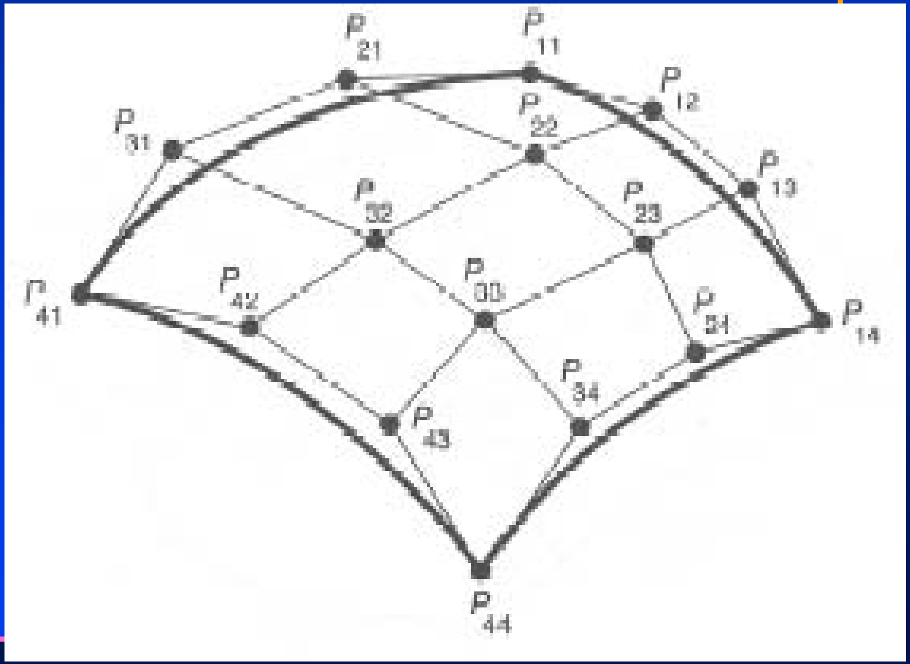
Parametric Curves and Surfaces II

Turbo Squid

Parametric Bicubic Surface

- $$Q(s,t) = \sum_{i=1}^{4}\sum_{j=1}^{4} \mathrm{P}_{ij} B_i(s) B_j(t)$$
- Cartesian product of two curves
- Each kind of curve has its 2D extension (e.g. Bézier curve)



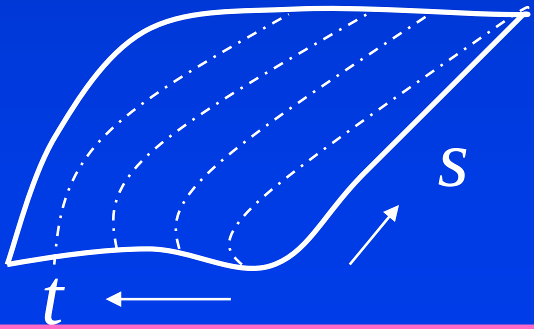
Matrix formulation

$$Q(s,t) = S \cdot M \cdot G(t) = S \cdot M \cdot \begin{bmatrix} G_1(t) \\ G_2(t) \\ G_3(t) \\ G_4(t) \end{bmatrix}$$

- For a fixed t, parametric cubic curve in s like before

- $G_i(t)$ are cubics

$$G_i(t) = T \cdot M \cdot \overline{G_i} = T \cdot M \cdot \begin{bmatrix} \overline{g_{i1}} \\ \overline{g_{i2}} \\ \overline{g_{i3}} \\ \overline{g_{i4}} \end{bmatrix}$$



Transpose :

