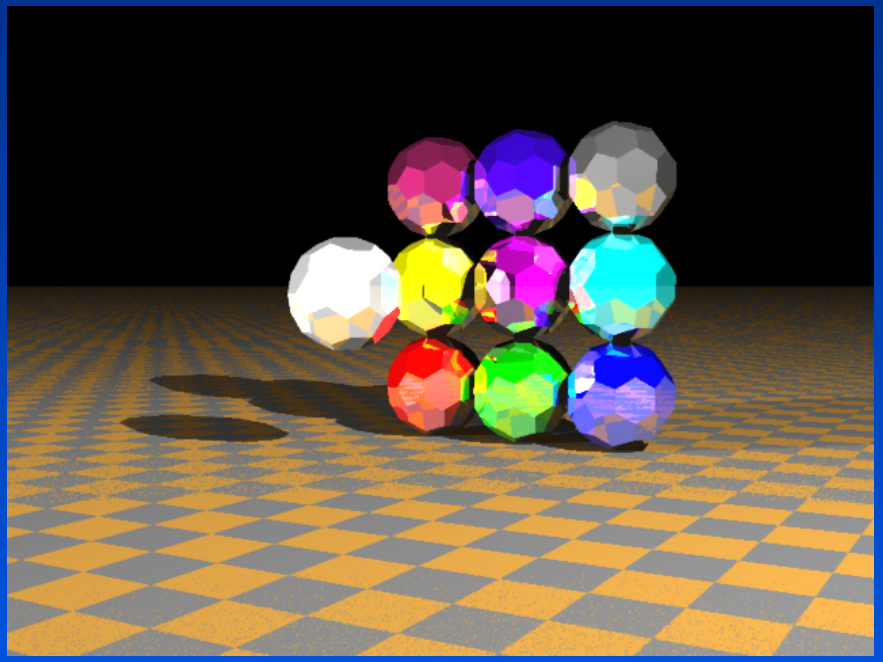
Visible Surface Algorithms

1. Back-face polygon culling
2. Z-buffer algorithms
3. 3-D depth-sort algorithm
4. Binary space partitioning trees
5. Visible-surface ray tracing



Visible Surface Determination

- Given a set of 3-D objects and a view specification, determine which lines or surfaces of the object are visible
  - From center of projection (perspective)
  - Along the direction of projection (parallel)
- Also called hidden surface removal
- Called visible-line determination when referring to lines only
- Note: lines themselves don't hide lines
  Lines must be edges of opaque surfaces to hide other lines

Image Precision

- Determine which of M objects is visible at each *N* pixels in image

  for each pixel in the image
  determine object closest to viewer that
  is intersected by the projector through the pixel;
  Draw the pixel in the appropriate color

- Performed at resolution of image
- Complexity: *O(NxM)*

Object Precision

- Compare objects directly with each other

  for each object in the world

  Compare with all other object to determine visible part

  Draw those parts in the appropriate color
- Performed at resolution of object
- Must be followed by scan conversion (usually image precision)
- Historically the first approach
- Complexity: $O(M^2)$♪

Costly operations

- Image precision :

  Which objects intersect a projector?

- Object precision :

  Which objects intersect each object?

Techniques for Efficient Visible Surface Algorithms♪

- Coherence: The degree to which parts of an environment exhibit logical similarities
  Reuse previous calculations (e.g., depth of adjacent pixels on a plane is related by difference equation)
- Perspective transformation: Transform environment into one in which points with same (x,y) lie on same projector
  Compare z's to determine the closest one
- Back-face culling: Remove surfaces that face away from point of view

- Extents and bounding volumes: Bound each complex object with a simpler one
  Use to cull possible overlaps
- Spatial partitioning: Break large problem into smaller ones, e.g., partition screen into regular grid
  Solve visible surface problem in each grid cell independently, only comparing with those in same grid
- Hierarchy: Build tree of nested extents
  If two internal nodes don't intersect, none of their children intersect

Coherence
"local similarity"♪

Use previous calculated value

- Object coherence
  - e.g. when 2 objects are entirely separated in x-y then need not consider polygons that make up the objects
- Face coherence
  - e.g. property at one part of face changes incrementally as you move across face
- Edge coherence
  - e.g. property along an edge changes incrementally (e.g. z)
- Implied edge coherence
  - e.g. edge formed from intersection of two faces exhibit edge coherence

- Area coherence
  - e.g. visibility at one pixel “similar” to visibility at neighboring pixel
- Depth coherence
  - E.g. depth (z) of one object similar

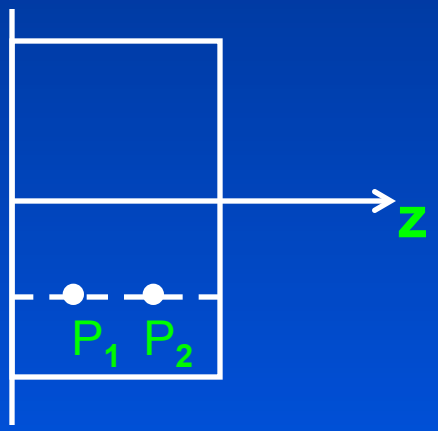
    Depth (z) of one part of an object can be determined by depth of a neighboring part
- Temporal coherence
  - E.g. properties of one frame (at a point in time) similar to neighboring frame

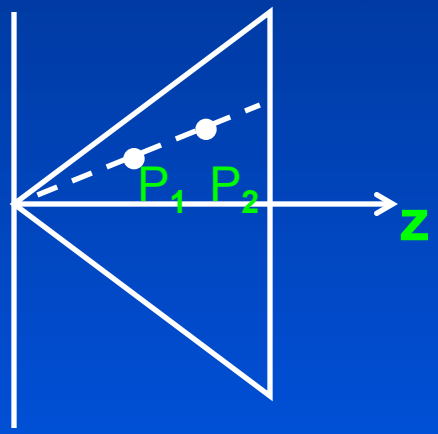
Perspective transformation♪

- Hidden surface problem: given p_1 and p_2, are p_1 and p_2 on the same projector?

  -Simple for parallel projection

  - Must divide by z for perspective projection





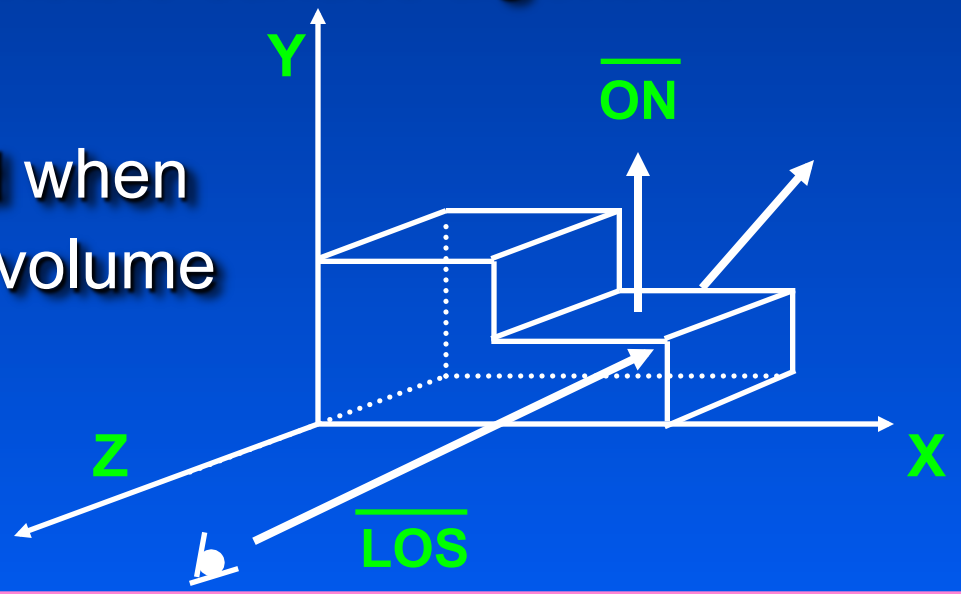
- After perspective transform, space transformed into parallel canonical view volume

