Quiz I

Wireless Communications, EEEN3008J

Q1. A "CAT-5" twisted-pair cable has a bandwidth of roughly 100 MHz. We would like to transmit information at a bit rate of 500 Mbps. Is a signal-to-noise ratio of 30 dB enough to reliably transmit this much information? Why or why not?

Solution: This question requires a simple application of the Shannon Theorem: $C = Blog_2(1 + SNR)$ In this case we are given C = 500 Mbps, B = 100 MHz, and SNR = 30 dB. In order to answer the question of whether or not the SNR is sufficient, we need to see if the capacity provided by the bandwidth and SNR is >= 500 Mbps.
Now, $SNR_{dB} = 10\log_{10} SNR$. So

$SNR = 10^{SNR_{dB}/10} = 10^{30/10} = 1000$

We can now apply the Shannon Theorem:

$C = B\log_2(1 + SNR) = 100MHz \times \log_2(1 + 1000) = 100MHz \times 9.967 \times (1Mbps/MHz)$
= 996.7 Mbps. Note the use of "Mbps/MHz" to underscore the fact that we are explicitly converting from "Hz" to "bps". Thus the Shannon Theorem tells us that the theoretical limit for transferring data over this medium at this SNR is approximately 1000 Mbps (or 1Gbps). Thus we can reliably transmit 500 Mbps over this connection.

Q2. What is the minimum signal-to-noise ratio, in decibels, that must be maintained in order to transmit a 600 Kbps signal over a medium with bandwidth 20,000 Hz?

Solution: Let us work around the Shannon Theorem to solve for the SNR:

$C = B\log_2(1 + SNR)$
$\frac{C}{B} = \log_2(1 + SNR)$
$2^{C/B} = 1 + SNR$
$SNR = 2^{C/B} - 1$
We can now solve this for the given capacity and bandwidth. Note that our capacity is in Kbps, but our bandwidth is in Hz. Since 20,000Hz = 20 KHz, we'll do the calculations in Kbps/KHz:
$SNR = 2^{(600/20)} - 1$
$SNR = 2^{30} - 1$

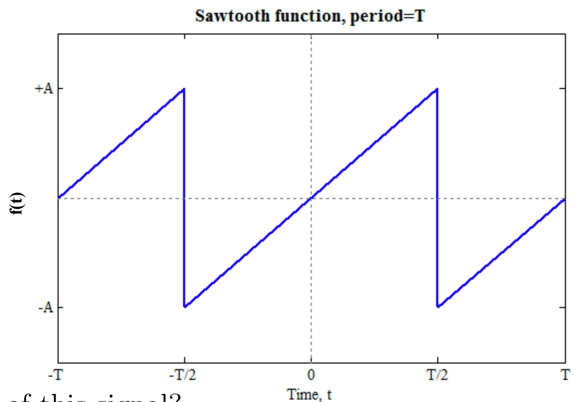
Therefore, SNR = 1073741823

The question however, asks for the SNR in decibels, so we need to convert:

$SNR_{dB} = 10 \log_{10} SNR$ $SNR_{dB} = 10 \log_{10} 1073741823$

$SNR_{dB} = 90.31$. So we would need at least 90.31 dB as a signal-to-noise ratio in order to transmit the desire capacity.

Q3. Consider the signal in the form of a sawtooth function shown below. The Fourier Series representation of the signal is given by: $f(t) = \sum_{n=1}^{n=\infty} \left(\frac{-2A}{n\pi}(-1)^n\right) \sin(2\pi n f_1 t)$



(a) What is the bandwidth of this signal?

(b) If the signal can be closely approximated with the first 3 terms of the series, what is the effective bandwidth of this signal if $f_1 = 5$ MHz?

(c) If the signal in part (b) is passed through a channel whose bandwidth is 200 kHz will it suffer flat fading or frequency-selective fading? Justify.

Solution: (a) The first term in the Fourier series is for $n = 1$ which is $\frac{2A}{\pi}\sin(2\pi f_1 t)$. Similarly the last term is for $n = \infty$, which is ∞. Therefore, ideally the signal has f_1 as the first frequency component and ∞ as the last frequency component and h0.2ence bandwidth is $\infty - f_1 = \infty$.

(b) If we approximate the signal with the first three terms of the series then the first term is $\frac{2A}{\pi}\sin(2\pi f_1 t)$ and the last term is $\frac{2A}{3\pi}\sin(2\pi(3f_1)t)$. So, the bandwidth is $3f_1 - f_1 = 2f_1$ Hz.

(c) Since bandwidth of signal is 10 MHz > 200 kHz the channel is frequency selective.

Q4. What are the main causes for wireless channel disturbances/corruptions?

Solution: Refere Lecture Notes

Q5. The received signal in a multi-path environment, in the absence of noise, is given as: $r(t) = \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c(t - \tau))$, where α_1 and α_2 are the amplitudes of the two components of the signal arriving from two multi-paths.

