EEEN3008J: Advance wireless communications

Tutorial 2

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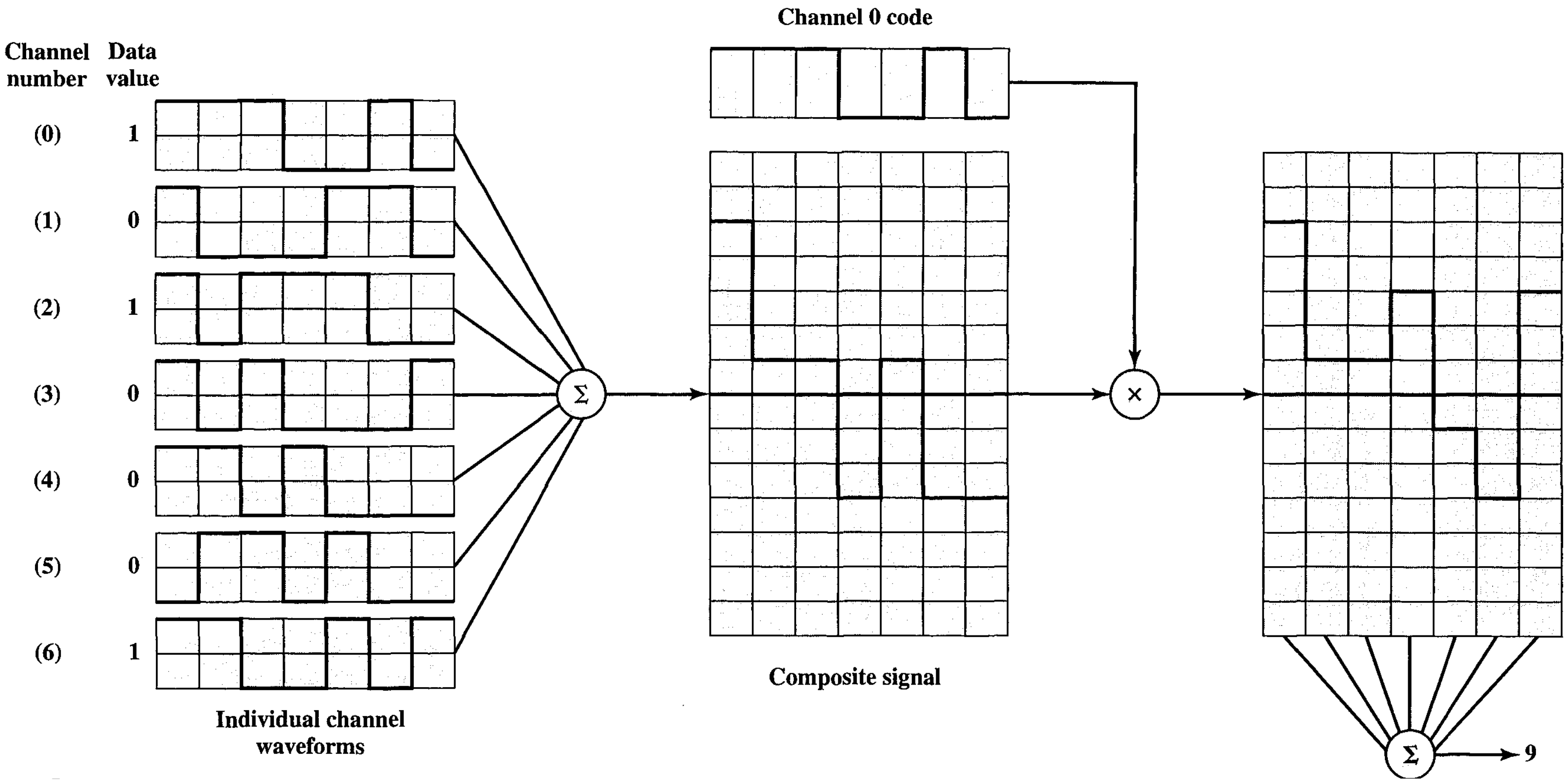
Problem 1

There are seven logical channels, all using DSSS with a spreading code of 7 bits. Assume that all sources are synchronized. To decode a given channel, the receiver multiplies the incoming composite signal by the spreading code for that channel, sums the result, and assigns binary 1 for a positive value and binary 0 for a negative value.



a. What are the spreading codes for the seven channels?
b. Determine the receiver output measurement for channel 1 and the bit value assigned.
c. Repeat part b for channel 2.

Solution to Problem 1

a. C0 = 1110010; C1 = 0111001; C2 = 1011100; C3 = 0101110; C4 = 0010111; C5 = 1001011; C6 = 1100101

b. C1 output = –7; bit value = 0

c. C2 output = +9; bit value = 1

If you multiply the composite signal = [5, 1, 1, -3, 1, -3, -3] with code for C1, to recover C1 message, the product will be = $[-1, 1, 1, 1, -1, -1, 1] * [5, 1, 1, -3, 1, -3, -3]^T = -7$ and since this is negative, channel 1 transmitted a 0. Same logic follows for part c.

Problem 2

Consider a receiver implementing microscopic diversity using two antennas. Let the envelope of the signals received by the two antennas at a given time instant be r_1 and r_2. Assume the noise at the antennas is uncorrelated and has the same power N. Compute the instantaneous SNR for this time instant at the output of the diversity combiner for selection diversity, equal gain combining, and maximum ratio combining respectively for the following two cases. Assume that u(t), the baseband equivalent transmitted signal, is one for the duration of transmission (this also means that P_t = 1).