Extents and Bounding volumes

- Cost of bounding box tests
- How well bounding box encloses object
  - e.g. compare bounding box of projected object on x-y plane
- If bounding box not a tight fit, may have many “false positives”

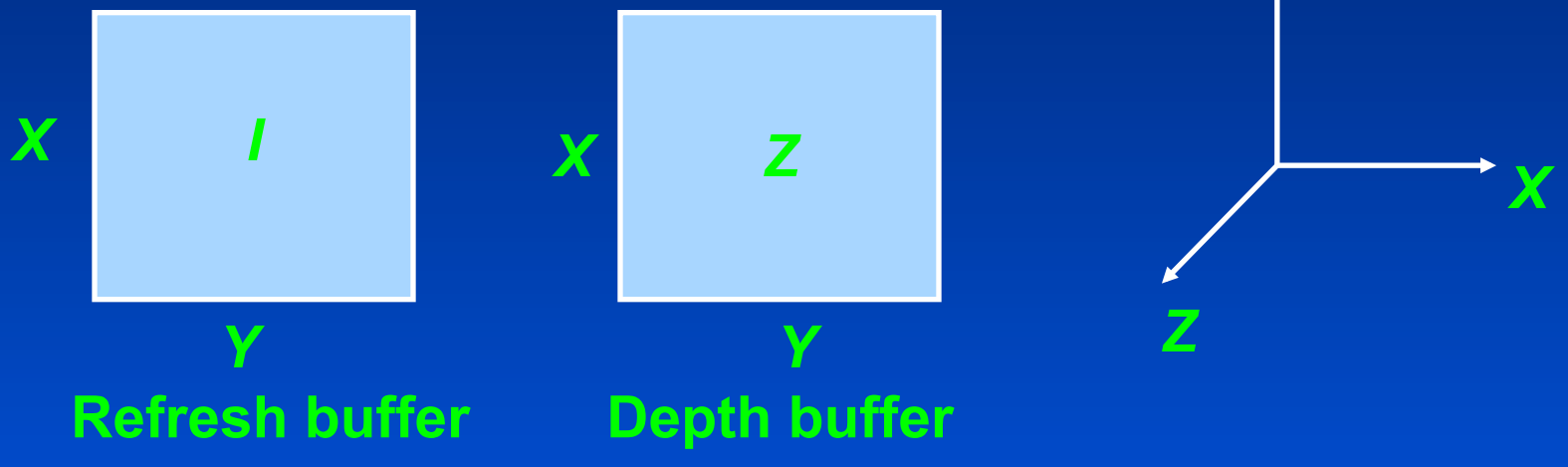
Back-Face Culling♪

- Assumes objects defined polyhedra
- When displaying a single convex polyhedron, back-face culling is the only visible surface algorithm needed!
- Dot product not needed when canonical parallel view volume is used



Z-Buffer Algorithm♪

- Requires two buffer



- Intensity buffer *I(x,y)* initialized to background
- Depth buffer Z*(x,y)* initialized to furthest z

- Polygons are scan-converted in arbitrary order

  1. Calculate the polygon depth z(x,y)
  2. If z(x,y) closer than z-buffer at (x,y), then
     a) Place z(x,y) into the z-buffer at (x,y), *Z(x,y)* and
     b) Place pixel value of polygon at z(x,y) into the refresh buffer at (x,y), *I(x,y)*
- Used in most graphics hardware
- Image precision
- Speed tends not to explode with scene complexity
- Use difference equation to calculate z incrementally♪

Calculation of z

- Interpolate along z_1-z_2

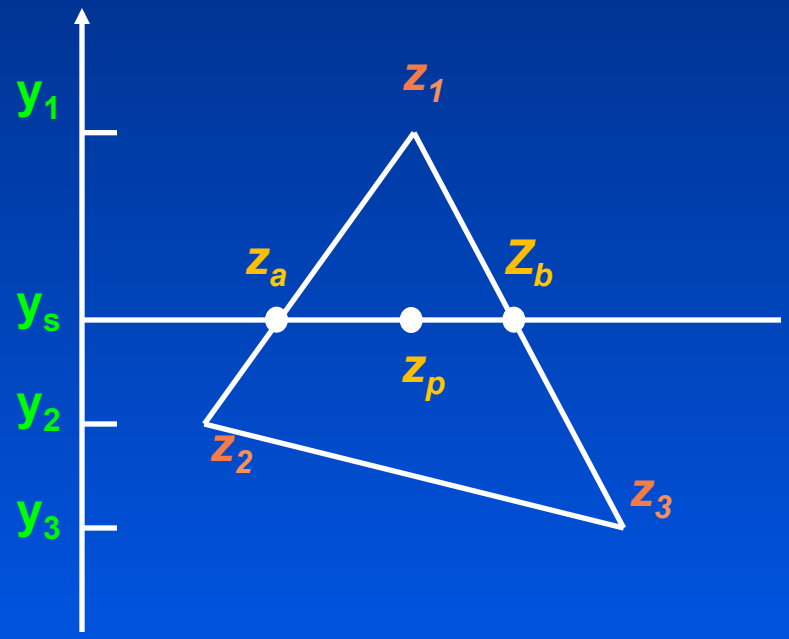
$$z_a = z_1 - (z_1 - z_2)\frac{y_1 - y_s}{y_1 - y_2}$$

- Interpolate along z_1-z_3

$$z_b = z_1 - (z_1 - z_3)\frac{y_1 - y_s}{y_1 - y_3}$$

- Interpolate along z_a-z_b

$$z_p = z_b - (z_b - z_a)\frac{x_b - x_p}{x_b - x_a}$$



- Can be done incrementally :
- If

$$Y_{i+1} = Y_i - 1$$

$$Z_{i+1} = Z_1 - \left(\frac{Z_1 - Z_2}{Y_1 - Y_2}\right)(Y_1 - Y_i) + \left(\frac{Z_1 - Z_2}{Y_1 - Y_2}\right)$$

$$= Z_i + \left(\frac{Z_1 - Z_2}{Y_1 - Y_2}\right)$$

similary if $X_{i+1} = X_i + 1$

Summary of z-buffer algorithms

- No upper limit on complexity of scene
- Simple
- Take advantage of image space coherence
- De-facto standard for hidden surface algorithms

- Inefficient: scan convert polygons that are not visible
- Transparency is a problem where you need to process the polygons in the order: far to near
- As number of polygons increases and size decreases, setup time for each polygon dominate over per pixel cost
  - Problem for large scenes that needs to be implemented in real-time
  - Need to do processing on the scene to make hidden surface easier