(i) Prove that the received signal can also be represented as: $r(t) = \alpha \cos(2\pi f_c t + \phi)$. Hence find the expressions for α and ϕ in terms of α_1, α_2, f_c, and τ.

(ii) Plot α against $f_c\tau$ (assuming $\alpha_1 = \alpha_2 = 2$) and discuss what you obtain about the wireless channel deep fading.

Solution: (i)

$$
\begin{aligned}
&r(t) \\
&= \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos\big(2\pi f_c (t-\tau)\big) \\
&= \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c t)\cos(2\pi f_c \tau) + \alpha_2 \sin(2\pi f_c t)\sin(2\pi f_c \tau) \\
&= (\alpha_1 + \alpha_2 \cos(2\pi f_c \tau))\cos(2\pi f_c t) + \alpha_2 \sin(2\pi f_c t)\sin(2\pi f_c \tau)
\end{aligned}
$$

Letting $\alpha = \sqrt{(\alpha_1 + \alpha_2\cos(2\pi f_c\tau))^2 + \alpha_2^2 \sin^2(2\pi f_c\tau)}$, $\frac{(\alpha_1+\alpha_2\cos(2\pi f_c\tau))}{\alpha} = cos\beta$, and $\frac{\alpha_2 \sin(2\pi f_c\tau)}{\alpha} = \sin\beta$, we have

$$
\begin{aligned}
r(t) &= \alpha\left(\frac{(\alpha_1 + \alpha_2\cos(2\pi f_c\tau))}{\alpha}\cos(2\pi f_c t) + \frac{\alpha_2 \sin(2\pi f_c\tau)}{\alpha}\sin(2\pi f_c t)\right) \\
&= \alpha(\cos\beta\cos(2\pi f_c t) + \sin\beta\sin(2\pi f_c t)) \\
&= \alpha\cos(2\pi f_c t - \beta)
\end{aligned}
$$

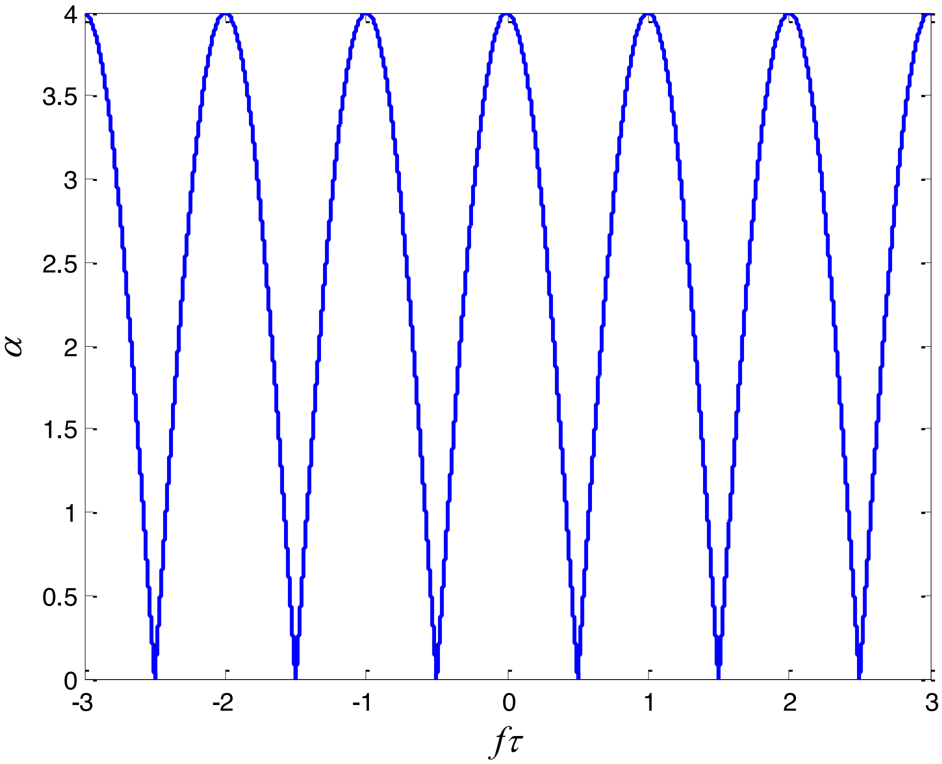
So, we get

$$
\begin{aligned}
\alpha &= \sqrt{(\alpha_1 + \alpha_2\cos(2\pi f_c\tau))^2 + \alpha_2^2\sin^2(2\pi f_c\tau)} \\
&= \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2\cos(2\pi f_c\tau)}
\end{aligned}
$$

and

$$
\phi = -\beta = -\tan^{-1}\left[\frac{\alpha_2\sin(2\pi f_c\tau)}{\alpha_1 + \alpha_2\cos(2\pi f_c\tau)}\right].
$$

(ii)


4
3.5
3
2.5
2
1.5
1
0.5
0
α
-3
-2
-1
0
1
2
3
$f\tau$