(a) $r_1 = r_2 = r$.

(b) (b) $r_1 \ll r_2 = r$.

Solution to Problem 2

(a) $r_1 = r_2 = r$

Selection: $\text{SNR=}\dfrac{r^2}{N} = \gamma$

Equal Gain: $\text{SNR=}\dfrac{(r+r)^2}{N+N} = 2\gamma$

Maximum Ratio: $\text{SNR=}\dfrac{r^2}{N} + \dfrac{r^2}{N} = 2\gamma$

(b) $r_1 \ll r_2 = r$

Selection: $\text{SNR=}\dfrac{r^2}{N} = \gamma$

Equal Gain: $\text{SNR=}\dfrac{r^2}{N+N} = \dfrac{1}{2}\gamma$

Maximum Ratio: $\text{SNR=}\dfrac{r^2}{N} = \gamma$

Problem 3

Consider a MIMO system with $N_R = N_T = N$ antennas and AWGN. The rank of the channel matrix $\boldsymbol{H}$ is N.

a. Show that the capacity

$$C = B \sum_{i=1}^{N} \log_2 \left(1 + \lambda_i \frac{P}{N\sigma^2} \right)$$

subject to the constraint that

$$\sum_{i=1}^{N} \lambda_i = \beta = \text{constant}$$

is maximized when $\lambda_i = \beta/N$ for $i = 1, 2, \ldots, N$, and hence

$$C = N \log_2 \left(1 + \frac{\beta P}{N^2 \sigma^2} \right)$$

b. If $\lambda_i = \beta/N$ for $i = 1, 2, \ldots, N$, show that $\boldsymbol{H}$ must be an orthogonal matrix that satisfies the condition

$$\boldsymbol{H}\boldsymbol{H}^H = \boldsymbol{H}^H\boldsymbol{H} = \frac{\beta}{N} \boldsymbol{I}_N$$

Solution to Problem 3

Solution: Given that the sum of singular values is bounded by $\sum_{i=1}^{r} \lambda_i \leq \bar{\lambda}$. Then we want to solve the following Lagrangian equation for all λ_i's such that they maximise C.

$$L = B \sum_{i=1}^{r} \log_2 \left(1 + \lambda_i \frac{P}{M\sigma^2}\right) - \mu \left(\sum_{i=1}^{r} \lambda_i - \bar{\lambda}\right)$$

which gives

$$\frac{\partial L}{\partial \lambda_i} = \frac{P/(M\sigma^2)}{\log_e 2 \left(1 + \lambda_i \frac{P}{M\sigma^2}\right)} - \mu \Rightarrow \lambda_i = \frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}$$

Hence all singular values have the same value $\lambda_i = \frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}$. Now $\sum_{i=1}^{r} \lambda_i \leq \bar{\lambda} \Rightarrow r \left(\frac{1}{(\log_e 2)\mu} - \frac{M\sigma^2}{P}\right) = \bar{\lambda}$. Therefore, $\mu = \frac{\log_e 2}{\frac{\bar{\lambda}}{r} + \frac{M\sigma^2}{P}}$.

(b) $\mathbf{H}\mathbf{H}^{\mathrm{H}} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{\mathrm{H}} = \frac{\beta}{N}\mathbf{Q}\mathbf{I}_N\mathbf{Q}^{\mathrm{H}} = \frac{\beta}{N}\mathbf{I}_N$, since $\mathbf{Q}\mathbf{Q}^{\mathrm{H}} = \mathbf{I}_N$.

Problem 4

A multipath fading channel has a multipath spread of $T_m = 1$ s and a Doppler spread $B_d = 0.01$ Hz. The total channel bandwidth at bandpass available for signal transmission is $W = 5$ Hz. To reduce the effects of intersymbol interference, the signal designer selects a pulse duration $T = 10$ s.

a. Determine the coherence bandwidth and the coherence time.

b. Is the channel frequency selective? Explain.

c. Is the channel fading slowly or rapidly? Explain.

d. Suppose that the channel is used to transmit binary data via (antipodal) coherently detected PSK in a frequency diversity mode. Explain how you would use the available channel bandwidth to obtain frequency diversity and determine how much diversity is available.

Solution to Problem 4

(a)

$$T_m = 1 \ \text{sec} \Rightarrow (\Delta f)_c \approx \frac{1}{T_m} = 1 \ Hz$$
$$B_d = 0.01 \ Hz \Rightarrow (\Delta t)_c \approx \frac{1}{B_d} = 100 \ \text{sec}$$

(b) Since $W = 5 \ Hz$ and $(\Delta f)_c \approx 1 \ Hz$, the channel is frequency selective.

(c) Since T=10 sec $<$ $(\Delta t)_c$, the channel is slowly fading.

(d) The desired data rate is not specified in this problem, and must be assumed. Note that with a pulse duration of $T = 10$ sec, the binary PSK signals can be spaced at $1/T = 0.1 \ Hz$ apart. With a bandwidth of W=5 Hz, we can form 50 subchannels or carrier frequencies. On the other hand, the amount of diversity available in the channel is $W/(\Delta f)_c = 5$. Suppose the desired data rate is 1 bit/sec. Then, ten adjacent carriers can be used to transmit the data in parallel and the information is repeated five times using the total number of 50 subcarriers to achieve 5-th order diversity. A subcarrier separation of 1 Hz is maintained to achieve independent fading of subcarriers carrying the same information.