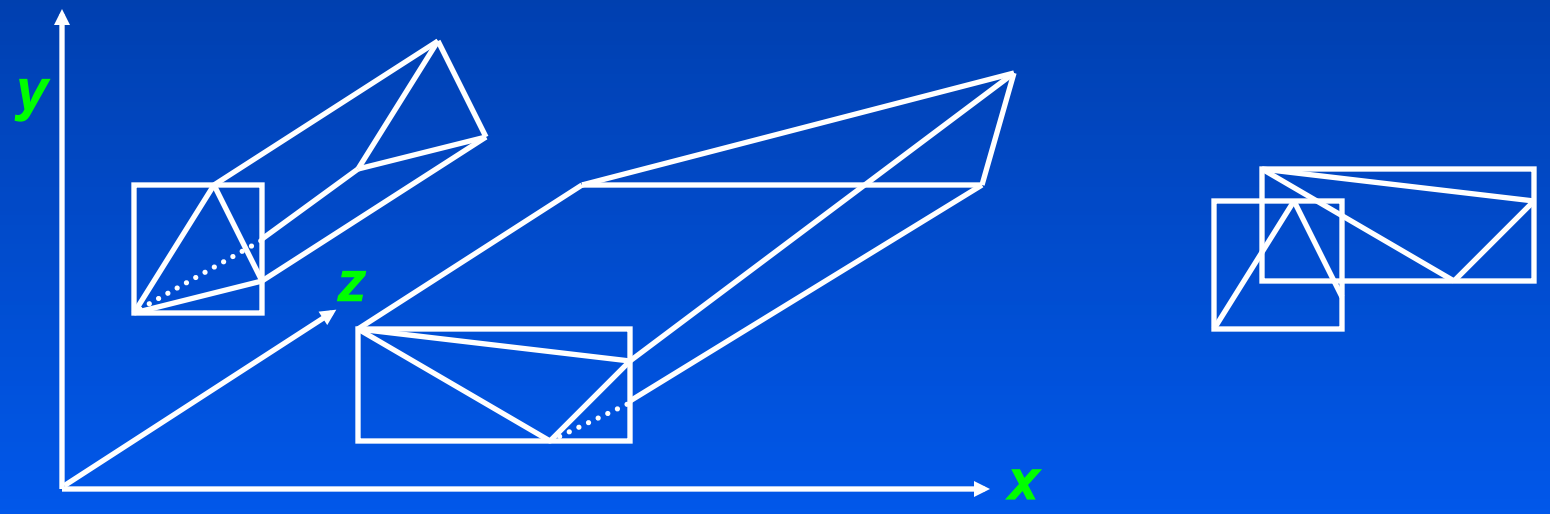
3-D Depth-Sort Algorithm♪

- Strategy is to work back to front : Sort polygons based on furthest z-coordinate
- Scan convert furthest polygon first, then work forward toward viewpoint
  resolve ambiguities as encountered
- Combine object-precision and image-precision operations

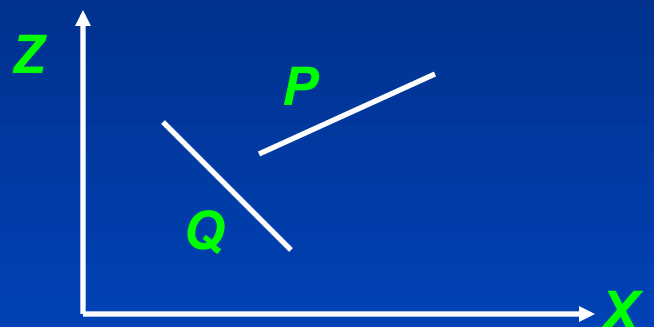
- Given preliminary sorted list of polygons (e.g. based on one vertex z value)
  - P: Polygon at end of list (furthest polygon in list)
  - {Q}: List of all polygons whose z-extent overlaps that of P
- Before scan-converting P, make up to five tests (in order of increasing complexity) to compare P with each polygon Q_i in {Q}
  - If all polygons pass at least one test, then scan-convert P
    There are no Q that is in front of P
  - Next polygon becomes new P♪

1. Extents in *x* do not overlap
2. Extents in *y* do not overlap

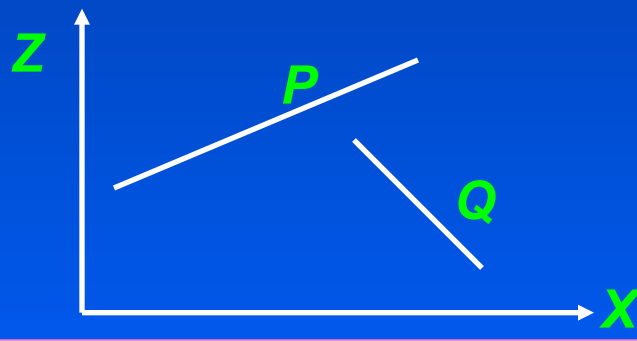
   Note-the xy-extent test can cause false rejections



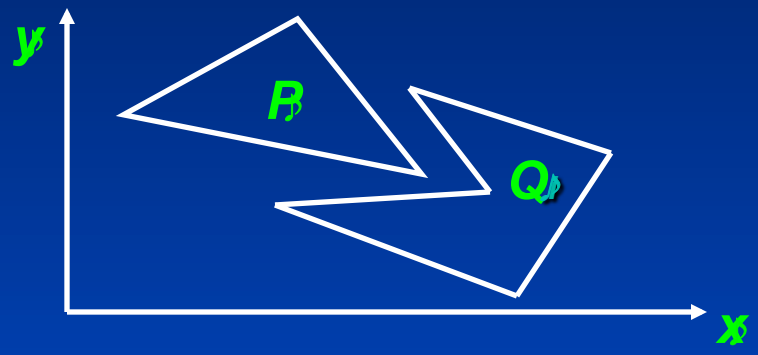
3. P is completely on the"back" side of Q_i, away from the viewpoint



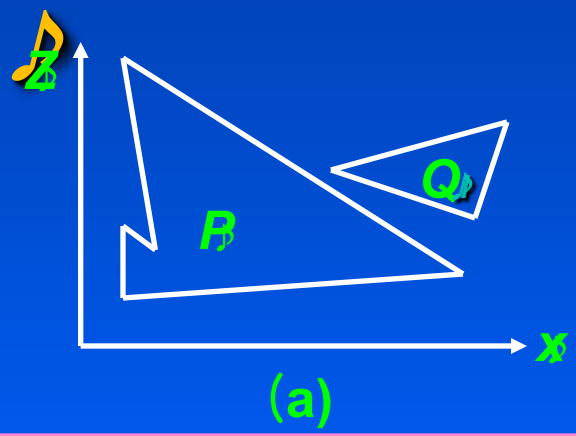
4. Q_i is completely on the "front" side of P, toward the view point

