	80.3	81.2	81.6	82.34	84.1	86.8	91.1
97	72.6	74.8	78.2	81.2	82.2	82.6	83.29	85.1	87.8	92.2
98	73.5	75.7	79.1	82.2	83.1	83.5	84.25	86.0	88.8	93.2
99	74.4	76.6	80.0	83.1	84.1	84.5	85.20	87.0	89.8	94.2
100	75.2	77.5	80.9	84.1	85.0	85.4	86.16	88.0	90.8	95.2
101	76.1	78.4	81.8	85.0	86.0	86.4	87.12	88.9	91.8	96.3
102	77.0	79.3	82.7	85.9	86.9	87.3	88.07	89.9	92.8	97.3
103	77.9	80.2	83.7	86.9	87.8	88.3	89.03	90.9	93.8	98.3
104	78.8	81.1	84.6	87.8	88.8	89.2	89.99	91.9	94.8	99.3
105	79.6	82.0	85.5	88.8	89.7	90.2	90.94	92.8	95.7	100.4
106	80.5	82.8	86.4	89.7	90.7	91.1	91.90	93.8	96.7	101.4
107	81.4	83.7	87.4	90.7	91.6	92.1	92.86	94.8	97.7	102.4
108	82.3	84.6	88.3	91.6	92.6	93.1	93.82	95.7	98.7	103.4
109	83.2	85.5	89.2	92.5	93.5	94.0	94.78	96.7	99.7	104.5
110	84.1	86.4	90.1	93.5	94.5	95.0	95.74	97.7	100.7	105.5