$$G_i(t) = \overline{G_i}^T M^T T^T$$

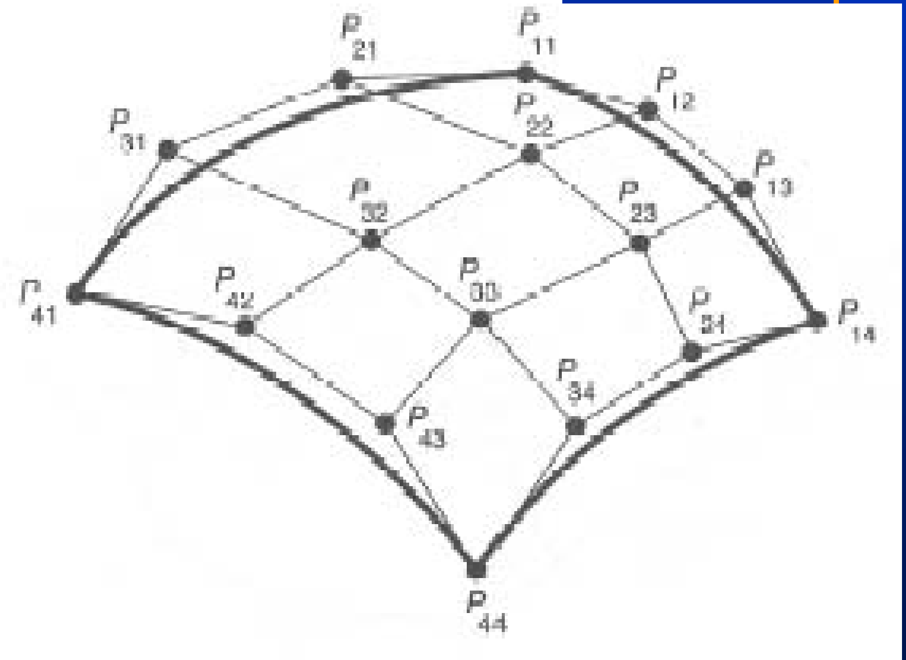
$$\therefore Q(s,t) = S \cdot M \cdot \underbrace{\begin{bmatrix} \overline{g_{11}} & \overline{g_{12}} & \overline{g_{13}} & \overline{g_{14}} \\ \overline{g_{21}} & \overline{g_{22}} & \overline{g_{23}} & \overline{g_{24}} \\ \overline{g_{31}} & \overline{g_{32}} & \overline{g_{33}} & \overline{g_{34}} \\ \overline{g_{41}} & \overline{g_{42}} & \overline{g_{43}} & \overline{g_{44}} \end{bmatrix}}_{\overline{G}} M^T T^T$$

Bezier surfaces

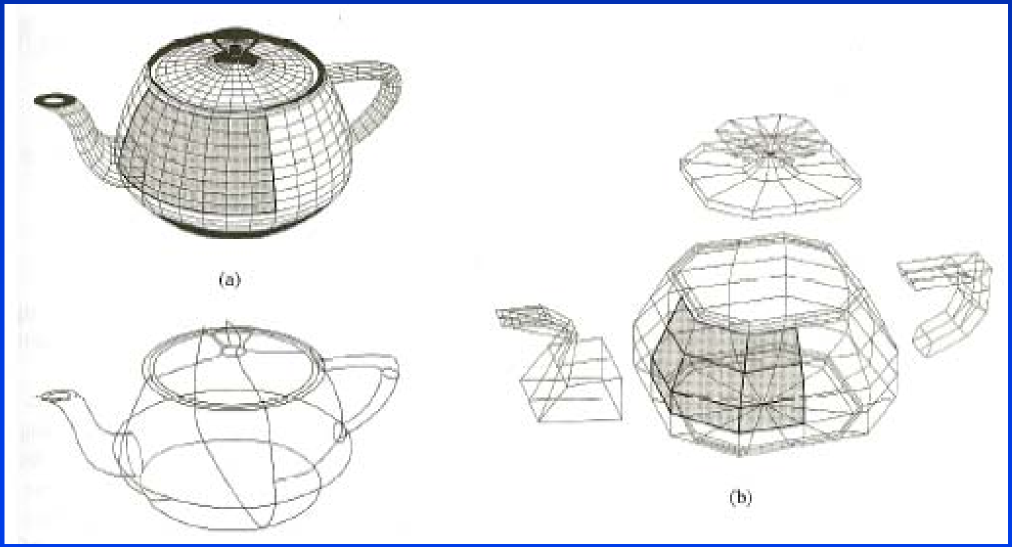
$\overline{G_{Bx}}$ – same derivation

16 - control points 4 corners of patch given

by $g_{11}, \; g_{41}, \; g_{44}, \; g_{14}$

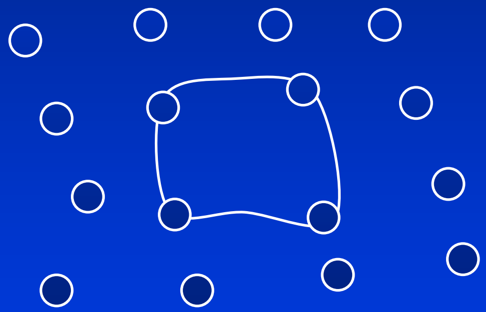


- Utah teapot as series of Bezier patches



B-Spline Surface

- “slide patch along grid of control points”