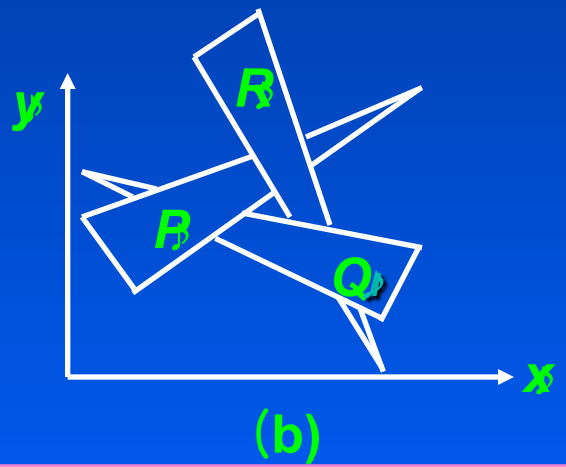


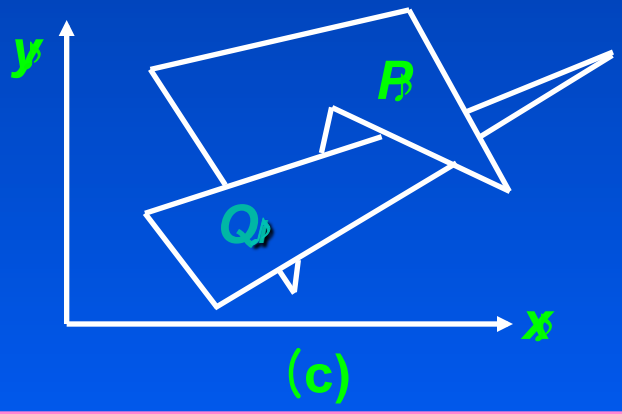
5. The projection of P and Q_i are disjoint



Sometimes all 5 tests fail







- IF all tests fail, move failed polygon Q_i to end of list, mark it to prevent infinite loops, repeat tests
  - Works in case (a) below, not in (b) or (c)
- If again all tests fail, don't move marked polygon, divide and conquer
  - Partition one polygon along plane of the other
- Once order has been determined, paint back-to-front (or front-to-back if can determine if a particular pixel has been written to)
- Can be used to do transparency correctly

Binary Space Partitioning (BSP) Trees♪

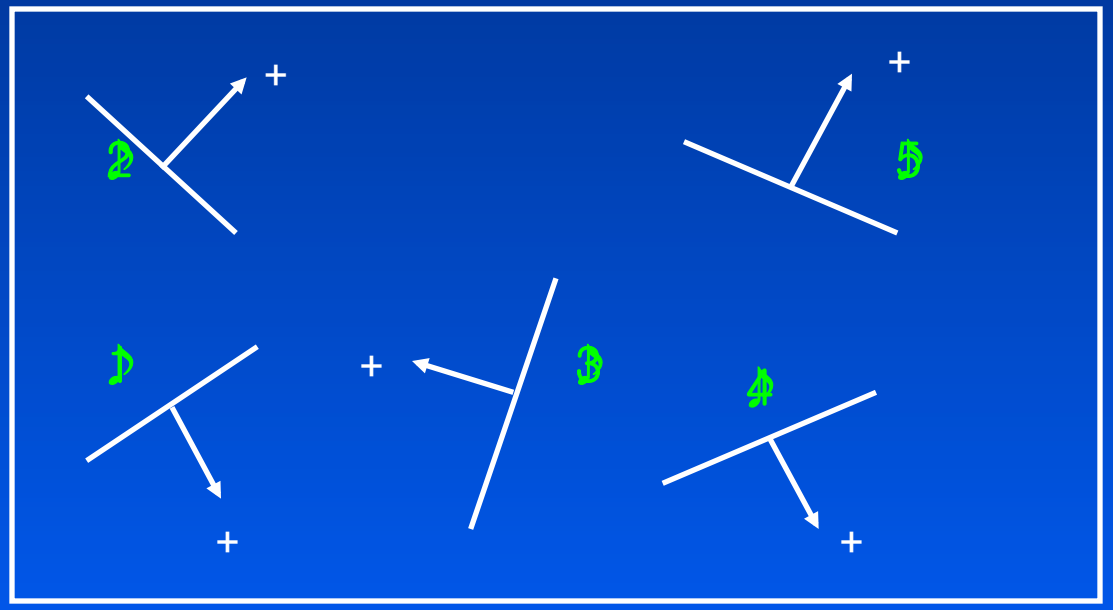
- Divide and conquer :
  - To order any polygon correctly, order all polygons on the “far”(relative to viewpoint) half-space relative to the polygon, then that polygon, then all polygons on the polygon’s “near” half-space
  - But how to order the polygons on a half-space correctly? Choose another polygon in that region and process it recursively!

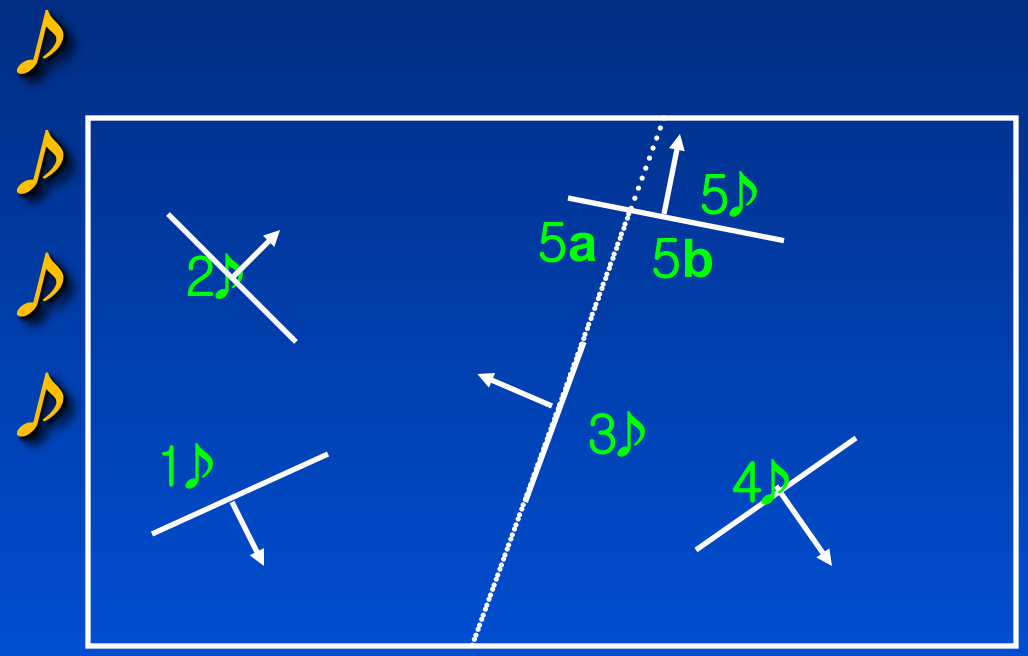
- Perform view-independent step once each time scene changes:
  - Recursively subdivide environment into a hierarchy of half-spaces by dividing polygons in a half-space by the plane of a selected polygon
  - Build a BSP tree representing this hierarchy
  - Each selected polygon is the root of the subtree

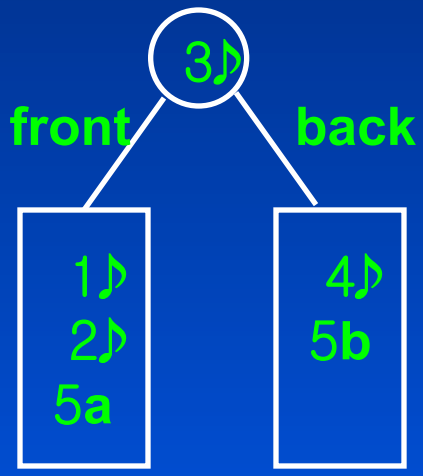
- Trades off view-independent preprocessing step(extra time and space) for low run-time overhead each time view changes
- Result is an ordered list back to front
  - Scan convert back-to-front (or front-to-back)
  - Good for transparency

Example

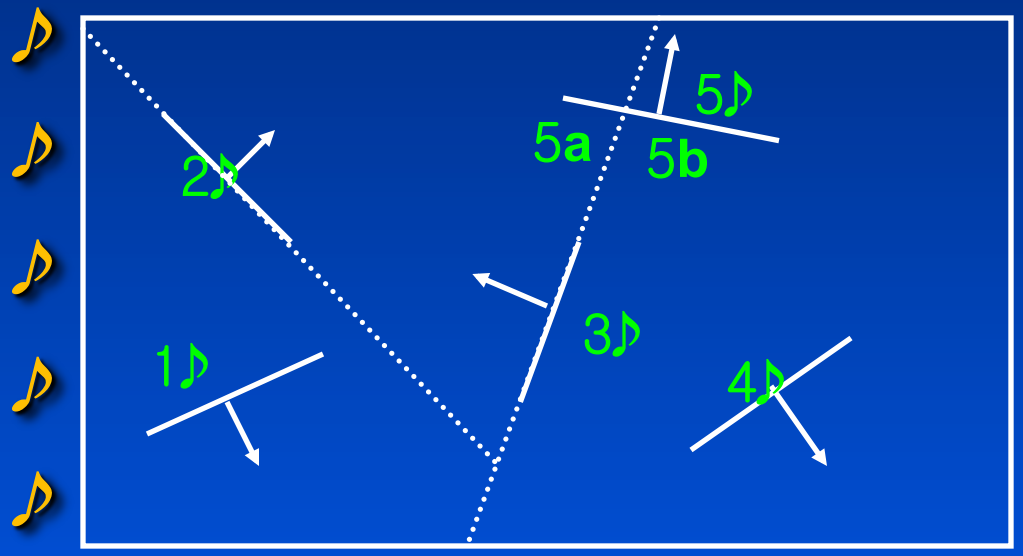
- Initial Scene looking from “above”
- Each polygon has a “+” side and a “-” side which is not related to position of camera

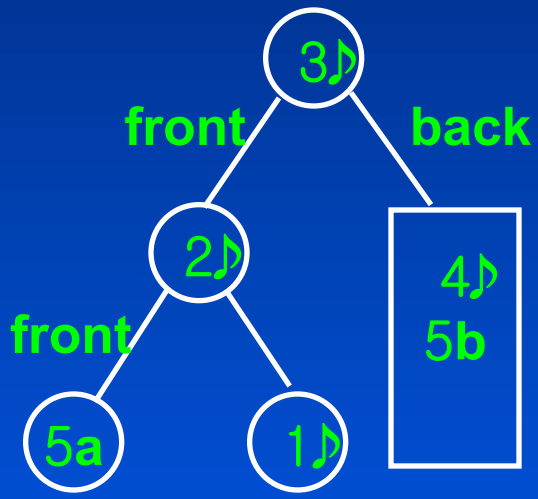




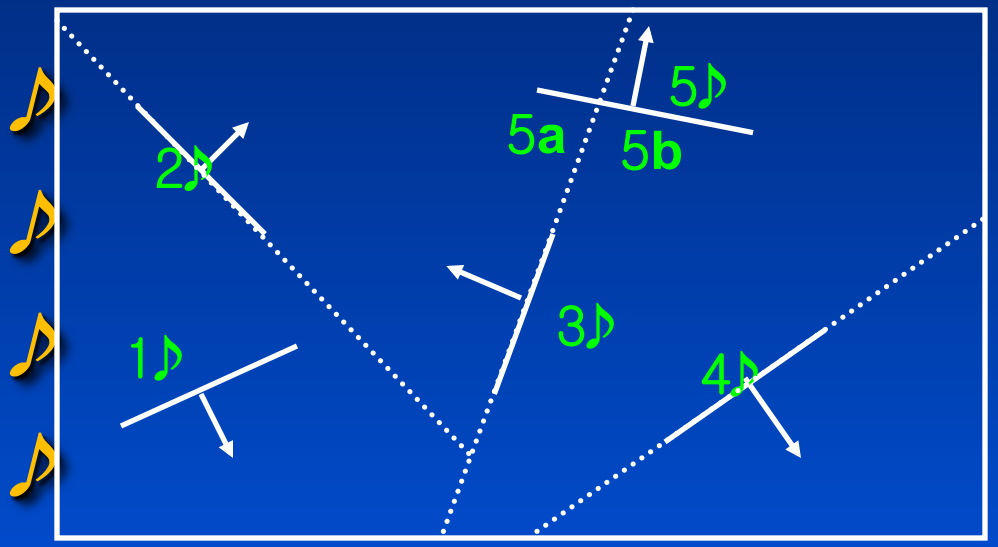


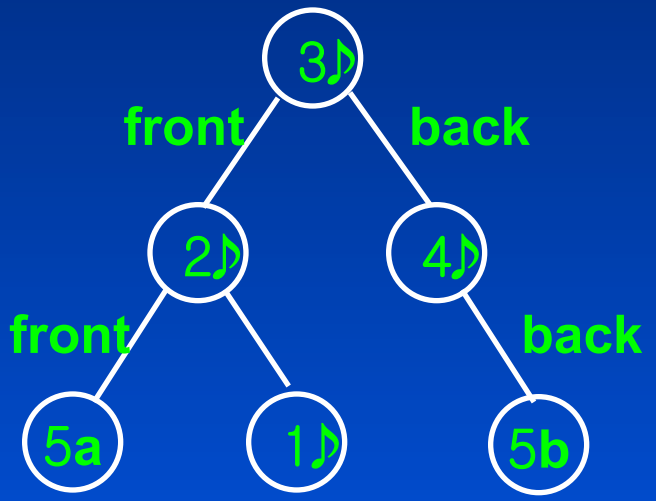
- Choose any polygon (e.g., polygon3) and subdivide space by its plane, splitting polygons when necessary