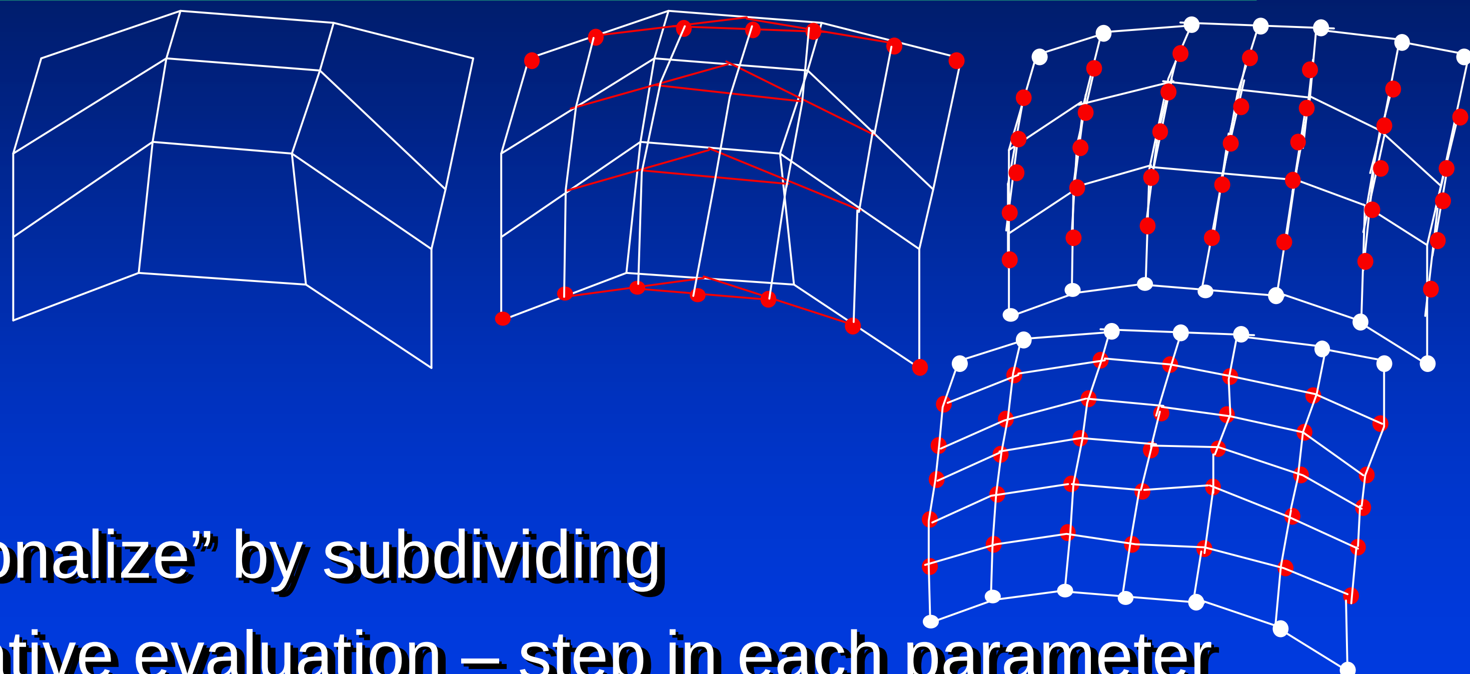
Normal of Surface

$$\hat{n}(s.t) = \frac{\partial}{\partial s} Q(s,t) \times \frac{\partial}{\partial t} Q(s,t)$$

s – tangent vector

t – tangent vector

Display



- “polygonalize” by subdividing
  - Iterative evaluation – step in each parameter
  - Recursive subdivision –termination criterion based on curvature

- Direct display: e.g. Ray-trace (find intersection of ray with surface)