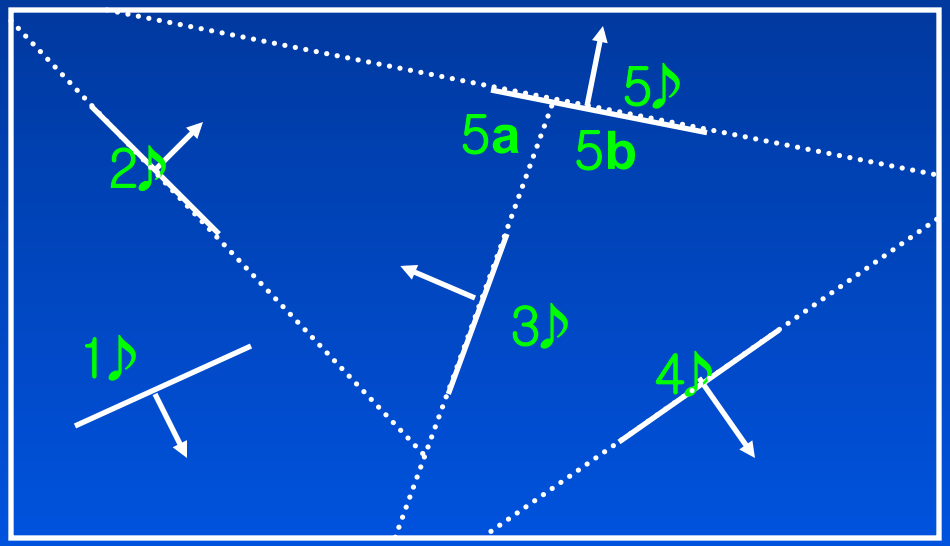
- Process front sub-tree recursively

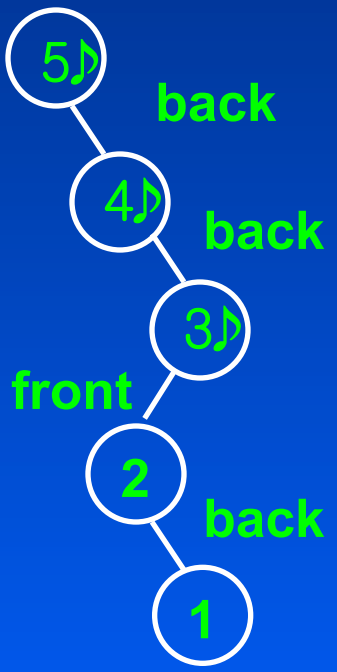




- Process back sub-tree recursively

- An alternative BSP tree with polygon 5 at the root♪

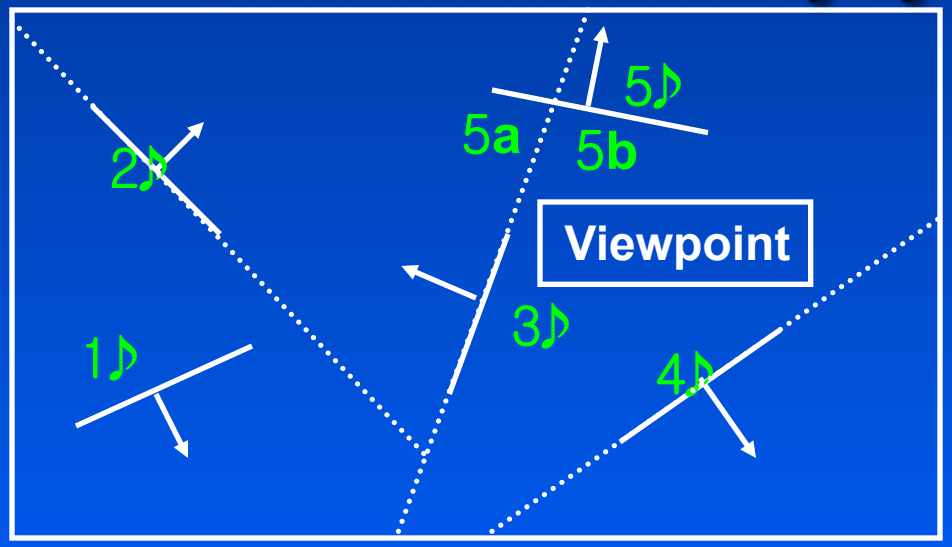


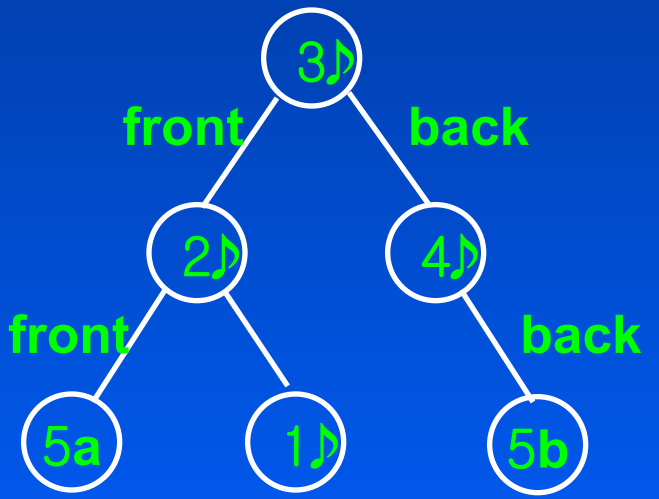


To Generate Ordered Polygons

- Perform BSP tree traversal (view-dependent step) once each time viewpoint changes
  - Nodes can be visited in back-to-front order using a modified in-order traversal of the tree
  - At any node, the far side of a node's polygon is the side that the viewpoint is not in

- Given the following viewpoint:
  - 1, 2, 5a, 3, 4, 5b
  - Order not necessarily by distance



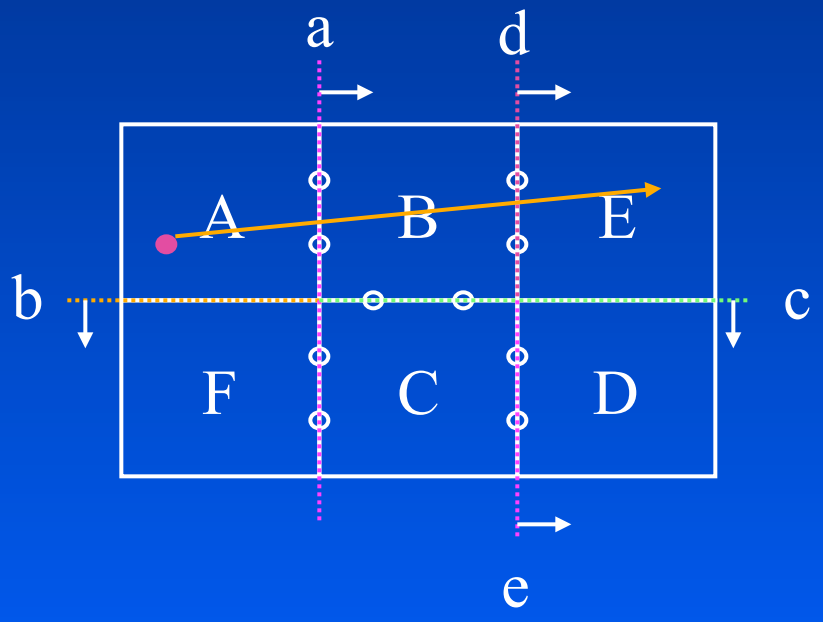


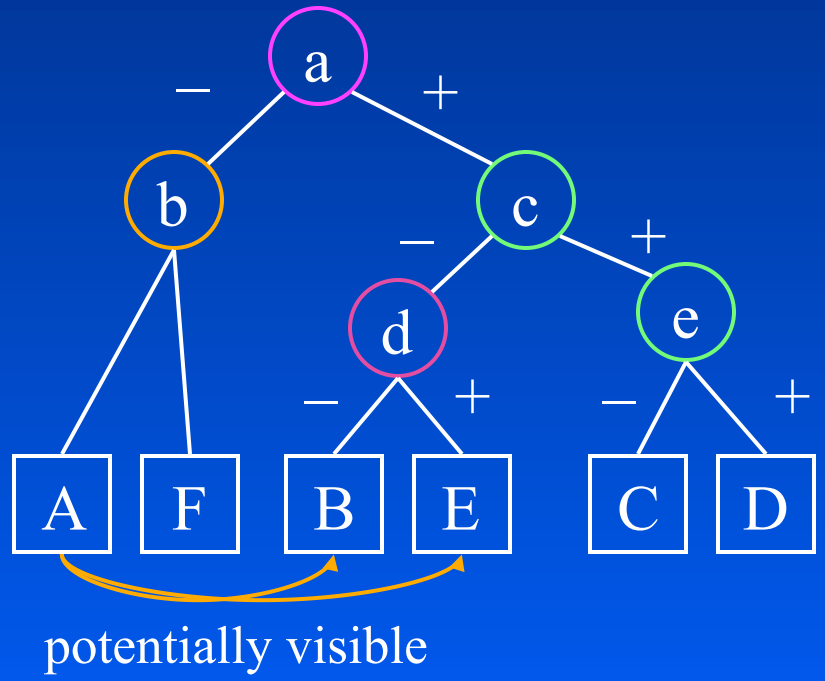
BSP Tree Traversal Algorithm

```
Procedure BSP_displayTree(tree :^BSP_tree)
Begin
If tree is not empty then
If viewer is in "+" side of root then
    begin
        {display "-" child, root, and "+" child}
        BSP_displayTree(tree^.plusChild);
        displayTree(tree^.root);
        BSP_displayTree(tree^.minusChild);
    end
    else
    begin
        {display "+" child, root, and "-" child}
        BSP_displayTree(tree^.plusChild);
        {ignore next line if back-face cullong
        desired)
        displayTree(tree^.root);
        BSP_displayTree(tree^.minusChild);
    end
end
```

Portal with BSP

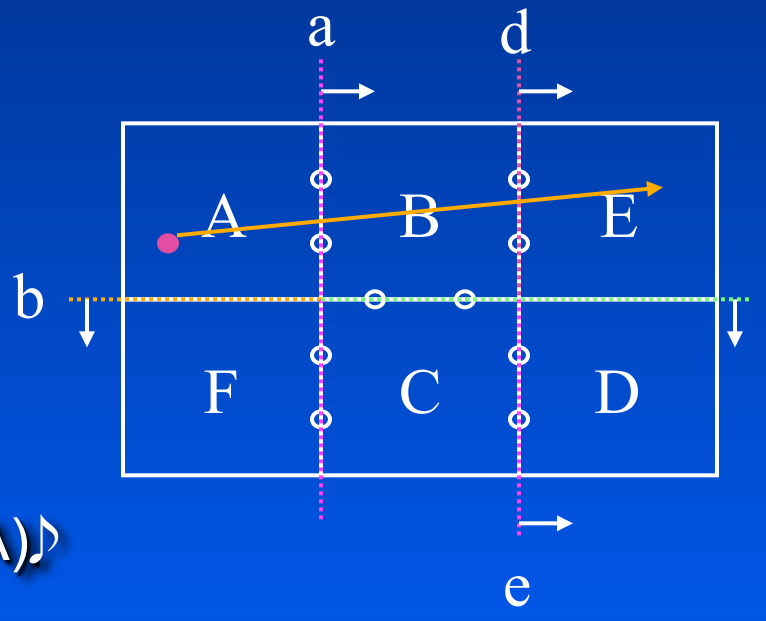
- Can see parts of other rooms through windows/doors
- PVS (potentially visible set): set of nodes visible from a node





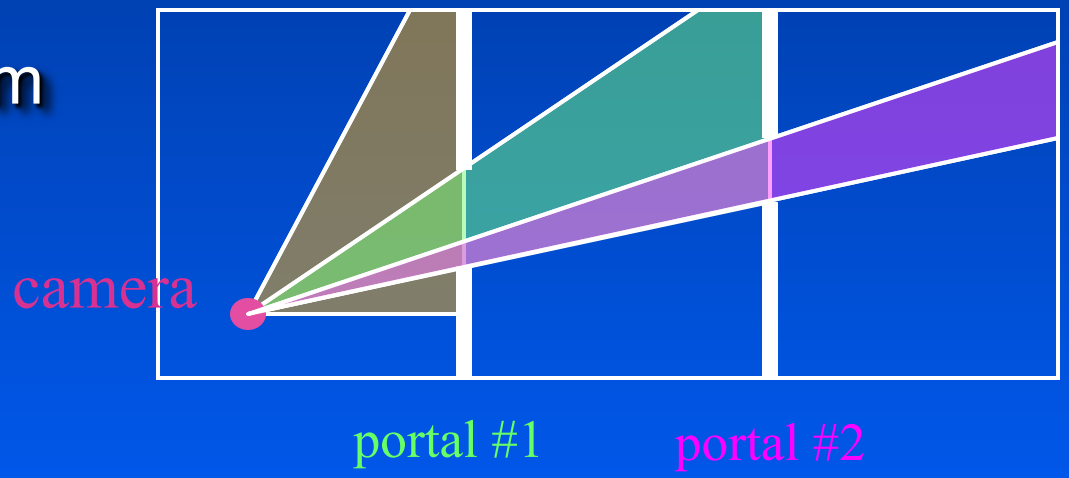
Portal, PVS

- Portal
  - Areas visible from a particular BSP node♪
- PVS : for a particular BSP node♪
  - Potentially visible nodes♪
  - E.g. : PVS(A) = { B, C, D, E }
- Example of use of PVS♪
  - Create PVS : pre-calculate
  - When rendering: draw A + PVS(A)♪



Portal without BSP tree

- Cull and render the room where camera is located
- If polygon tagged as portal, recurse into next room and cull (against reduced view frustrum) and render until no more portals encountered
- Mirrors portals to room itself



Visible-Surface Ray Tracing♪

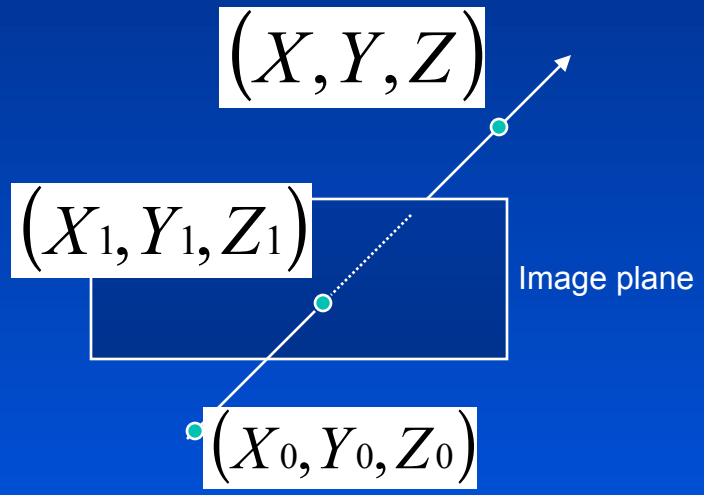
- Shoots a ray through each pixel
- Visible object is closest intersected by ray
- This is the naïve image-precision algorithm described before
- We will revisit ray-tracing with recursive ray-tracing

Select center of Projection and window on view plane;

```
for each scan line in image do
    for each pixel in scan line do
    begin
        {determine ray from center of projection through pixel}
        for each object in scene do
            if object is intersected and
                                is closest considered thus far then
            begin
                record intersection and object name;
                set pixel's color to that at closest object intersection
            end
end
```

Ray-object intersection

- Parametric ray representation:



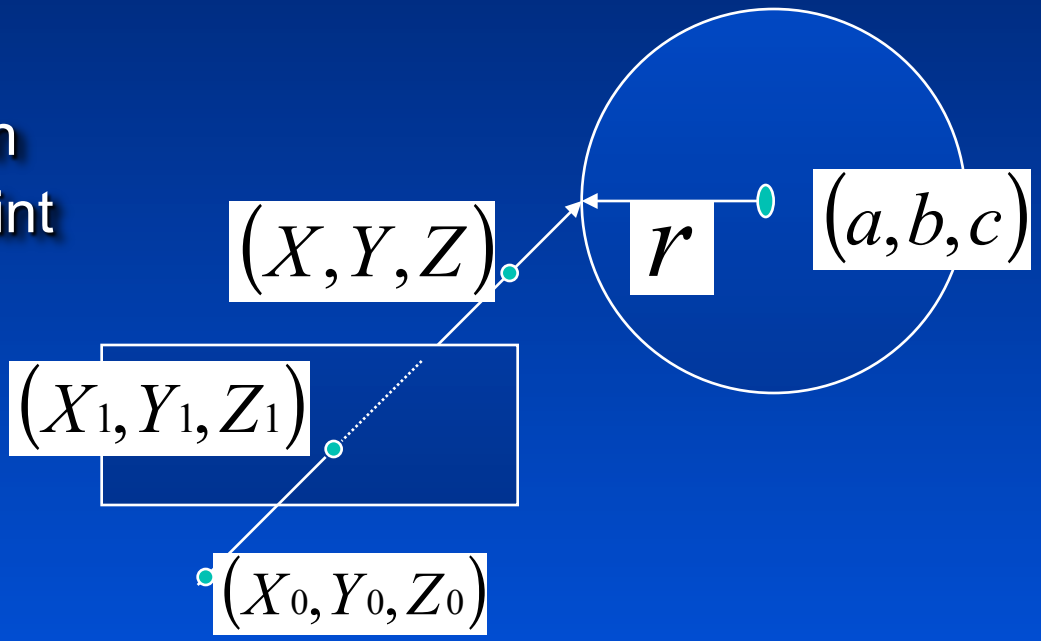
$$X = X_0 + t(X_1 - X_0)$$
$$Y = Y_0 + t(Y_1 - Y_0)$$
$$Z = Z_0 + t(Z_1 - Z_0)$$

or

$$X = X_0 + t\Delta X$$
$$Y = Y_0 + t\Delta Y$$
$$Z = Z_0 + t\Delta Z$$

- (x_0, y_0, z_0) is the center of projection
- (x_1, y_1, z_1) is the center of a pixel on the view plane
- t is a measure of the distance along the ray from the origin

- Sphere
- Plug in expression for (x, y, z) of a point on the ray to give a quadratic in t with constant coefficients
- Ready to be solved using quadratic formula



$$(X-a)^2+(Y-b)^2+(Z-c)^2=r^2$$

$$X^2-2aX+a^2+Y^2-2bY+b^2+Z^2-2cZ+c^2=r^2$$

- Most useful for modeling reflections, refraction, shadows, when rays are traced recursively from closest intersection
- Expensive: intersect each ray with every object

  E.g., for a small scene:

  1024 * 1024 * 1024 << 1,000,000,000

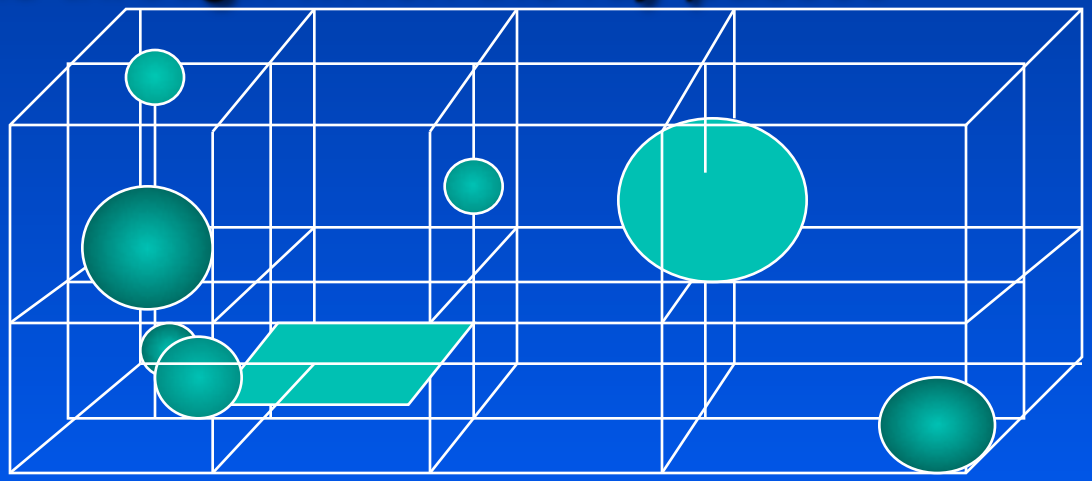
  scan lines — pixels on a line — objects — intersections
- Must use techniques such as spatial partitioning and hierarchies to cut down number of intersections

Optimizing intersection calculation♪

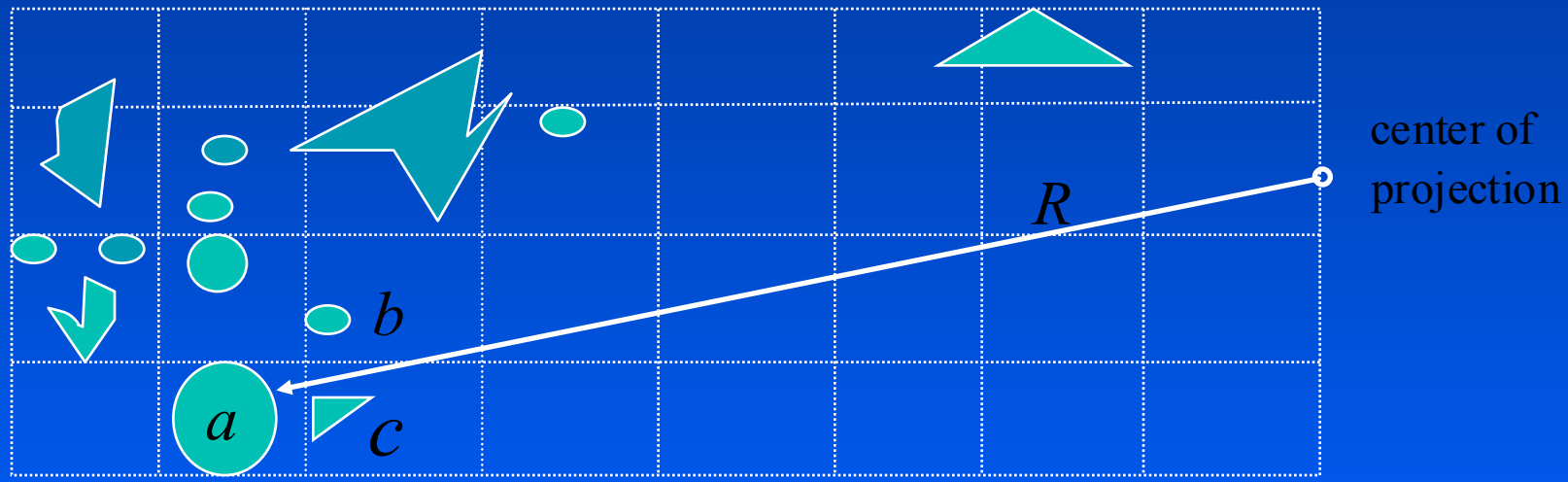
- Transform so that ray lie along z axis
- Bounding volumes
- Spatial partitioning
  - Regular voxels
  - Hierarchical partitioning (Octrees)

Regular Spatial Partitioning

- Environment is partitioned into regular grid of equal-sized volumes
- Ray intersections are calculated with objects in the partitions through which the ray passes



- Ray R need only be intersected with objects a, b, and c since the other partitions through which R passes are empty
- Use modified line rasterizer to determine the next cell



Next: Illumination Models